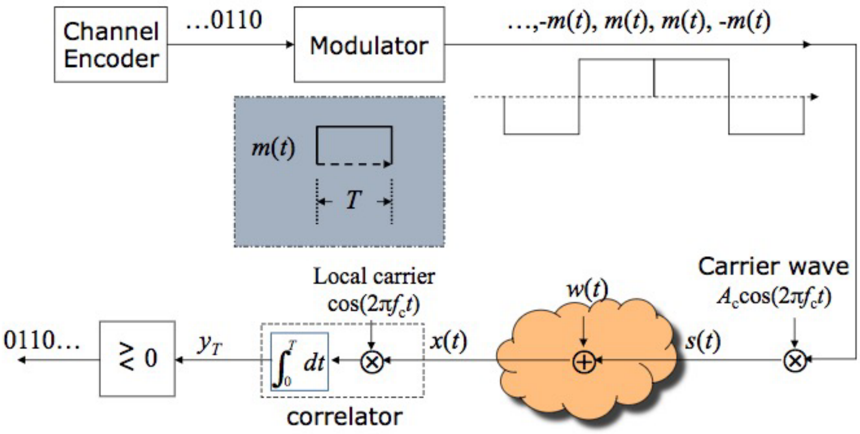
Sample Problems for the Third Quiz

1. For the exemplified ideal PSK digital communication system below, show that $y_T = \pm\frac{1}{2}A_cT$, provided that $w(t) = 0$ and f_c is a multiple of $1/T$.



2. For a sequence '*abddc daaab dcdab cdaab bcaab*' of length 25, determine the following quantities.

   - Let $A = \{a, b\}$. Determine $N_{25}(A)$, i.e., the number of occurrences of 'a' or 'b' in the sequence of length 25.
   - Let $B = \{b, c\}$. Determine $N_{25}(B)$, i.e., the number of occurrences of 'b' or 'c' in the sequence of length 25.
   - Determine $N_{25}(A \cap B)$.
   - Is event A independent of event B from the empirical distribution of the sequence?

3. Prove that $P(A|B) = P(A)$ if, and only if, $P(B|A) = P(B)$.

4. An experiment is performed on a random phenomenon and five measurements are made, which result -6.8, -2.1, 0.5, 1.9 and 9.2.

   - Draw the empirical *cumulative distribution function (cdf)* based on these measurements.

     Hint: Each measurement is assigned a probability mass of $1/5$.
   - Use intervals of length 1 to draw the empirical *probability mass function (pmf)* of this experiment.

5. Prove that if random process $X(t)$ is stationary, its autocorrelation function $R_X(t_1, t_2)$ is equal to $R_X(t_1 - t_2, 0)$.

   Hint: A random process is said to be *stationary* if its statistical property is independent of the time, including its pdf.

6. Prove that we can relate the autocorrelation function

$$R_X(t_1, t_2) \triangleq E[X(t_1))(X(t_2)]$$

   with the autocovariance funciton

$$C_X(t_1, t_2) \triangleq E[(X(t_1) - \mu_X(t_1))(X(t_2) - \mu_X(t_2))]$$

   as

$$C_X(t_1, t_2) = R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2).$$

7. Give the two conditions of the wide-sense stationarity on the mean function and autocovariance function. What will the two conditions change to, if cyclostationarity is considered instead?

8. Prove that the autocorrelation function of a WSS random process always peaks at zero.

9. Determine the mean function and autocorrelation function of $X(t) = \cos(2\pi f_c t + \Theta)$, where Θ is uniformly distributed over $[-\pi, \pi)$.

10. Determine the mean function and autocorrelation function of $X(t) = \sum_{n=-\infty}^{\infty} I_n \cdot p(t - nT - t_d)$, where t_d is uniformly distributed over $[0, T)$ and $\{I_n\}_{n=-\infty}^{\infty}$ are independent and identically distributed with $\Pr[I_n = -1] = \Pr[I_n = 1] = \frac{1}{2}$.

11. Determine the cross-correlation function of $X_I(t) = X(t)\cos(2\pi f_c t + \Theta)$ and $X_Q(t) = X(t)\sin(2\pi f_c t + \Theta)$, where $X(t)$ is a WSS random process with autocorrelation function $R_X(\tau)$.