1. An unmodulated carrier of amplitude A_c and frequency f_c and band-limited white noise are summed and then passed through an ideal envelope detector. Assume the noise spectral density to be of height $N_0 / 2$ and bandwidth 2W, centered about the carrier frequency f_c. Determine the output signal-to-noise ratio for the case when the carrier-to-noise ratio is high.

Sol:

The received signal is

$$
\begin{aligned}
x(t) &= A_c \cos(2\pi f_c t) + n(t) \\
&= A_c \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \\
&= [A_c + n_c(t)] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)
\end{aligned}
$$

The envelope detector output is therefore

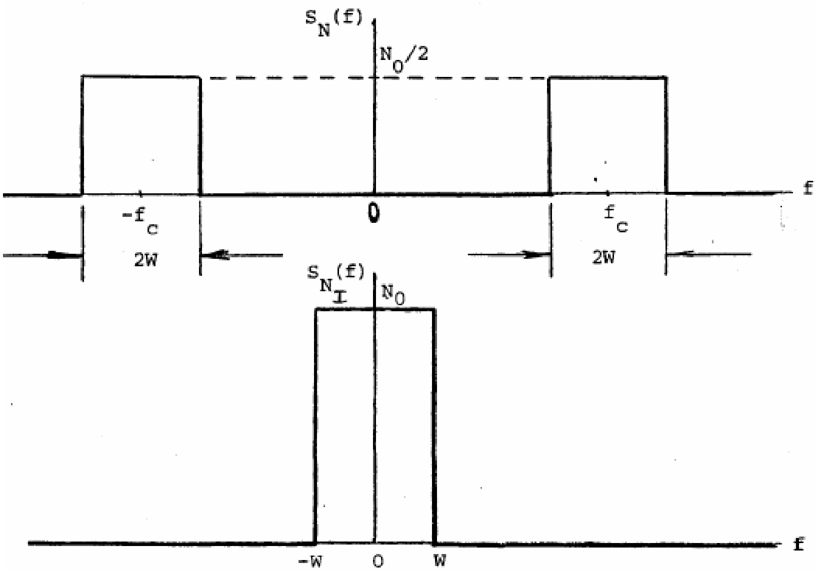
$$a(t) = \{[A_c + n_c(t)]^2 + n_s^2(t)\}^{1/2}$$

For the case when the carrier-to-noise ratio is high, we may approximate this result as

$$a(t) \simeq A_c + n_c(t)$$

The term A_c represents the useful signal component. The output signal power is thus A_c^2.

The power spectral densities of n(t) and $n_I(t)$ are as shown below:



The output noise power is $2N_0W$. The output signal-to-noise ratio is therefore

$$(SNR)_0 = \frac{A_c^2}{2N_0W}$$

2. Suppose that the received signal in FM system contains some residual amplitude modulation of positive amplitude a(t),as shown by

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)]$$

Where f_c is the carrier frequency. The phase $\phi(t)$ is related to the modulating signal $m(t)$ by

$$\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau$$

Where k_f is a constant. Assume that the signal s(t) is restricted to a frequency band of width B_T, centered at f_c, where B_T is the transmission bandwidth of the FM signal in the absence of amplitude modulation, and that the amplitude modulation is slowly varying compared with $\phi(t)$. Show that the output of an ideal frequency discriminator produced by s(t) is proportional to $a(t)m(t)$. Hint: Use the complex notation described in Appendix 2 to represent the modulated wave $s(t)$.

Sol:

The complex envelope of the modulated wave s(t) is

$$\tilde{s}(t) = a(t) \exp[j\phi(t)]$$

Since a(t) is slowly varying compared to $\exp[j\phi(t)]$, the complex envelope $\tilde{s}(t)$ is restricted effectively to the frequency band $-B_T/2 \le f \le B_T/2$. An ideal frequency discriminator consists of a differentiator followed by an envelope detector. The output of the differentiator, in response to $\tilde{s}(t)$, is

$$\tilde{v}_o(t) = \frac{d}{dt}\tilde{s}(t)$$

$$= \frac{d}{dt}\{a(t) \exp[j\phi(t)]\}$$

$$= \frac{da(t)}{dt}\exp[j\phi(t)] + j\frac{d\phi(t)}{dt} a(t) \exp[j\phi(t)]$$

$$= a(t) \exp[j\phi(t)] \left[\frac{1}{a(t)}\frac{da(t)}{dt} + j\frac{d\phi(t)}{dt}\right]$$

Since a(t) is slowly varying compared to $\phi(t)$, we have

$$\left|\frac{d\phi(t)}{dt}\right| \gg \left|\frac{1}{a(t)}\frac{da(t)}{dt}\right|$$

Accordingly, we may approximate $\tilde{v}_o(t)$ as

$$\tilde{v}_o(t) \simeq j\, a(t) \frac{d\phi(t)}{dt} \exp[j\phi(t)]$$

However, by definition

$$\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$$

Therefore,

$$\tilde{v}_o(t) = j2\pi k_f\, a(t)\, m(t) \exp[j\phi(t)]$$

Hence, the envelope detector output is proportional to a(t) m(t) as shown by

$$|\tilde{v}_o(t)| = 2\pi k_f\, a(t)\, m(t)$$

3. What are the definitions of SNR_I、SNR_O、SNR_C and figure of merit?

Input signal-to-noise ratio (SNR_I)

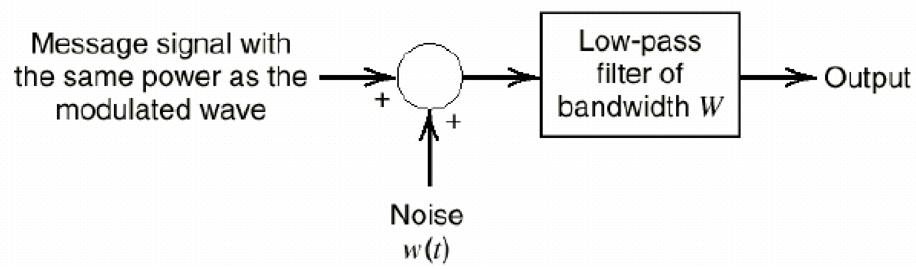
- The ratio of the average power of the modulated signal $s(t)$ to the average power of the filtered noise $n(t)$.

Output signal-to-noise ratio (SNR_O)

- The ratio of the average power of the demodulated message signal to the average power of the noise, measured at the receiver output.

Channel signal-to-noise ratio (SNR_C)

- The ratio of the average power of the modulated signal $s(t)$ to the average power of the channel noise in the message bandwidth, measured at the receiver input (as illustrated below).



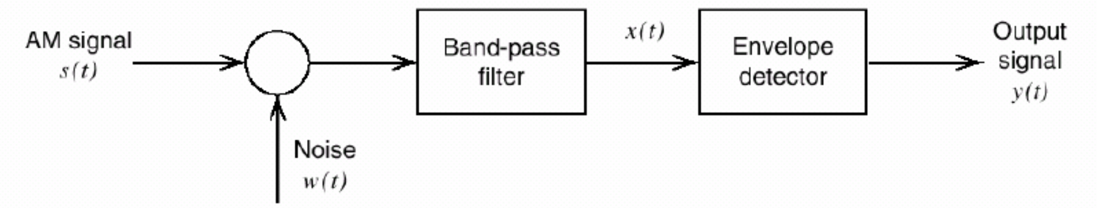
$$\text{figure of merit} = \frac{SNR_O}{SNR_C}$$

4. In a full AM signal, both sidebands and the carrier wave are transmitted, as shown by

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

where $A_c \cos(2\pi f_c t)$ is the carrier wave, $m(t)$ is the message signal, and k_a is a constant that determines the percentage modulation. Please compute the SNRc for this AM signal $s(t)$. (Note: assume $m(t)$ is zero mean and the message bandwidth is W.)

sol:



$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} E[s^2(t)]dt = A_c^2 E\left[(1 + k_a m(t))^2\right]\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2(2\pi f_c t)dt$$

$$= \frac{A_c^2}{2}(1 + k_a^2 P) \quad \text{(Assume } m(t) \text{ zero mean.)}$$

$$\text{Also, } \int_{-W}^{W} S_w(f)df = \int_{-W}^{W}\frac{N_0}{2}df = WN_0$$

$$SNR_{C,\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2WN_o}$$