Principles of Communication Engineering II

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http://moser.cm.nctu.edu.tw/nctu/pce2/ Due on: 26 May 2009

Problem 1 *Budget Link Calculation*

A direct broadcast satellite (DBS) system has the following specifications:

- satellite EIRP: 57 dBW
- downlink carrier frequency: 12.5 GHz
- data rate: 10 Mb/s
- required $\frac{\mathcal{E}_b}{N_0}$ at the receiving earth terminal: 10 dB
- Security margin: $\mathcal{M} = 5$ dB
- Distance of the satellite from the receiving earth terminal: 41'000 km

Calculate the minimum diameter of the dish antenna needed to provide satisfactory TV reception, assuming that the dish has an efficiency of 55 percent and it is located alongside the home where the temperature is 310 K. For this calculation, assume that the operation of the DBS system is essentially downlink-limited.

Problem 2 *GSM*

a) Consider Global System for Mobile (GSM), which is a TDMA/FDD system that uses 25 MHz for the forward link, which is broken into radio channels of 200 kHz. On a single radio channel 8 speech channels are supported. If no guard band is assumed, find the number of simultaneous users that can be accommodated in GSM.

b) On a single radio channel GSM uses a frame structure where each frame consists of eight time slots. Each time slot contains 156.25 bits, and data is transmitted at 270.833 kbps in the channel. Find

   i) the time duration of a bit,
   ii) the time duration of a slot,
   iii) the time duration of a frame, and
   iv) how long must a user occupying a single time slot wait between two successive transmissions,

v) the raw data rate provided for each user.

c) One time slot of 156.25 bits consists of 6 trailing bits, 8.25 guard bits, 26 training bits, and two traffic bursts of 58 bits of data. Find the frame efficiency. What is the effective data rate for each user?

Problem 3 — ***Maximum Ratio Combiner***

In this problem we study the maximum ratio combiner. Consider a set of noisy signals $x_\ell(t)$, $\ell = 1, \ldots, \mathsf{L}$, where

$$X_\ell(t) = S_\ell(t) + N_\ell(t), \qquad \ell = 1, \ldots, \mathsf{L}.$$

Assume the following:

- The signal components $s_\ell(t)$ are locally coherent, *i.e.*,

$$S_\ell(t) = z_\ell M(t), \qquad \ell = 1, \ldots, \mathsf{L},$$

  where the z_ℓ are positive real numbers, and $M(t)$ denotes a message signal with unit power.

- The noise components $N_\ell(t)$ have zero mean, and they are statistically independent, *i.e.*,

$$\mathsf{E}[N_\ell(t) N_{\ell'}(t)] = \begin{cases} \sigma_\ell^2 & \text{for } \ell = \ell', \\ 0 & \text{otherwise.} \end{cases}$$

The output of the linear combiner is defined by

$$X(t) = \sum_{\ell=1}^{\mathsf{L}} \alpha_\ell X_\ell(t)$$

where the parameters α_ℓ are to be determined.

a) Show that the output signal-to-noise ratio is

$$\text{SNR}_{\text{out}} = \frac{\left(\sum_{\ell=1}^{\mathsf{L}} \alpha_\ell z_\ell\right)^2}{\sum_{\ell=1}^{\mathsf{L}} \alpha_\ell^2 \sigma_\ell^2}.$$

b) Set

$$\mu_\ell \triangleq \alpha_\ell \sigma_\ell,$$
$$\nu_\ell \triangleq \frac{z_\ell}{\sigma_\ell},$$

and reformulate the expression for SNR_{out}. Hence, applying the Schwarz inequality to this reformulation, show that

$$\text{SNR}_{\text{out}} \le \sum_{\ell=1}^{\mathsf{L}} \text{SNR}_\ell$$

where $\text{SNR}_\ell = \nu_\ell^2$.

c) Show that the optimum values of the combiner's coefficients are

$$\alpha_\ell = \frac{z_\ell}{\sigma_\ell^2},$$

in which case the Schwarz inequality is satisfied with equality.

Problem 4 ***Typical Error Event and Diversity***

Consider a BPSK system. If the modulated signal x is transmitted over a standard additive Gaussian noise channel

$$Y = x + Z,$$

we have seen in class that the probability of error (assuming coherent detection) is given as

$$\Pr(\text{error}) = \mathcal{Q}\left(\sqrt{2\textsc{snr}}\right).$$

This means that the probability of error decays exponentially fast in the SNR at high SNR.
If we have a Rayleigh fading channel instead, then the signal is additionally suffering from multiplicative noise. This noise has a very strong impact: the probability of error becomes

$$\Pr(\text{error}) = \mathsf{E}\left[\mathcal{Q}\left(\sqrt{2|H_0|^2\textsc{snr}}\right)\right] \approx \frac{1}{4\textsc{snr}}, \tag{1}$$

which decays only like $1/\textsc{snr}$ and not exponentially fast. The reason for this poor performance is that with high probability the multiplicative noise has small amplitude and is therefore "destroying" the good signal-to-noise ratio at high SNR, *i.e.*, the **typical error event** is not large additive noise, but small multiplicative noise.

a) To investigate this point more in detail, compute the probability that the multiplicative noise has small amplitude, *i.e.*, compute

$$\Pr\left[|H_0|^2 < \frac{1}{\text{SNR}}\right] \tag{2}$$

where $H_0 \sim \mathcal{N}_{\mathbb{C}}(0,1)$. Compare this to (1).

Note: If $H_0 \sim \mathcal{N}_{\mathbb{C}}(0,1)$ then $f_{|H_0|^2}(t) = e^{-t}$, $t \geq 0$.

We have seen that one way to combat this poor performance is to make use of diversity. For example we could transmit the same information L times at different times or over different bands, etc., just making sure that the fading that occurs in the different paths is independent of each other. The optimum receiver then is a *maximum ratio combiner*: it basically takes the received signals from all paths, rotates the phase to align all of them and then adds all parts together. The probability of error then becomes

$$\Pr(\text{error}) = \mathsf{E}\left[\mathcal{Q}\left(\sqrt{2\sum_{i=1}^{\mathsf{L}}|H_i|^2\textsc{snr}}\right)\right]$$

where $\{H_i\}$ are IID $\sim \mathcal{N}_{\mathbb{C}}(0,1)$.

b) For this new scenario, compute the probability that the multiplicative noise has small amplitude, *i.e.*, compute

$$\Pr\left[\sum_{i=1}^{\mathsf{L}}|H_i|^2 < \frac{1}{\text{SNR}}\right].$$

Compare this to (2).

Note: The non-negative random variable $T = \sum_{i=1}^{\mathsf{L}}|H_i|^2$ has density $f_T(t) = \frac{1}{(\mathsf{L}-1)!}t^{\mathsf{L}-1}e^{-t}$, $t \geq 0$.

The implementation of a maximum ratio combiner is often costly because of high complexity: all L channel taps need to be estimated, the received signal must be phase rotated and combined. In practice, it is thus often desirable to use a less complex receiver structure. A receiver based on *selection combining* checks all diversity branches and then only selects the strongest one to decode the signal, ignoring the rest.

c) Compare the performance of this suboptimal selection combiner with the optimal maximum ratio combiner by computing again the typical error event:

$$\Pr\left[\max_{i\in\{1,\ldots,\mathsf{L}\}} |H_i|^2 < \frac{1}{\text{SNR}}\right].$$

Compare with the results from above.

Note: For small values of t the density of $T = \max_{i\in\{1,\ldots,\mathsf{L}\}} |H_i|^2$ can be approximated by $f_T(t) \approx \mathsf{L}t^{\mathsf{L}-1}$, $t \geq 0$, t small.