

By extended Fourier transform,

$$\begin{aligned}
 \int_{-\infty}^{\infty} \operatorname{sgn}(f) e^{-a|f|+j2\pi ft} df &= \int_0^{\infty} e^{-a|f|+j2\pi ft} df - \int_{-\infty}^0 e^{-a|f|+j2\pi ft} df \\
 &= \int_0^{\infty} e^{-(a-j2\pi t)f} df - \int_{-\infty}^0 e^{(a+j2\pi t)f} df \\
 &= \frac{1}{a-j2\pi t} - \frac{1}{a+j2\pi t} \\
 &= \frac{j4\pi t}{a^2+4\pi^2 t^2}
 \end{aligned}$$

$$\operatorname{sgn}(f) \xrightarrow{\text{InverseFourier}} \lim_{a \downarrow 0} j \frac{4\pi t}{a^2+4\pi^2 t^2} = \begin{cases} \frac{j}{\pi t}, & t \neq 0 \\ 0, & t = 0 \end{cases}$$

$$2u(f) = 1 + \operatorname{sgn}(f) \xrightarrow{\text{InverseFourier}} \delta(t) + \frac{j}{\pi t} \cdot \mathbf{1}\{t \neq 0\}$$