

$$P(\text{Error}) = \int_0^{\infty} \Phi\left(-\sqrt{2\gamma}\right) \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma$$

(Let $u(\gamma) = \Phi\left(-\sqrt{2\gamma}\right)$ and $v(\gamma) = -e^{-\gamma/\gamma_0}$.)

Apply $\int u \cdot dv = u \cdot v - \int v \cdot du$.)

$$= \Phi\left(-\sqrt{2\gamma}\right) \left(-e^{-\gamma/\gamma_0}\right) \Big|_0^{\infty} - \int_0^{\infty} \left(-e^{-\gamma/\gamma_0}\right) \left(-\frac{1}{\sqrt{2\gamma}} \frac{1}{\sqrt{2\pi}} e^{-\gamma}\right) d\gamma$$

$$= \frac{1}{2} - \int_0^{\infty} \frac{1}{\sqrt{4\pi\gamma}} e^{-\gamma(1+1/\gamma_0)} d\gamma \quad (x = \gamma(1 + 1/\gamma_0))$$

$$= \frac{1}{2} + \frac{1}{\sqrt{1 + 1/\gamma_0}} \int_0^{\infty} \left(-\frac{1}{\sqrt{4\pi x}} e^{-x}\right) dx \quad (\text{This is exactly } u'(x).)$$

$$= \frac{1}{2} + \frac{1}{\sqrt{1 + 1/\gamma_0}} \Phi\left(-\sqrt{2x}\right) \Big|_0^{\infty}$$

$$= \frac{1}{2} - \frac{1}{2\sqrt{1 + 1/\gamma_0}}$$