A Generalization of the Fano Metric and its Effect on Sequential Decoding Using a Stack[1]

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I. Introduction

In this work, we consider the application of sequential decoding to codes of practically short code length, such as few hundreds bits, as the performance is conventionally deviated from the optimal maximum-likelihood (ML) performance. By re-formulating the original Fano metric as an equally weighted sum of two portions: *the ML metric portion to previously received code bits*, and *an estimate portion on the ML metric of upcoming code bits*, we found that the sequential decoding can be viewed as a special case of the Algorithm A [1]. One can then vary weights on these two metric portions. Empirical investigation shows that by a proper choice of the weighting factor, employing the generalized (or weighted) Fano metric in sequential decoding can yield a markedly smaller bit-error-rate than the conventional (equally weighted) one, and the computational complexity is only moderately increased.

II. Generalized Fano Metric

Let G be a code tree of an (n,k) convolutional code with root s, where each codeword is represented by a code path from root s to a leaf. Suppose that a cost $c(n_i,n_j)$ is associated with the branch between nodes n_i and n_j. Define the cost of a path as the sum of the branch costs of all the branches it contains. Algorithm A then searches the code tree to locate the code path with the maximum cost. For Algorithm A, the search on G is guided by an evaluation function $f(n_\ell)$ defined for every node n_ℓ in G. In its definition, $f(n_\ell)$ is divided into two parts: $f(n_\ell)=g(n_\ell)+h(n_\ell)$. The first part, $g(n_\ell)$, is equal to the cost of the path from root node s to node n_ℓ. The second function, $h(n_\ell)$, estimates the maximum cost among all paths from node n_ℓ to a leaf node.

By adding the constant $\sum_{j=0}^{N-1}\Pr(r_j)$ to the Fano metric, we obtain the decomposition:

$$\sum_{j=0}^{\ell n-1}[\log_2\Pr(r_j|v_j)-R]+\sum_{j=\ell n}^{N-1}\log_2\Pr(r_j).$$

These two terms can be interpreted as the g-function and the h-function, defined for the end node of the path labelled by $\boldsymbol{v}_{(\ell n-1)}=(v_0,v_1,\ldots,v_{(\ell n-1)})$, where $\boldsymbol{r}=(r_0,r_1,\ldots,r_{N-1})$ is the received vector, and R is the code rate of the (n,k) convolutional code with code length N. A *weighted* evaluation function between these two terms can be defined by respectively multiplying the above g-function and h-function by ω and $1-\omega$. This weighted evaluation function can then be transformed back to its equivalent Fano metric

$$M_\omega\left(\boldsymbol{v}_{(\ell n-1)}\right)\triangleq\sum_{j=0}^{\ell n-1}\left(\log_2\frac{\Pr(r_j|v_j)^\omega}{\Pr(r_j)^{1-\omega}}-\omega R\right).$$

Obviously, when $\omega=1/2$, the generalized Fano metric reduces to the conventional one (with a multiplicative constant, 1/2). Taking $\omega=1$ makes the generalized Fano metric equivalent to the ML cost, and the stack algorithm using the new metric becomes an ML decoding algorithm. As anticipated, in this case the algorithm will have the best word error probability but also a very high computational complexity. It is interesting to note that for discrete symmetric channels, the generalized Fano metric is equivalent to the metric proposed in [2]; however, they are not equal for other types of channel such as AWGN channels [3].

References

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[1]This work was supported by NSC of Taiwan, ROC, under the project of NSC89-2213-E-260-002, by NSF under Grants CCR-97-32959 and CCR-00-98029, and by the Hawaii Center for Advanced Communications (HCAC).

0-7803-7501-7/02/$17.00©2002 IEEE.