

1. Prove spectrum properties(N is WSS) :

(1) $R_{N_i}(\tau) = R_{N_o}(\tau)$ and $R_{N_i, N_o}(\tau) = -R_{N_o, N_i}(\tau)$

(2) $R_N(\tau) = R_{N_i}(\tau)\cos(2\pi f_c\tau) - R_{N_o, N_i}(\tau)\sin(2\pi f_c\tau)$

(3) $R_{\tilde{N}}(\tau) = R_{N_i}(\tau) + jR_{N_o, N_i}(\tau)$

(4) $R_N(\tau) = \text{Re}\{R_{\tilde{N}}(\tau)\exp(j2\pi f_c\tau)\}$

(5) $S_{\tilde{N}}(f)$ is real valued.

(6) $S_{N_i}(f) = S_{N_o}(f)$ and $S_{N_i, N_o}(f) = -S_{N_o, N_i}(f)$

(7) $S_N(f) = \frac{1}{2}(S_{\tilde{N}}(f - f_c) + S_{\tilde{N}}(-f - f_c))$

2. Consider a square-law detector, using a nonlinear device whose transfer characteristic is defined by

$$v_2 = a_1 v_1(t) + a_2 v_1^2(t)$$

where a_1 and a_2 are constants, $v_1(t)$ is the input, and $v_2(t)$ is the output. The input consists of the AM wave

$$v_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

(1) Evaluate the output $v_2(t)$.

(2) Find the conditions for which the message signal $m(t)$ may be recovered from $v_2(t)$.