1.

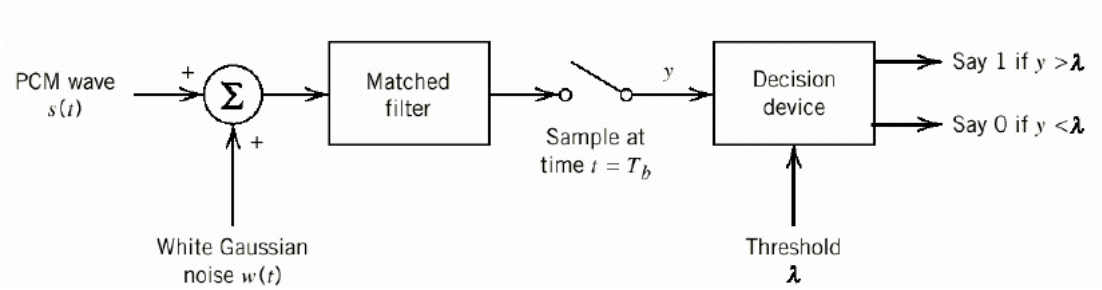
(a)Describe what is matched filter

(b)Prove that matched filter maximizes the output signal to noise ratio under AWGN

Answer : slide 4-4~4-9

2.

For polar non-return-to-zero system with optimal matched filter receiver over AWGN channel. The pulse shaping function g(t) is real.(Hint : slide 4-13~4-19)



(a)

Derive y(t) where

$$h(\tau)=g^{*}(T-\tau),t=T,I\in\{+1,-1\},E_g=\int_{-\infty}^{\infty}|g(\tau)|^2\,d\tau\ ,n=\int_{-\infty}^{\infty}g^{*}(\tau)w(\tau)d\tau$$

(b)

Derive $E[n^2]$

(c)

Suppose transmit in equal probability, determine the threshold λ that achieves optimal decision. (Do Not write down the threshold λ only.)

(d)

Prove that $BER_{opt}=\frac{1}{2}erfc(\sqrt{\frac{E_g}{N_0}})$

3.

Let $X(t)=\sum_{-\infty}^{\infty}a_k\delta(t-kT_b)$, X(t) is not WSS, please find the *Time Average*

Autocorrelation Function $\overline{R}_X(\tau)$. (Note: $\{a_k\}$ is i.i.d.) (Hint: slide 4-58)

Sol:

$$\overline{R}_a(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} E\left[\left(\sum_{k=-\infty}^{\infty} a_k\delta(t-kT_b)\right)\left(\sum_{j=-\infty}^{\infty} a_j\delta(t+\tau-jT_b)\right)\right]dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sum_{k=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} E[a_j a_k]\delta(t+\tau-jT_b)\delta(t-kT_b)dt$$

$$\overline{R}_a(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\sum_{k=-\infty}^{\infty}\delta(t-kT_b)\delta(t+\tau-kT_b)\right)dt$$

$$= \delta(\tau)\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(\sum_{k=-\infty}^{\infty}\delta(t-kT_b)\right)dt$$

$$= \frac{1}{T_b}\delta(\tau) \quad \Rightarrow \overline{S}_Y(f) = \frac{1}{T_b}|G(f)|^2$$