1.

Y(t) = X(t) $*$ h(t)

$= \int_{-\infty}^{\infty} h(\tau)X(t-\tau)d\tau$

$$\mu_Y(t) = E[Y(t)] = E\left[\int_{-\infty}^{\infty} h(\tau)X(t-\tau)d\tau\right]$$

$$= \int_{-\infty}^{\infty} h(\tau)E[X(t-\tau)]d\tau = \mu_X \int_{-\infty}^{\infty} h(\tau)d\tau$$

$$R_Y(t,u) = E[Y(t)Y(u)]$$

$$= E\left[\int_{-\infty}^{\infty} h(\tau_1)X(t-\tau_1)d\tau_1 \cdot \int_{-\infty}^{\infty} h(\tau_2)X(u-\tau_2)d\tau_2\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h(\tau_2)E\left[X(t-\tau_1)X(u-\tau_2)\right]d\tau_2 d\tau_1$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h(\tau_2)R_X(t-\tau_1, u-\tau_2)d\tau_2 d\tau_1$$

$\Rightarrow$ If $X(t)$ WSS,

then $R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h(\tau_2)R_X(\tau-\tau_1+\tau_2)d\tau_2 d\tau_1$

➔ **Y(t) is also WSS.**

2.

$R_Y(\tau) = E[Y(t+\tau)Y(t)]$

$= E[X(t+\tau)\cos(2\pi f_c t + 2\pi f_c \tau + \Theta)X(t)\cos(2\pi f_c t + \Theta)]$

$= E[X(t+\tau)X(t)]E[\cos(2\pi f_c t + 2\pi f_c \tau + \Theta)\cos(2\pi f_c t + \Theta)]$

$= \frac{1}{2}R_X(\tau)E[\cos(2\pi f_c \tau) + \cos(4\pi f_c t + 2\pi f_c \tau + 2\Theta)]$

$= \frac{1}{2}R_X(\tau)\cos(2\pi f_c \tau)$

$S_Y(f) = \frac{1}{4}[S_X(f - f_c) + S_X(f + f_c)]$

3.

(a) The power spectral density consists of two components:

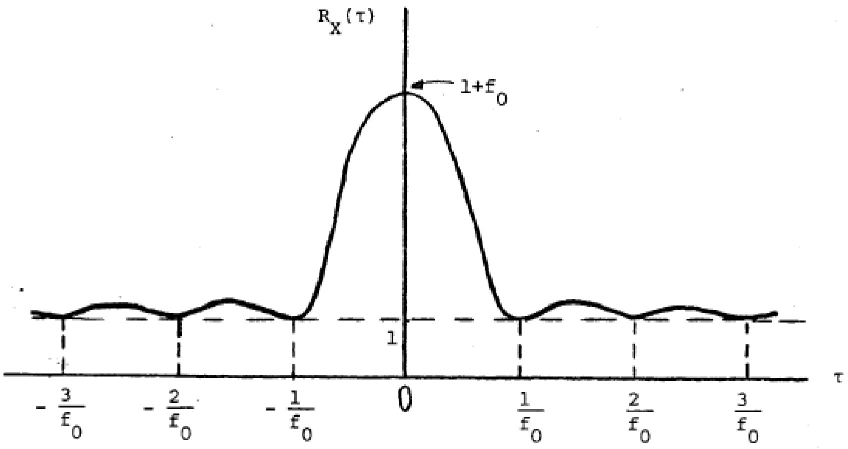
(1) A delta function $\delta(t)$ at the origin, whose inverse Fourier transform is one.

(2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \, \text{sinc}^2(f_0\tau)$.

Therefore, the autocorrelation function of X(t) is

$$R_X(\tau) = 1 + f_0 \, \text{sinc}^2(f_0\tau)$$

which is sketched below:



(b) Since $R_X(\tau)$ contains a constant component of amplitude 1, it follows that the dc power contained in X(t) is 1.

(c) The mean-square value of X(t) is given by

$$E[X^2(t)] = R_X(0)$$
$$= 1 + f_0$$

The ac power contained in X(f) is therefore equal to f_0.