1. Please describe the function of simply AM signal in most general form. And point out two required conditions on amplitude sensitivity, if violate these conditions what problems will happen?

   Sol:

   Carrier $c(t) = A_c \cos(2\pi f_c)$

   Baseband $m(t)$

   Modulated Signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$,

   where $k_a$ is amplitude sensitivity or modulation index

   - Two required conditions on amplitude sensitivity
     - $1 + k_a m(t) \geq 0$, which is ensured by $|k_a m(t)| \leqq 1$.
       - The case of $|k_a m(t)| > 1$ is called *overmodulation*.
       - The value of $|k_a m(t)|$ is sometimes represented by "percentage" (because it is limited by 1), and is named $(|k_a m(t)| \times 100)\%$ modulation.
     - $f_c >> W$, where $W$ is the message bandwidth.
       - Violation of this condition will cause **nonvisualized envelope**.

2. A DSB-SC modulated signal is demodulated by applying it to a coherent detector.
   (a) Evaluate the effect of a frequency error $\Delta f$ in the local carrier frequency of the detector, measured with respect to the carrier frequency of the incoming DSB-SC signal.
   (b) For the case of a sinusoidal modulating wave, show that because of the frequency error, the demodulated signal exhibits beats at the error frequency.

Sol:

(a)

$$S_1(t) = A_c m(t)\cos(2\pi f_c t)\cos[2\pi(f_c + \Delta f)t]$$

$$= \frac{A_c}{2} m(t)\{\cos(2\pi \Delta f t) + \cos[2\pi(2 f_c + \Delta f)t]\}$$

After LPF:

$$S_2(t)\frac{A_c}{2} m(t)\cos(2\pi \Delta f t)$$

Thus the output is the message signal modulated by sinusoidal of frequency $\Delta f$

(b)

If $m(t) = \cos(2\pi f_m t)$,

Then $S_2(t) = \frac{A_c}{2}\cos(2\pi f_m t)\cos(2\pi \Delta f t)$

Therefore, the demodulated signal exhibits beats at the error frequency $\Delta f$ .

3. Using the definition of the Hilbert transform, show that a SSB modulated signal

resulting from the message signal m(t) and carrier $\cos(2\pi f_c t)$ of unit amplitude is given by

$$s(t) = \frac{1}{2} m(t) \cos(2\pi f_c t) \pm \frac{1}{2} \hat{m}(t) \sin(2\pi f_c t)$$

Where the minus sign corresponds to the transmission of the upper sideband and the plus sign corresponds to the transmission of the lower sideband.

Sol:

Consider the modulated signal s(t),

$$s(t) = \frac{1}{2} m(t) \cos(2\pi f_c t) - \frac{1}{2} \hat{m}(t) \sin(2\pi f_c t) \text{ ------------------------------------------------(1)}$$

Applying the Fourier eq(1), we obtain

$$S(f) = \frac{1}{4}[M(f - f_c) + M(f + f_c)] - \frac{1}{4j}[\hat{M}(f - f_c) - \hat{M}(f + f_c)] \text{ ----------------------(2)}$$

From the definition of Hilbert transform, we have

$$\hat{M}(f) = -j\,\mathrm{sgn}(f) M(f)$$

Equivalently, we may write

$$-\frac{1}{j}\hat{M}(f - f_c) = \mathrm{sgn}(f - f_c) M(f - f_c)$$

$$-\frac{1}{j}\hat{M}(f + f_c) = \mathrm{sgn}(f + f_c) M(f + f_c)$$

(i) for $f \geq 0$ & $f > f_c$

$\mathrm{sgn}(f - f_c) = \mathrm{sgn}(f + f_c) = +1$

Thus, eq(2) reduces to

$$S(f) = \frac{1}{4}[M(f - f_c) + M(f + f_c)] + \frac{1}{4}[M(f - f_c) - M(f + f_c)] = \frac{1}{2} M(f - f_c)$$

In words, we may thus state that, except for a scaling factor, the spectrum of the modulated signal s(t) defined in eq(1) is the sane as that of the DSB-SC modulated signal for f> fc.

(ii) for $f \geq 0$ & $f < f_c$ we have

$\mathrm{sgn}(f - f_c) = -1$

$\mathrm{sgn}(f + f_c) = +1$

Correspondingly, eq(2) reduces to

$$S(f)=\frac{1}{4}[M(f-f_c)+M(f+f_c)]+\frac{1}{4}[-M(f-f_c)-M(f+f_c)]=0$$

Combining the result for (i) and (ii), the modulated signal s(t) of eq(1) represents a single sideband modulated signal containing only the upper sideband. This result was derived for f>0. This result also holds for f<0.

By a procedure similar to that described above, the modulated signal

$$s(t)=\frac{1}{2}m(t)\cos(2\pi f_c t)+\frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)$$

Represents a single sideband modulated signal containing only the lower sideband.