1. Consider a sinusoidal signal with random phase, defined by

$$X(t) = A\cos(2\pi f_c t + \Theta)$$

where A and f_c are constants and Θ is a random variable that is uniformly distributed over the interval $[-\pi, \pi]$, that is,

$$f_\Theta(\theta) = \begin{cases} \dfrac{1}{2\pi} & -\pi \le \theta \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

Please derive the autocorrelation function of X(t) using the time difference parameter τ.

2. Consider a pair of stationary processes X(t) and Y(t). Show that the cross-correlations $R_{XY}(\tau)$ and $R_{YX}(\tau)$ of these processes have the following properties：

(a) $R_{XY}(\tau) = R_{YX}(-\tau)$

(b) $|R_{XY}(\tau)| \le \frac{1}{2}[R_X(0) + R_Y(0)]$

where $R_X(\tau)$ and $R_Y(\tau)$ are the autocorrelation functions of X(t) and Y(t), respectively.

3. Briefly describe the strictly stationary and wide sense stationary.

4. A random process X(t) is defined by

$$X(t) = A\cos(2\pi f_c t)$$

where A is a Gaussian-distributed random variable of zero mean and variance σ_A^2. This random process is applied to an ideal integrator, producing the output

$$Y(t) = \int_0^t X(\tau)d\tau$$

(a) Determine whether or not Y(t) is stationary

(b) Determine whether or not Y(t) is ergodic.