1. Prove spectrum properties(N is WSS)：

(1) $R_{N_I}(\tau) = R_{N_Q}(\tau)$ and $R_{N_I,N_Q}(\tau) = -R_{N_Q,N_I}(\tau)$

(2) $R_N(\tau) = R_{N_I}(\tau)\cos(2\pi f_c\tau) - R_{N_Q,N_I}(\tau)\sin(2\pi f_c\tau)$

(3) $R_{\tilde{N}}(\tau) = R_{N_I}(\tau) + jR_{N_Q,N_I}(\tau)$

(4) $R_N(\tau) = \mathrm{Re}\{R_{\tilde{N}}(\tau)\exp(j2\pi f_c\tau)\}$

(5) $S_{\tilde{N}}(f)$ is real valued.

(6) $S_{N_I}(f) = S_{N_Q}(f)$ and $S_{N_I,N_Q}(f) = -S_{N_Q,N_I}(f)$

(7) $S_N(f) = \frac{1}{2}(S_{\tilde{N}}(f - f_c) + S_{\tilde{N}}(-f - f_c))$

Sol: 請參考講義 1-219~1-224.

2. Consider a square-law detector, using a nonlinear device whose transfer characteristic is defined by

$$v_2 = a_1 v_1(t) + a_2 v_1^2(t)$$

where a_1 and a_2 are constants, $v_1(t)$ is the input, and $v_2(t)$ is the output. The input consists of the AM wave

$$v_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

(1) Evaluate the output $v_2(t)$.

(2) Find the conditions for which the message signal $m(t)$ may be recovered from $v_2(t)$.

Sol:

Let

$$v_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

(a) Then the output of the square-law device is

$$\begin{aligned} v_2(t) &= a_1 v_1(t) + a_2 v_1^2(t) \\ &= a_1 A_c[1 + k_a m(t)]\cos(2\pi f_c t) \\ &+ \frac{1}{2} a_2 A_c^2 [1 + 2k_a m(t) + k_a^2 m^2(t)]\,[1 + \cos(4\pi f_c t)] \end{aligned}$$

(b) The desired signal, namely $a_2 A_c^2 k_a m(t)$, is due to the $a_2 v_1^2(t)$ - hence, the name "square-law detection". This component can be extracted by means of a low-pass filter. This is not the only contribution within the baseband spectrum, because the term 1/2 $a_2 A_c^2 k_a^2 m^2(t)$ will give rise to a plurality of similar frequency components. The ratio of wanted signal to distortion is $2/k_a m(t)$. To make this ratio large, the percentage modulation, that is, $|k_a m(t)|$ should be kept small compared with unity.