How to derive $\Pr\{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2 > \|\boldsymbol{x}-\boldsymbol{s}_j\|^2 | \boldsymbol{s}_i \text{ transmitted}\}$ for arbitrary dimension?

Since $\boldsymbol{x} = \boldsymbol{s}_i + \boldsymbol{w}$ when $\boldsymbol{s}_i$ was transmitted, we have

$$\begin{aligned}
&\Pr\left\{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2 > \|\boldsymbol{x}-\boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted}\right\} \\
&= \Pr\left\{\|(\boldsymbol{s}_i+\boldsymbol{w})-\boldsymbol{s}_i\|^2 > \|(\boldsymbol{s}_i+\boldsymbol{w})-\boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted}\right\} \\
&= \Pr\left\{\|\boldsymbol{w}\|^2 > \|\boldsymbol{w}+(\boldsymbol{s}_i-\boldsymbol{s}_j)\|^2 \middle| \boldsymbol{s}_i \text{ transmitted}\right\} \\
&= \Pr\left\{\|\boldsymbol{w}\|^2 > \|\boldsymbol{w}\|^2 + \|\boldsymbol{s}_i-\boldsymbol{s}_j\|^2 + 2(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w} \middle| \boldsymbol{s}_i \text{ transmitted}\right\} \\
&= \Pr\left\{(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w} < -\frac{1}{2}\|\boldsymbol{s}_i-\boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted}\right\} \\
&= \Pr\left\{n < -\frac{1}{2}\|\boldsymbol{s}_i-\boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted}\right\}
\end{aligned}$$

where $n \triangleq (\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w}$.

Observe that $\boldsymbol{w}$ is zero-mean Gaussian distributed with covariance matrix $E[\boldsymbol{w}\boldsymbol{w}^T] = \frac{N_0}{2}\mathbb{I}$, where $\mathbb{I}$ is the identity matrix. Hence, $n \triangleq (\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w}$ is Gaussian distributed with

$$E[n] = E\left[(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w}\right] = (\boldsymbol{s}_i-\boldsymbol{s}_j)^T E[\boldsymbol{w}] = (\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{0} = 0$$

and

$$\begin{aligned}
E[n^2] &= E\left[(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w}\cdot((\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w})^T\right] \\
&= E\left[(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\boldsymbol{w}\boldsymbol{w}^T(\boldsymbol{s}_i-\boldsymbol{s}_j)\right] \\
&= (\boldsymbol{s}_i-\boldsymbol{s}_j)^T E\left[\boldsymbol{w}\boldsymbol{w}^T\right](\boldsymbol{s}_i-\boldsymbol{s}_j) \\
&= \frac{N_0}{2}(\boldsymbol{s}_i-\boldsymbol{s}_j)^T\mathbb{I}(\boldsymbol{s}_i-\boldsymbol{s}_j) \\
&= \frac{N_0}{2}\|\boldsymbol{s}_i-\boldsymbol{s}_j\|^2.
\end{aligned}$$

This implies that $w \triangleq n/\|\boldsymbol{s}_i-\boldsymbol{s}_j\|$ is Gaussian distributed with mean zero and variance $N_0/2$.

As a result,

$$
\begin{aligned}
&\Pr\left\{ n < -\frac{1}{2}\|\boldsymbol{s}_i - \boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted} \right\} \\
&= \Pr\left\{ \|\boldsymbol{s}_i - \boldsymbol{s}_j\| w < -\frac{1}{2}\|\boldsymbol{s}_i - \boldsymbol{s}_j\|^2 \middle| \boldsymbol{s}_i \text{ transmitted} \right\} \\
&= \Pr\left\{ w < -\frac{1}{2}\|\boldsymbol{s}_i - \boldsymbol{s}_j\| \middle| \boldsymbol{s}_i \text{ transmitted} \right\} \\
&= \Pr\left\{ w > \frac{1}{2}\|\boldsymbol{s}_i - \boldsymbol{s}_j\| \middle| \boldsymbol{s}_i \text{ transmitted} \right\},
\end{aligned}
$$

where the last equality is valid because the probability density function of a zero-mean Gaussian random variable is symmetric with respect to $w = 0$ (hence, $\Pr[w > a] = \Pr[w < -a]$ for any $a > 0$).

Note: There is a typo on Slide 5-54, $d_{ij}$ should be $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|$, not $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|^2$.