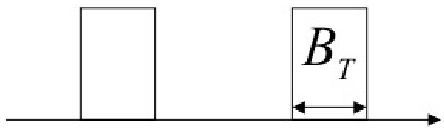
1. Question: On Slide 2-119, why the PSD of the (ideally) filtered while noise is given by the below figure? Should it be the **spectrum** of the filtered output, rather the **PSD** of the filtered output?



Answer: For a linear time-invariant (LTI) filter with impulse response $h(\tau)$, Fourier transforms of input $x(t)$ and output $y(t)$ satisfy:

$$Y(f) = X(f)H(f).$$

In addition, we have shown in our lectures that the time-averaged PSDs of LTI filter input and LTI filter output satisfy

$$\bar{S}_Y(f) = \bar{S}_X(f)|H(f)|^2.$$

For an ideal bandpass filter,

$$H(f) = |H(f)|^2 = \begin{cases} 1, & |f - f_c| < W; \\ 0, & \text{otherwise} \end{cases}$$

Thus, **both** the spectrum and the time-averaged PSD of the filtered output satisfy

$$\begin{cases} \text{remain the same}, & |f - f_c| < W; \\ 0, & \text{otherwise} \end{cases}$$

2. Question: How does a mixer work in a superheterodyne receiver?

Answer: A mixer in a superheterodyne receiver translates the antenna-received RF frequency f_{RF} to a system-desired intermediate frequency f_{IF}. With this purpose in mind, what a mixer does is basically to multiply the received sinusoidal signal $\cos(2\pi f_{\text{RF}} t)$ by $\cos(2\pi f_{\text{LO}} t)$, where f_{LO} is the local oscillator frequency. Usually, $f_{\text{IF}} < f_{\text{RF}}$ and $f_{\text{LO}} < f_{\text{RF}}$ but in general, no assumptions are made on their orders.

Suppose $f_{\mathrm{IF}} < f_{\mathrm{RF}}$. Then, we can choose $f_{\mathrm{LO}} = f_{\mathrm{RF}} - f_{\mathrm{IF}}$ and yield

$$\begin{aligned} & \cos(2\pi f_{\mathrm{RF}} t) \underbrace{\cos(2\pi f_{\mathrm{LO}} t)}_{\text{mixer}} && (1) \\ = & \ \frac{1}{2}\big[\cos(2\pi(f_{\mathrm{RF}} + f_{\mathrm{LO}})t) + \cos(2\pi(f_{\mathrm{RF}} - f_{\mathrm{LO}})t)\big] \\ = & \ \frac{1}{2}\big[\cos(2\pi(f_{\mathrm{RF}} + f_{\mathrm{LO}})t) + \cos(2\pi f_{\mathrm{IF}} t)\big] && (2) \end{aligned}$$

We can then use a bandpass filter (centered at f_{IF}) to remove $\cos(2\pi(f_{\mathrm{LO}}+f_{\mathrm{RF}})t)$ and get the desired $\cos(2\pi f_{\mathrm{IF}} t)$.

On the other hand, we can choose $f_{\mathrm{LO}} = f_{\mathrm{RF}} + f_{\mathrm{IF}}$ and yield

$$\begin{aligned} & \cos(2\pi f_{\mathrm{RF}} t) \underbrace{\cos(2\pi f_{\mathrm{LO}} t)}_{\text{mixer}} \\ = & \ \frac{1}{2}\big[\cos(2\pi(f_{\mathrm{RF}} + f_{\mathrm{LO}})t) + \cos(2\pi(f_{\mathrm{RF}} - f_{\mathrm{LO}})t)\big] \\ = & \ \frac{1}{2}\big[\cos(2\pi(f_{\mathrm{LO}} + f_{\mathrm{RF}})t) + \cos(2\pi(f_{\mathrm{LO}} - f_{\mathrm{RF}})t)\big] \\ = & \ \frac{1}{2}\big[\cos(2\pi(f_{\mathrm{LO}} + f_{\mathrm{RF}})t) + \cos(2\pi f_{\mathrm{IF}} t)\big] && (3) \end{aligned}$$

We can again use a bandpass filter (centered at f_{IF}) to remove $\cos(2\pi(f_{\mathrm{LO}}+f_{\mathrm{RF}})t)$ and get the desired $\cos(2\pi f_{\mathrm{IF}} t)$. The above explanation is from the standpoint of how f_{LO} should be selected, which leads to the conclusion that

$$f_{\mathrm{LO}} = f_{\mathrm{RF}} \pm f_{\mathrm{IF}}. \quad (4)$$

Now from the standpoint of determining the **image (RF) interference**, we have from (4) that $f_{\mathrm{RF}} = f_{\mathrm{LO}} \pm f_{\mathrm{IF}}$, by which we know that after fixing f_{LO} at the mixer, f_{IF} at the IF section and a bandpass filter centered at f_{IF}, two f_{RF}'s will survive at the output of the bandpass filter: $f_{\mathrm{LO}} + f_{\mathrm{IF}}$ and $f_{\mathrm{LO}} - f_{\mathrm{IF}}$. One of the two is a wanted signal and the other is an unwanted signal. Imagine that a mirror is placed at f_{LO}. Then the "image" of $f_{\mathrm{LO}} + f_{\mathrm{IF}}$ is exactly located at $f_{\mathrm{LO}} - f_{\mathrm{IF}}$ (and vice versa).

Since it is never required that $f_{\mathrm{LO}} > f_{\mathrm{IF}}$, the general formula for the two f_{RF} that survive at the output of the bandpass filter centered at f_{IF} is

$$\text{either } \begin{cases} f_{\text{LO}} + f_{\text{IF}} \\ f_{\text{LO}} - f_{\text{IF}} & \text{if } f_{\text{LO}} > f_{\text{IF}} \end{cases} \text{ or } \begin{cases} f_{\text{IF}} + f_{\text{LO}} \\ f_{\text{IF}} - f_{\text{LO}} & \text{if } f_{\text{LO}} < f_{\text{IF}} \end{cases}$$

where for the latter, the mirror should be imagined to be located at f_{IF}. This results in the general formula on Slide 2-113, i.e., $f_{\text{RF}} = |f_{\text{IF}} \pm f_{\text{LO}}|$.

3. Question: On Slide 2-113, why is 1.56 MHz the image interference?

   Answer: Please refer to my answer to the previous question. Since $f_{\text{LO}} > f_{\text{IF}}$, the "mirror" should be imagined to be located at f_{LO}, and the two RF frequencies that survive at the output of the bandpass filter centered at f_{IF} are $f_{\text{LO}} + f_{\text{IF}} = 1.56$ MHz and $f_{\text{LO}} - f_{\text{IF}} = 0.65$ MHz. Because 0.65 MHz is the wanted signal, 1.56 MHz is the image inference.

4. Question: On Slide 2-114, how to know that f_{IF} is equal to $f_{\text{LO}} - f_{\text{RF}}$ and f_{image} is $f_{\text{LO}} + f_{\text{RF}}$?

   Answer: Sorry for the confusion. $f_{\text{image}} = f_{\text{LO}} + f_{\text{RF}}$ is used to indicate the **image frequency** generated due to mixing or superheterodying. Again, after mixing operation, two frequencies will be generated (cf. Eqs. (2) and (3)): one of them is the wanted signal and the other is often referred to as its image (where the "mirror" should be imagined to be placed at f_{LO}). Since this image frequency can be removed easily by a bandpass filter, we do not need to worry about it. What troubles us is $f_{\text{interference}} - f_{\text{LO}}$, which happens to be equal to f_{IF}. In such case, a bandpass filter centered at f_{IF} cannot tell apart $f_{\text{interference}} - f_{\text{LO}}$ and $f_{\text{LO}} - f_{\text{RF}}$; so conceptually, it is suggested to suppress $f_{\text{interference}}$ before the mixing operation.