Optimum Transmission Range for Wireless Ad Hoc Networks

Jing Deng
Dept. of EECS
Syracuse Univ.
Syracuse, NY 13244
jdeng01@ecs.syr.edu

Yunghsiang S. Han
Dept. of CSIE
National Chi Nan Univ.
Taiwan, R.O.C.
yshan@csie.ncnu.edu.tw

Po-Ning Chen
Dept. of CE
National Chiao-Tung Univ.,
Taiwan, R.O.C.
poning@faculty.nctu.edu.tw

Pramod K. Varshney
Dept. of EECS
Syracuse Univ.
Syracuse, NY 13244
varshney@ecs.syr.edu

Abstract— **The transmission range that achieves the most economical use of energy in wireless ad hoc networks is studied under homogeneous node distribution. By assuming the knowledge of node location, we first proposed a transmission strategy to ensure the progress of data packets toward their final destinations. Then the average packet progress for a transmission range universal for all nodes is derived, which is accordingly used to determine the optimal transmission range that gives the maximum efficiency of energy consumption. Different from some previous work, our analysis does not make the assumption of large nodal density in the wireless ad hoc networks studied. Numerical and simulation results are presented to examine our analysis for wireless ad hoc networks.**

I. INTRODUCTION

The research on wireless ad hoc networks has experienced a rapid growth over the last few years. Unique properties of ad hoc networks, such as operation without pre-existing infrastructure, fast deployment, and self-configuration, make them suitable for communication in tactical operations, search and rescue missions, and home networking. While most studies in this area have concentrated on the design of routing protocols, medium access control protocols, and security issues, we investigate the efficiency of energy consumption of wireless ad hoc networks in this work.

Due to their portability and their deployment in potentially harsh scenarios, nodes in ad hoc networks are usually powered by batteries with finite capacity. It is always desirable to extend the lifetime of ad hoc network nodes without sacrificing their functionality. Thus, the study of energy-efficient mechanisms is of significant importance.

In wireless ad hoc networks, the major energy consumption at each node is due to system operation, data processing, and wireless transmission and reception. While there are studies on increasing battery capacity and reducing the energy consumption of system operation and data processing, a study on energy consumption economy of radio transceivers is also necessary to achieve a more energy-efficient system design [1]. In some previous work, the radio transmission range of nodes in wireless networks is optimized, based on their local neighborhood information, to establish desirable network topologies and lower transmission interference [2], [3], [4], [5], and [6]. In this work, we consider the radio transmission range as a pre-determined system parameter, which should be defined *a priori* during system design, and used throughout the lifetime of the wireless ad hoc network.

When two communicating nodes are not in the range of each other in wireless ad hoc networks, they need to rely on multi-hop transmissions. Under such conditions, packet forwarding, or routing, becomes necessary. The value of the radio transmission range affects network topology and energy consumption considerably. A larger transmission range increases the distance progress of data packets toward their final destinations. This is unfortunately achieved at the expense of higher energy consumption per transmission. On the other hand, a shorter transmission range uses less energy to forward packets to the next hop, although a larger number of hops is needed for packets to reach their destinations.

There have been some published studies that have concentrated on the optimization of the radio transmission range of wireless networks. In [7], the optimal transmission radii that maximize the expected progress of packets in desired directions were determined for different transmission protocols in multihop packet radio networks with randomly distributed terminals. The optimal transmission radii were expressed in terms of the number of terminals in range. It was found that the

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optimal transmission radius for slotted ALOHA without capture covers eight nearest neighbors in the direction of packet's final destination. The study concentrated on limiting transmission interference to improve throughput performance in wireless networks under heavy traffic condition. Energy consumption, however, was not considered in the paper.

Similar assumptions were made in [8], which further allowed all nodes to adjust their transmission radii independently at any time. It was found that higher throughput and progress could be obtained by transmitting packets to the nearest neighbor in the forward direction and using the lowest possible transmission power for each transmission.

A distributed position-based network protocol that minimizes energy consumption was proposed in [9]. The protocol was proved to be self-reconfigurating and stay close to the minimum energy solution when applied to mobile networks.

The optimization of transmission range as a system design issue was studied in [10]. The wireless network was assumed to have nodes with relatively low mobility and short range, and high nodal density. Assuming nodes without power control capability, the authors argued that the optimum range could be set at the system design stage. It was shown that the optimum one-hop transmission distance is independent of the physical network topology, the number of transmission sources, and the total transmission distance. It only depends on the propagation environment and radio transceiver device parameters. The study assumed that intermediate nodes are always available at the desired locations when they are needed to route a packet forward. This assumption can only be justified in networks with high nodal density.

A bit-meter-per-joule metric for energy consumption in wireless ad hoc sensor networks was studied in [11]. The paper presented a system-level characterization of energy consumption for sensor networks. Power efficiency metric in average watt-per-meter for each radio transmission was calculated, and was subsequently extended to determine global energy consumption. The analysis showed how the overall energy consumption varies with transceiver characteristics, node density, data traffic distribution, and base-station location. The study was based on the assumptions that the sensor network has a relay architecture and all the traffic is sent from sensor nodes toward the base station which is assumed to be far away. Furthermore, the source always chooses the relay neighbor that is with the lowest bit-meter-per-joule among all neighbors.

In summary, compared with [5], [8], and [9], our study determines a single optimum transmission range for all the nodes in the network, and this value can be set in the pre-deployment phase as long as the expected nodal density is known. Compared with [10], our study does not make the assumption that a relaying node that is closest to the destination can always be found. Thus, the wireless networks we study do not need to be highly densed. Compared with [11], our network does not have any base station or common receiver and does not assume that the destination is far away from the source. Our study takes into consideration the node density as well as the probability of finding an intermediate node to relay data packets toward their destinations, while nodes are distributed in the network according to a two-dimension Poisson distribution.

As in [10] and [11], we do not model the extra energy consumption due to packet retransmissions. We assume a light traffic load in the networks we study and a data packet collision-free Medium Access Control scheme is utilized [12] [13].

Our paper is organized as follows: Our analysis and transmission strategy are presented in Section II. Numerical and simulation results along with some discussion are provided in Section III. Section IV concludes our work.

II. ANALYSIS OF SINGLE-TRANSMISSION DISTANCE-ENERGY EFFICIENCY

In this section, we analyze and optimize the distance-energy efficiency for a wireless ad hoc network with randomly distributed nodes at the time of the *first* transmission, even if a multi-hop transmission is subsequently required for the transmitted packet to reach its ultimate destination. Specifically, the *single-transmission distance-energy efficiency* is defined as the ratio of *the average progress* of the transmitted packet during the first transmission and the *energy consumption of a single transmission*.

A. Network Model and Transmission Strategy

Suppose that a source node, S, is located at the center of a circle of radius x, where x is the largest possible distance between S and any destination. In other words, the source node does not send any packet to nodes outside the circle. The destination node D, to which S intends to transmit a data packet, is assumed uniformly distributed over the entire circle.

Due to the limited radio range (or equivalently, limited transmission energy), the packet from the source node to the destination node may need to be sequentially routed by a certain number of intermediate nodes, which are termed *routers*. All nodes, including the source node and the intermediate nodes, employ a common transmission radius r. Consequently, direct transmission occurs only when the destination node is within distance r from the source node. To avoid the trivial case, we assume that $x > r$.

Any node within the transmission range of a node is called its *neighbor*. We assume that each node knows the locations of all its neighbors and the location of the destination node. Based on this assumption, a transmission strategy can be designed as follows:

(i) The source node S transmits a packet to the destination node D directly, if D is located within distance r from S.

(ii) When the destination node D is outside the transmission range of the source node S, the source node S sends the packet to a neighbor s.t. (a) the neighbor is closer in distance to the destination node D than the source node S, and (b) if multiple such nodes exist then the neighbor closest to D is selected.

(iii) Since the source node S knows the locations of all the neighbor nodes and the destination node, it does not send out the packet when no neighbor that satisfies the condition in (ii) is available, and postpones the transmission until such a neighbor appears due to nodal mobility[1].

The probability that n nodes appear in an area of size A is given by $(\rho A)^n e^{-\rho A}/n!$, where ρ is the density parameter for this two-dimensional Poisson point process [7]. The appearance of nodes in any two non-overlapping areas are assumed independent.

The energy consumption corresponding to each transmission can be formulated as [10]:

$$E_\mathrm{t}(r) = k_1 r^\omega + k_2,$$

where r is the radio transmission range, ω is the path loss exponent, k_1 is determined by the characteristic of the transmitter and the channel, and k_2 is the transceiver energy consumption that is not related to r. Let E_r be the energy consumption of receiving, decoding, and processing data packets at the receiver. As a result of the above formulation, the *single-transmission consumed energy* is fixed for a given r and is given by $E_\mathrm{t}(r) + E_\mathrm{r}$.

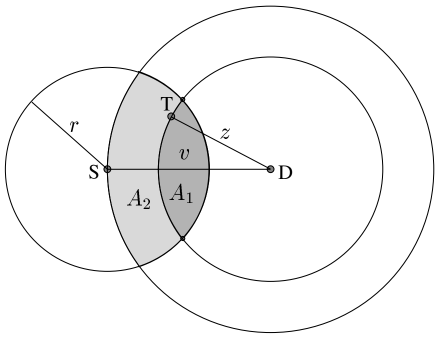

Fig. 1. Illustration of relaying nodes.

To complete the analysis of the distance-energy efficiency, it remains to determine *the average progress* of the transmitted packet in a single hop.

B. Average Single-Transmission Progress

Denote the distance between the source node S and the destination node D by v. When $v \le r$, direct transmission to the destination node D can be attained; hence, the distance progress of the transmitted packet to the destination node D is v. In the situation that $v > r$, the source node has to locate an appropriate neighbor for subsequent packet routing. Define *distance progress* as the difference between the original distance between the sender and the destination and the distance between the relaying node and the destination [11]. The distance progress of the transmitted packet toward the destination node D is therefore equal to $(v - z)$, where z is the distance between the first-hop router T and the destination node D (cf. Fig. 1).

Denote by $\boldsymbol{P}$ the random variable corresponding to the distance progress for a single transmission. Let $\boldsymbol{V}$ and $\boldsymbol{Z}$ be the random variables corresponding to v and z discussed above, respectively, i.e., $\boldsymbol{V}$ represents the distance between the source and the destination while $\boldsymbol{Z}$ represents the distance between the first-hop router T and the destination node D. Define a new random variable $\boldsymbol{H}$ as:

$$\boldsymbol{H} = \begin{cases} 1, & \text{if a neighbor satisfying the condition in strategy (ii) exists;} \\ 0, & \text{otherwise.} \end{cases}$$

Note that, according to the strategy described in the previous subsection, the source node will send the packet directly to the destination node whenever $\boldsymbol{V} \le r$ without regard to the value of $\boldsymbol{H}$.

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[1] An alternative design is to increase the transmission radius in order to force the appearance of such a neighbor or even to reach the destination node directly. Extra energy consumption thus becomes necessary. However, the probability of having no relaying nodes is usually negligibly small (cf. Appendix). We accordingly exclude this alternative in this work.

With the above notations and assumptions, the problem of finding the average single-transmission progress is equivalent to the derivation of the expected value of $\boldsymbol{P}$ under the condition that $[(\boldsymbol{V} \leq r) \cup ((\boldsymbol{V} > r) \cap (\boldsymbol{H} = 1))]$. Note that if $[(\boldsymbol{V} \leq r) \cup ((\boldsymbol{V} > r) \cap (\boldsymbol{H} = 1))]$ is false, no transmission will take place according to (iii) of the transmission strategy; hence, no energy will be consumed (and the progress is certainly zero). We can, therefore, formulate the relation between $\boldsymbol{P}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, and $\boldsymbol{H}$ as:

$$\boldsymbol{P} = \begin{cases} \boldsymbol{V}, & \text{if } \boldsymbol{V} \leq r; \\ \boldsymbol{V} - \boldsymbol{Z}, & \text{if } (\boldsymbol{V} > r) \cap (\boldsymbol{H} = 1), \end{cases}$$

provided that $[(\boldsymbol{V} \leq r) \cup ((\boldsymbol{V} > r) \cap (\boldsymbol{H} = 1))]$ is true. It can be derived that the expected value of $\boldsymbol{P}$ under $[(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)]$ is:

$$\begin{aligned} & E\left[\boldsymbol{P} \,|(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\right] \\ = & \frac{3x^2 r - r^3 - 6 \displaystyle\int_0^r \int_r^x v e^{-\rho A_1(v-p,v,r)} dv dp}{3\left(x^2 - 2 \displaystyle\int_r^x v e^{-\rho A_{SD}(v,r)} dv\right)}, \end{aligned}$$

where A_{SD} denotes the area of the overlapping region of the circle centered at S with radius r and the circle centered at D with radius v, i.e., the shaded region in Fig. 1. A_{SD} is split into two regions by the circle centered at D with radius z. The areas of these two regions are denoted as A_1 and A_2, respectively, as shown in Fig. 1.

Obviously, the value of A_{SD} is a function of v and r:

$$\begin{aligned} A_{SD}(v,r) &= r^2 \cos^{-1}\left(\frac{r}{2v}\right) + v^2 \cos^{-1}\left(1 - \frac{r^2}{2v^2}\right) \\ & \quad - \frac{1}{2} r \sqrt{(2v+r)(2v-r)}, \end{aligned}$$

Similarly, the value of A_1 is a function of z, v, and r. $A_1(z,v,r)$ is:

$$\begin{aligned} & r^2 \cos^{-1}\left(\frac{r^2+v^2-z^2}{2rv}\right) z^2 \cos^{-1}\left(\frac{z^2+v^2-r^2}{2vz}\right) \\ & - \frac{1}{2}\sqrt{(r+v+z)(v+z-r)(r+v-z)(r+z-v)}. \end{aligned}$$

The single-transmission distance-energy efficiency $e(r)$ is then given by:

$$\frac{3x^2 r - r^3 - 6 \displaystyle\int_0^r \int_r^x v e^{-\rho \cdot A_1(v-p,v,r)} dv dp}{3\left(k_1 r^\omega + k_2 + E_{\mathrm{r}}\right)\left(x^2 - 2 \displaystyle\int_r^x v e^{-\rho \cdot A_{SD}(v,r)} dv\right)}.$$

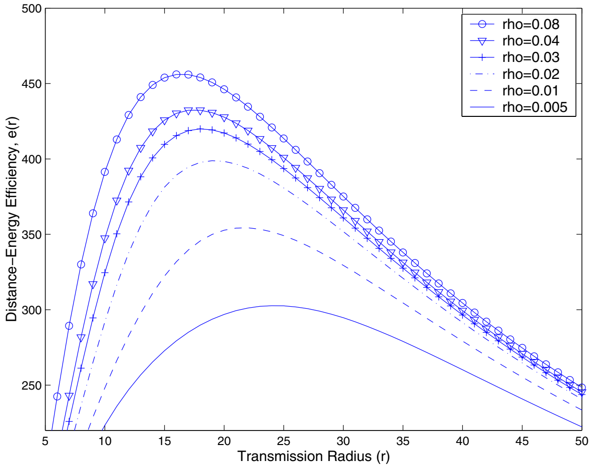

Fig. 2. Distance-energy efficiency ($\omega = 2$).

III. NUMERICAL AND SIMULATION RESULTS

Numerically evaluated distance-energy efficiency results and simulation results for its verification are summarized in this section.

A. *Numerical Results*

Fig. 2 compares the analytical single-transmission distance-energy efficiencies for different nodal densities. The network coverage area is assumed to be a circle with a radius of 100 meters (i.e., $x = 100$). The path loss exponent ω is assumed to be 2. Quantities k_1 and $k_2 + E_r$ are assumed to be 6.6319×10^{-5} and 1.476×10^{-2}, respectively.[2] Nodal density varies from 0.005 to 0.08, which corresponds, on an average, to a range of 157 to 2512 nodes in a circle with radius 100 meters.

It can be observed from Fig. 2 that the single-transmission distance-energy efficiency improves initially as r increases, and then degrades after r exceeds a certain value. Fig. 2 also shows that the distance-energy efficiency in a network with higher nodal density is higher. The explanation of this result is that the probability of finding relaying nodes closer to the final destination is higher when there are more nodes in the network. Each hop of transmission in this case makes more progress toward the final destination, thereby improving the distance-energy efficiency.

In Fig. 3, we compare the optimum transmission range that maximizes the distance-energy efficiency, $e(r)$, for different nodal density and path loss exponent. When

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[2] These parameters are chosen to be the same as in [10] for the purpose of comparison. Other values of parameters may lead to different but similar results.

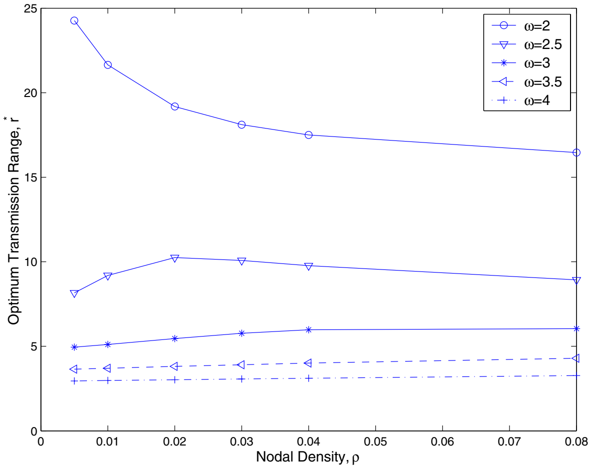

Fig. 3. Optimum Transmission Range.

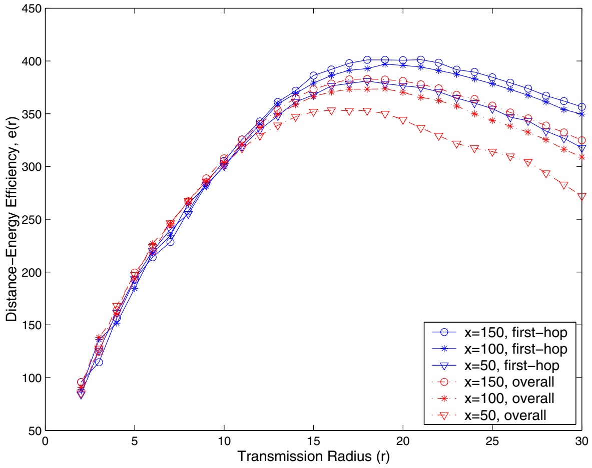

Fig. 4. Distance-energy efficiency for $\rho = 0.02$.

ω is 2, a decrease of optimum transmission range, r^*, is observed, with the increase of nodal density. As ρ decreases, it is less likely to find a neighbor node to relay data packets efficiently toward the final destinations. Thus, the optimum transmission range r^* increases with decreasing ρ. However, this trend can not be observed when ω is 4. In the $\omega = 4$ scenario, the optimum transmission range r^* remains relatively flat for different nodal density. This is due to the prohibitively expensive cost of increasing the transmission range when the path loss exponent is high. When path loss exponent ω falls in between these two values, the change of r^* as a function of ρ is less predictable, which could be caused by the combined effects of the two factors discussed above. Note that our results of r^* with large ρ and $\omega = 2$ agree with that found in [10] under the assumption that a source node can always find a neighbor at the required location to forward its data packet.

B. Simulation Results

Simulations built in C language have been performed to verify the accuracy of our analytical results. To facilitate the simulations, the network nodes are distributed on a circle according to a two-dimensional Poisson counting distribution. The circle is centered at $(0, 0)$ with radius x ranging from 50 to 150 meters. A source node is fixed at $(0, 0)$, while destination nodes are randomly chosen from the circle. The source node transmits packets to the destination node in accordance to our transmission strategy. We measured the average first-hop distance-energy efficiency of each pair of source and destination. All results are the average of 500 runs, each of which selects 100 destinations randomly.

Fig. 4 presents the simulation results when nodal density ρ is 0.02. As shown in this figure, the optimum transmission radius for single-transmission distance-energy efficiency is about 18 meters. This result coincides with our analytical result. The values of the distance-energy efficiency $e(r)$ also agree with those shown in Fig. 2. The value of x only mildly affects the distance-energy efficiency as indicated by Fig. 4. In general, a larger x results in a little higher overall distance-energy efficiency. Such a slight difference could be due to the increase of the number of hops for a larger x. From our simulations, the average number of hops for packets to reach the final destination is larger for a larger x; so the influence of a smaller last hop progress is less significant.

In addition, we simulated the overall distance-energy efficiency (not just the first-hop, but the entire path from the source to the final destination). As shown in Fig. 4, the first hop distance-energy efficiency and the overall distance-energy efficiency are approximately the same when r is small. Their difference is more noticeable when r is larger. Besides, the first hop distance-energy efficiency is a little larger than the overall energy efficiency. This is anticipated as the last hop is usually not as efficient as all other hops.[3]

IV. CONCLUSIONS

The radio transmission range as a system parameter affects the overall energy consumption of wireless ad

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[3] Based on uniform selection of the traffic destination, the expected value of the last hop progress is approximately $r/2$, which is smaller than the average hop progress when adequate number of nodes are present.

hoc networks. On the one hand, a longer transmission range increases the expected progress of the data packet toward its final destination at the expense of a higher energy consumption per transmission. On the other hand, a shorter transmission range consumes less per-transmission energy, but it requires a larger number of hops for the data packet to reach its destination.

Based on the underlying device energy consumption model and a two-dimensional Poisson node distribution, we propose an analytical model to investigate the optimal value of the radio transmission range. With the knowledge of node locations, a transmission strategy is designed to ensure the packet progress toward the final destination. An optimum transmission range is then determined. We conclude that the optimum transmission radius is influenced more by the nodal density than the network coverage area. Our results can be used to determine the proper radio transmission power for wireless ad hoc networks or micro-sensor networks in the pre-deployment phase.

In the future, we plan to compare our scheme with existing transmission control schemes using the NS2 simulator. We also plan to relax the assumption of light traffic load in our future work.

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APPENDIX: PROBABILITY OF NO RELAYING NODES

In this appendix, we study the probability that suitable relaying nodes cannot be found in our transmission strategy. According to the notations introduced in Section II, this probability is equal to

$$\begin{aligned}1 &- \left(\Pr\{\boldsymbol{D} \le r\} + \Pr\{\boldsymbol{D} > r \cap \boldsymbol{H} = 1\}\right)\\ &= \Pr\{\boldsymbol{D} > r \cap \boldsymbol{H} = 0\},\end{aligned}$$

and can be upper-bounded by $\Pr\{\boldsymbol{H} = 0\}$. The probability of $\Pr\{\boldsymbol{H} = 0\}$ is equal to the probability that the shaded area in Fig. 1 does not have any node inside.

Observe in Fig. 1 that the area of the shaded region $A_{SD}(v,r)$ increases with v, the distance between the source and the destination. Thus, $A_{SD}(v,r) \ge A_{SD}(r,r)$ when $v \ge r$.

The probability that no relaying nodes can be found is thus upper-bounded by:

$$P_0(v,r) = e^{-\rho\cdot A_{SD}(v,r)} \le e^{-\rho\cdot A_{SD}(r,r)} = e^{-\left(\frac{2}{3}\pi - \frac{\sqrt{3}}{2}\right)\rho r^2}$$

Denote $N_\rho = \rho\pi r^2$ as node degree, the average number of nodes within transmission range r. Then, $P_0(v,r) \le e^{-0.39N_\rho}$. From the upper bound, it can be calculated that $P_0(v,r)$ is lower than 15% when N_ρ is 5. When N_ρ increases up to 10 and 15, $P_0(v,r)$ decreases to 2.1% and 0.3%, respectively. Since the values shown previously are the upper bounds for $P_0(v,r)$, the actual values are even smaller as v increases.

We therefore conclude that with a moderately large number of nodes within the transmission range, the probability that suitable relaying nodes cannot be found is negligibly small in our transmission strategy.

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