

In order to give you more insight about the answer of Problem 2-d), let me further explain it in the following. An MSK passband signal can be formulated as:

$$\begin{aligned}
s(t) &= \sqrt{\frac{2E_b}{T_b}} \cos \left( 2\pi f_c t + \sum_{k=-\infty}^{n-1} I_k \pi h + I_n \pi h \left( \frac{t - nT_b}{T_b} \right) \right) \\
&= \sqrt{\frac{2E_b}{T_b}} \cos \left[ 2\pi \left( f_c + \frac{1}{4T_b} I_n \right) t + \theta_n - \frac{\pi}{2} n I_n \right] \\
&= \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos (2\pi f_1 t + q_1), & I_n = -1 \\ \sqrt{\frac{2E_b}{T_b}} \cos (2\pi f_2 t + q_2), & I_n = 1, \end{cases}
\end{aligned}$$

for  $nT_b \leq t < (n+1)T_b$ , where

$$\begin{cases} \theta_n = \frac{\pi}{2} \sum_{k=-\infty}^{n-1} I_k \\ f_1 = f_c - \frac{1}{4T_b} \\ f_2 = f_c + \frac{1}{4T_b} \\ q_1 = \theta_n + \frac{\pi}{2} n \\ q_2 = \theta_n - \frac{\pi}{2} n. \end{cases}$$

This formula indicates that an MSK signal is actually a binary FSK signal with “undue” phase at each transmission interval  $T = T_b$ . So, one may “ignore” the “undue” phase (that is added solely for the sake of making phase continuous) and only work on the demodulation of  $f_1$  and  $f_2$  in a noncoherent fashion.

Now the question is “Will  $f_2 - f_1 = 1/(2T_b)$  affect the noncoherent possibility?” Assume that  $2f_1$  and  $2f_2$  are multiples of  $1/T_b$ , which is definitely possible for MSK modulations, and assume without loss of generality that

$\cos(2\pi f_1 t + q_1)$  is the transmitted signal,

$$\begin{aligned}
\ell_1^2 &= \frac{2E_b}{T_b} \left( \int_0^{T_b} \cos(2\pi f_1 t + q_1) \cos(2\pi f_1 t) dt \right)^2 \\
&\quad + \frac{2E_b}{T_b} \left( \int_0^{T_b} \cos(2\pi f_1 t + q_1) \sin(2\pi f_1 t) dt \right)^2 \\
&= \frac{E_b}{2T_b} \left( \int_0^{T_b} \cos(4\pi f_1 t + q_1) dt + \int_0^{T_b} \cos(q_1) dt \right)^2 \\
&\quad + \frac{E_b}{2T_b} \left( \int_0^{T_b} \sin(4\pi f_1 t + q_1) dt - \int_0^{T_b} \sin(q_1) dt \right)^2 \\
&= \frac{T_b E_b}{2}
\end{aligned}$$

and

$$\begin{aligned}
\ell_2^2 &= \frac{2E_b}{T_b} \left( \int_0^{T_b} \cos(2\pi f_1 t + q_1) \cos(2\pi f_2 t) dt \right)^2 \\
&\quad + \frac{2E_b}{T_b} \left( \int_0^{T_b} \cos(2\pi f_1 t + q_1) \sin(2\pi f_2 t) dt \right)^2 \\
&= \frac{E_b}{2T_b} \left( \int_0^{T_b} \cos(2\pi(f_2 + f_1)t + q_1) dt + \int_0^{T_b} \cos(2\pi(f_2 - f_1)t - q_1) dt \right)^2 \\
&\quad + \frac{E_b}{2T_b} \left( \int_0^{T_b} \sin(2\pi(f_2 + f_1)t + q_1) dt + \int_0^{T_b} \sin(2\pi(f_2 - f_1)t - q_1) dt \right)^2 \\
&\approx \frac{E_b}{2T_b} \left( \int_0^{T_b} \cos(\pi T_b t - q_1) dt \right)^2 + \frac{E_b}{2T_b} \left( \int_0^{T_b} \sin(\pi T_b t - q_1) dt \right)^2,
\end{aligned}$$

where we may assume that  $f_1 + f_2 \gg 1/T_b$ ; hence,  $\int_0^{T_b} \cos(2\pi(f_2 + f_1)t + q_1) dt$  and  $\int_0^{T_b} \sin(2\pi(f_2 + f_1)t + q_1) dt$  are respectively much less than the other

summands. We then notice that

$$\begin{aligned}
& \left( \int_0^{T_b} \cos(\pi T_b t - q_1) dt \right)^2 + \left( \int_0^{T_b} \sin(\pi T_b t - q_1) dt \right)^2 \\
&= \left( \int_0^{T_b} [\cos(q_1) \cos(\pi T_b t) + \sin(q_1) \sin(\pi T_b t)] dt \right)^2 \\
&\quad + \left( \int_0^{T_b} [\cos(q_1) \sin(\pi T_b t) - \sin(q_1) \cos(\pi T_b t)] dt \right)^2 \\
&= \left( \cos(q_1) \frac{\sin(\pi T_b^2)}{\pi T_b} + \sin(q_1) \frac{1 - \cos(\pi T_b^2)}{\pi T_b} \right)^2 \\
&\quad + \left( \cos(q_1) \frac{1 - \cos(\pi T_b^2)}{\pi T_b} - \sin(q_1) \frac{\sin(\pi T_b^2)}{\pi T_b} \right)^2 \\
&= \frac{\sin^2(\pi T_b^2) + (1 - \cos(\pi T_b^2))^2}{\pi^2 T_b^2} \\
&= \frac{4 \sin^2(\pi T_b^2/2)}{\pi^2 T_b^2},
\end{aligned}$$

which implies

$$\ell_2^2 \approx \frac{2E_b \sin^2(\pi T_b^2/2)}{\pi^2 T_b^3} \approx \frac{2E_b (\pi T_b^2/2)^2}{\pi^2 T_b^3} = \frac{E_b T_b}{2},$$

where the second approximation follows from  $\sin(\phi) \approx \phi$  for  $\phi$  small. So, under **completely** error-free transmission, it is possible demodulate  $f_1$  non-coherently as long as  $\ell_1^2 \neq \ell_2^2$ . But since  $\ell_1^2 \approx \ell_2^2$ , such demodulation is vulnerable after the introduction of noise.

This will give you a clear picture why the noncoherent demodulation for FSK signals require orthogonality among signal frequencies.