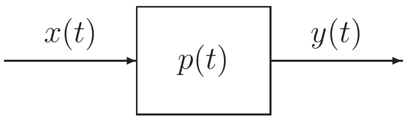
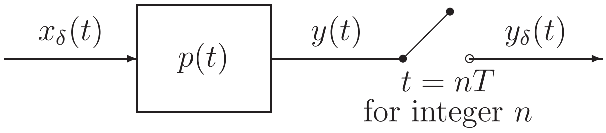
Sample Problems for the Tenth Quiz

1. (Alternative proof for Nyquist Criterion)

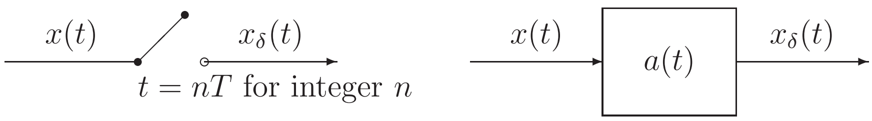
   (a) Find the LTI filter design with impulse response $p(t)$ that guarantees $y(t) = x(t)$.



   (b) Find the LTI filter design with impulse response $p(t)$ that guarantees $y_\delta(t) = x_\delta(t)$, where $x_\delta(t) = \sum_{k=-\infty}^{\infty} x(nT)\delta(t - nT)$.



2. Is a sampler linear? Can we find an equivalent linear time-invariant filter with impulse response $a(t)$ such that $x_\delta(t) = x(t) \star a(t)$?



   Hint: $x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$.

3. (a) Show that the time-averaged PSD of

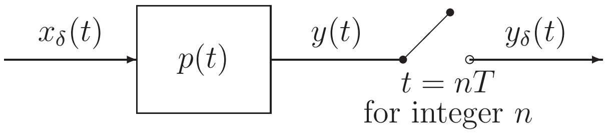
$$s(t) = \sum_{n=-\infty}^{\infty} a_n \, g(t - nT)$$

   for WSS $\{a_n\}_{n=-\infty}^{\infty}$ is

$$\overline{\text{PSD}}_s(f) = |G(f)|^2 \left(\frac{1}{T} \sum_{n=-\infty}^{\infty} R_a(n) e^{-\imath 2\pi f n T} \right)$$

   where $G(f)$ is the Fourier transform of $g(t)$ and $R_a(n) = E[a_{n+m} a_n^*]$ is the autocorrelation function of WSS $\{a_n\}_{n=-\infty}^{\infty}$.

(b) Use (a) to find the time-averaged PSD of $x_\delta(t)$ and $y(t)$ below, where $x_\delta(t) = \sum_{k=-\infty}^{\infty} a_k\, \delta(t-kT)$ and $\{a_n\}_{n=-\infty}^{\infty}$ WSS. Note that here $p(t)$ is a general LTI filter and may not satisfy the Nyquist Criterion (and is possibly a design of correlative level coding).



(c) Continue from (b). We note that $y_\delta(t) = \sum_{n=-\infty}^{\infty} c_n\, \delta(t - nT)$, where $c_n = y(nT)$. If $c_n = a_n + a_{n-1}$ as the duo-binary correlative level coding, is $\{c_n\}_{n=-\infty}^{\infty}$ also WSS? Represent the time-averaged PSD of $y_\delta(t)$ in terms of that of $x_\delta(t)$.

4. From the slides, we know that the error propagation of correlative level coding can be eliminated by pre-coding. Two examples are given in our lectures, which are:

$$\text{Duo-binary} \begin{cases} b_k = \tilde{b}_k \oplus \tilde{b}_{k-1} \\ a_k = 2\tilde{b}_k - 1 \\ c_k = a_k + a_{k-1} \end{cases}$$

$\Rightarrow b_k$ can be fully determined by c_k.

$$\text{Modified Duo-binary} \begin{cases} b_k = \tilde{b}_k \oplus \tilde{b}_{k-2} \\ a_k = 2\tilde{b}_k - 1 \\ c_k = a_k - a_{k-2} \end{cases}$$

$\Rightarrow b_k$ can be fully determined by c_k.

Give one pre-coding scheme for

$$\begin{cases} b_k = ?? \\ a_k = 2\tilde{b}_k - 1 \\ c_k = a_k + 2a_{k-1} + a_{k-2} \end{cases} \qquad \Rightarrow b_k \text{ can be fully determined by } c_k.$$

Hint: $b_k = (\alpha\tilde{b}_k) \oplus (\beta\tilde{b}_{k-1}) \oplus (\gamma\tilde{b}_{k-2})$ for some $(\alpha, \beta, \gamma) \in \{0, 1\}^3$. There are eight possible candidates for precoding design.