Simple median-based EP PP scheme for enhancement of reconstructed Bayer colour filter array images

Yi-Hong Yang1, Peng-Hua Wang2, Po-Ning Chen1

1Dept. of Electrical and Computer Engineering, National Chiao-Tung Univ., Hsinchu City 30010, Taiwan
2Dept. of Communications Engineering, National Taipei University, New Taipei City 23741, Taiwan

Abstract: In this paper, the median-based edge-preserving (EP) modification is revisited and has been shown to improve effectively the quality of a reconstructed Bayer CFA image in terms of the composite peak signal-to-noise ratio (CPSNR) performance index. Along this research direction, we propose an EP-modified signal-correlation-based (SCB) post-processing, called EP-SCB, as an enhancement to any existing interpolation method. The Bayer image reconstruction system we consider thus consists of two operational phases. The first phase performs an initial estimation of the missing RGB colors by using an existing interpolation method, while the second phase applies EP-SCB post-processing. Since a certain class of images may not fulfill the premise of having small variations in local color difference, which is assumed by SCB-type interpolation, thereby resulting in a deterioration in CPSNR after EP-SCB post-processing, a threshold test on local variance ratio is also devised to conditionally switch off the second phase. Experimental results show that the EP-SCB post-processing with variance ratio test gives a worse average CPSNR than the original interpolation methods tested in none of the Kodak and IMAX image groups experimented.

1. Introduction

Charge-coupled devices (CCDs) and complementary metal oxide semiconductor (CMOS) sensors are widely used for converting light into electronic signals. Due to technology advances of these devices, digital cameras and digital video recorders nowadays mostly equip with CCDs or CMOS sensors as their image sensors.

For digital cameras and digital video recorders, it would be perfect to use three image sensors to collect the color information of a pixel: one for the red channel, one for the green channel, and the remaining one for the blue channel. In such a three-sensor structure, a light path of equal length for each color channel and a color-splitting mechanism such as trichroic prisms are usually necessary. As a consequence, the camera may become bulky and its cost increases.

An alternative and perhaps more cost-economic design for a camera is to use one image sensor for each pixel instead of three. In this structure, only one color is recorded for a pixel, and the remaining two colors are estimated in terms of the available color information of nearby pixels. Its implementation then requires placing tiny color filters over an image sensor, where the arrangement of these color filters constitutes the so-called color filter array (CFA).

Since human eyes are more sensitive to green color, it is technically reasonable for a CFA to have more green filters than either red or blue ones. A typical CFA pattern, called the Bayer CFA,
After the light passes through the CFA, only one color is sensed and converted to a digital value for each pixel. The camera must restore the digital values of the other two colors by an interpolation technique. For this reason, the study of an efficient and accurate interpolation method is of both theoretical and practical importance.

Two basic interpolation methods for Bayer CFA images are the bilinear (BI) interpolation and the edge-directed interpolation [1, 2, 3]. These two methods regard the three colors as independent information and estimate the values of the missing colors by the known values of the same color around them.

It can be anticipated that the three colors in a real image exhibit a certain degree of correlation. Thus exploiting the inter-channel information will lead to an interpolation algorithm that generates better images. Two correlation models have been used for the CFA data: the color difference model and the color ratio model. The color difference model asserts that the pairwise differences between red, green and blue color values vary only slightly in a small local area. On the other hand, the color ratio model, as its name reveals, is based on the premise that the pairwise ratios between different color values in a pixel remain constant in a small local area, for which a well-known example is the normalized color-ratio modeling proposed by Lukac and Plataniotis in 2004 [4]. Experiments show that both correlation models are effective in characterizing Bayer CFA images when images exhibit mostly low spatial frequency content, but the color difference model is more widely used in practice because it usually leads to a more computationally efficient interpolation approach.

Examples of applying the color difference model to the CFA data are presented below. In 1988, Freeman proposed to use a median filter upon one-dimensional color differences [5]. In 2003, based on the color difference model, a linear interpolation method was devised by Pei and Lam, and is called the signal-correlation-based (SCB) interpolation [1]. In the same year, Lu and Tan offered another color-difference-based linear interpolation method, where interpolative weighting coefficients are calculated based on edge sensing information [6]. Very recently, Yu et al. proposed a color image demosaicking scheme using joint intra and inter channel information in terms of color difference modeling [7]. A comparison of these interpolation methods except the most recent one can be found in [8].

Alternatively, other researchers, such as Pekkucuksen and Altunbasak [9], proposed to simul-
taneously stress edges in the interpolation. The model that Pekkucuksen and Altunbasak used is often referred to as the spectral-spatial correlation (SSC) model and can be regarded as an extension of the inter-channel correlation model. Further studies based on the SSC model are given below. In 2007, an SSC-based interpolation was proposed by Tsai and Song in their two-stage approach [10]. In 2008, Chung et al. proposed to combine the SSC approach with the Sobel-mask-based gradient edge detection [11]. In 2009, Li and Randhawa combined several techniques, such as the consideration of weighted median with high order polynomial interpolation in different directions, into their proposed interpolation approach, and confirmed by simulations its effectiveness in preserving edge patterns [12].

None of the above works incorporate statistical or adaptive techniques. However, in certain situations, statistical or adaptive techniques may provide additional help to interpolation results. An example of the former is the interpolation approach proposed by Chang and Chen in 2007, where the missing color values are statistically estimated from the color differences available [13]. An example of the latter, as proposed by Paily et al., is the one that employs spatially adaptive interpolation for both noiseless and noisy CFA data [14], which is referred to as the local polynomial approximation combining with intersection of confidence intervals (LPA-ICI) in their paper and has been shown to provide very good color interpolation. An example of a mixture of both techniques is the adaptive window estimation scheme proposed by Kim et al. [15].

Recently, new techniques have been introduced to the restoration of CFA images such as the residual interpolation (RI) [16] and minimized-Laplacian residual interpolation (MLRI) [17], where residuals from tentative estimation of missing color values are used in interpolation operation instead of color differences. The residual interpolation accordingly can be regarded as an extension of color difference interpolation. Based on the same notion, interactive residual interpolation (IRI) [18] was later proposed with iterative deployment to the three color channels in order to generate a more accurate reconstruction of the target image. By noting that color differences are not necessarily lowpass signals, Jaiswal et al. exploited the correlations among color channels separately for high and low frequency components through the use of high pass and low pass filters, and proposed the inter-color correlation (ICC) [19] interpolation in late 2014. In particular situations where local geometry of images is ambiguous, Duran and Buades showed that improvement of image quality can be acquired by balancing local channel correlation with self-similarity of non-local image content, and referred their approach as self-similarity and spectral correlation adaptive algorithm (SSCA) [20]. Challenged the problem from a different angle, Wu et al. employed the information of efficient regression priors and devised a directional difference regression based demosaicing scheme [21].

In this paper, we first revisit our previously proposed edge-preserving (EP) modification to BI and SCB interpolations in [22], and show by more extensive examinations its effectiveness in preserving the edge patterns of high spatial frequency nature. Specifically, EP modification turns the original linear BI and SCB interpolations into nonlinear EP-BI and EP-SCB. It uses the median value in place of the mean value whenever the latter appears in interpolation computations. In order to realize why and how effectively the simple EP modification can preserve edge patterns, we apply our proposed approach onto circular zone plate (CZP) images and square zone plate (SZP) images. The CZP images contain a variety of spatial frequency components along every directions and hence can be used to examine how an interpolation approach responds to different spatial frequency components in a Bayer CFA image. Experiments show that such a simple modification considerably improves the edge-preserving effect of the original interpolation methods without the necessity of an explicit direction-detection mechanism; hence, it is called the edge-preserving (EP)
modification.

After confirming the effectiveness of EP modification in terms of the performance index of composite peak signal-to-noise ratio (CPSNR) through simulations over CZP images, we seek to provide an analytical footing. Since the pixel elements of an arrayed digital image are aligned in rows and columns, the interpolation manipulation is actually performed over a local rectangular-shape region. Thus, analysis of CPSNRs for a rectangular array of pixel quantities through CZP images that have constant circular contours is computationally involved. We then demonstrate that by using an SZP image instead, which varies its spatial frequency components in a similar manner to CZP images, together with mean-absolute-value (MAE) analysis, theoretical evidence for the effectiveness of EP modification can be obtained.

The above MAE analysis however cannot be applied to real images. An alternative examination is thus adopted to compare the capabilities of interpolation approaches in restoring image contents of different spatial frequencies for real images. Specifically, we pass the tested square image of size $N \times N$ through a two-dimensional ideal lowpass filter with cutoff frequency $f_c$ varying from 0, $\pi/N$, $2\pi/N$, ..., up to $\pi$. We then apply the interpolation approach to filtered images and confirm the anticipation that when the cutoff frequency increases, the MAE ratio of EP-BI against BI visually decreases.

We next turn to the proposed EP-SCB post-processing with variance ratio test. After verifying that EP-SCB is a better interpolation approach than SCB via simulations, we further found, as a follow-up work to [23], that EP-SCB can co-work in a tandem manner with any existing interpolation approach to further improve the image quality. Since the edge-preserving effectiveness is generally considered a key index for most interpolation methods, and due to the simplicity of its implementation, applying EP-SCB onto an image restored by an existing interpolation method may help improving the image quality without adding too much computational effort. This motivates our proposal of using EP-SCB as a post-processing for other interpolation approaches.

Preliminary experiments nevertheless suggest that there exists a certain class of images violating our anticipation, where EP-SCB post-processing indeed degrades slightly the reconstruction quality of the interpolated image. The key reason for this anti-anticipation is that SCB interpolation assumes the color difference model; therefore, for images that do not fulfill the premise of such model, EP-SCB post-processing might result in a rebuilt image of reduced quality, particularly for images with high color-difference variations in local areas. We thus devise a simple local variance ratio test on color differences that can help deciding which of the two reconstructed images respectively obtained before and after EP-SCB post-processing should be retained, and which should be disregarded.

In order to examine the effectiveness of the proposed EP-SCB post-processing with local variance ratio test, we use 24 Kodak images [24] and 18 IMAX images [25, 26] in our experiment. Five existing interpolation methods are tested, which are HA [27], LPA-ICI [14], MLRI [17], ICC [19] and EP-SCB itself. Note that HA is the acronym of Hamilton and Adams, who authored the patent of adaptive color plan interpolation, and is commonly used as a benchmark in the literature of image reconstruction technology [16, 17, 20]. Experimental results indicate that the EP-SCB post-processing with variance ratio test gives a worse average CPSNR than the original interpolation methods tested in none of the Kodak and IMAX image groups experimented.

The rest of the paper is organized as follows. Section 2 introduces the background relevant to SCB interpolation. Section 3 presents the EP modification on SCB, followed by the experimental results in Section 4. Section 5 addresses the proposed EP-SCB post-processing with local variance ratio test. The experimental results regarding EP-SCB post-processing and local variance ratio test
Figure 2. A sample Bayer CFA pattern used for SCB interpolation. Thirteen nearby pixels with color background are involved for the generation of the missing B7 and G7 values at the center position.

are given in Section 6. Conclusions are drawn in Section 7.

2. Preliminary

For completeness, the necessary background on SCB interpolation [1] is addressed in this section, based on which the proposed EP-SCB post-processing can be presented in later sections.

A straightforward method to obtain the missing color values in a Bayer CFA pattern, often referred to as bilinear (BI) interpolation, is to average all the nearby pixel values of the same color. However, since it considers no signal correlation among red, green and blue channels, such a straightforward method, although very simple in its implementation, produces less accurate estimates and the edges in resulting images may become jagged after being BI-interpolated from their Bayer CFA counterparts.

In order to alleviate the unwanted color patterns caused by separate interpolation across red, green and blue channels, Pei and Tam proposed to interpolate the Bayer CFA image according to an empirical color difference model of RGB signal correlation [1]. In particular, they proposed to create auxiliary $K_r$ and $K_b$ channels as exemplified below:

\[
\begin{align*}
K_r^2 &= \text{unused} \\
K_b^2 &= (G_b + G_e + G_3 + G_6)/4 - B_2 \\
K_r^3 &= G_3 - (R_1 + R_7)/2 \\
K_b^3 &= G_3 - (B_2 + B_4)/2 \\
K_r^6 &= G_6 - (R_5 + R_7)/2 \\
K_b^6 &= G_6 - (B_2 + B_10)/2 \\
K_r^7 &= (G_3 + G_6 + G_8 + G_11)/4 - R_7 \\
K_b^7 &= \text{unused}
\end{align*}
\]

where the pixel positions in the above formulas are defined in Figure 2. As seen from Eqs. (1)–(4), $K_r$ is exactly the estimate of the color difference between green and red channel values, while $K_b$
estimates the color difference between green and blue channels. Note that no computations are necessary for $K_r^2$ and $K_b^7$ since they are unused in SCB interpolation.

With these auxiliary $K_r$ and $K_b$ channel values, the missing color values for positions $B_2$ and $R_7$ are respectively given by:

\[
\begin{align*}
\hat{G}_2 &= B_2 + (K_b b + K_b e + K_b 3 + K_b 6)/4 \\
\hat{R}_2 &= \hat{G}_2 - (K_r a + K_r 1 + K_r 5 + K_r 7)/4
\end{align*}
\] (5)

and

\[
\begin{align*}
\hat{G}_7 &= R_7 + (K_r 3 + K_r 6 + K_r 8 + K_r 11)/4 \\
\hat{B}_7 &= \hat{G}_7 - (K_b 2 + K_b 4 + K_b 10 + K_b 12)/4
\end{align*}
\] (6)

The missing $R$ and $B$ channel values for position $G_3$ are similarly calculated as:

\[
\begin{align*}
\hat{R}_3 &= G_3 - (K_r 1 + K_r 7)/2 \\
\hat{B}_3 &= G_3 - (K_b 2 + K_b 4)/2
\end{align*}
\] (7)

Note that the interpolated red and blue channel values for a position like $G_6$ can be likewise obtained as (7) and hence we omit their formulas.

Compared with BI interpolation that uses only 3-by-3 pixel mask to reconstruct the missing colors without auxiliary channels, SCB interpolation uses thirteen pixels to perform the same task. As an example in Figure 2, the thirteen colored pixels in positions $R_1, R_5, R_7, R_9, R_3, G_3, G_6, G_8, G_11, B_2, B_4, B_10$ and $B_12$ are involved for the generation of $\hat{G}_7$ and $\hat{B}_7$. As a consequence, SCB interpolation generally results in better reconstruction images than BI interpolation.

3. Edge-preserving (EP) modification

Both EI and SCB interpolation methods introduced in the previous section are based on average operation. Hence, they both have the so-called edge-aliasing side effect. However, we found that this side effect can be considerably alleviated simply by removing the outliers in the average operation. Specifically, when averaging more than two values, we propose to alternatively delete the largest and the smallest ones until either one or two values remain. The resulting interpolated value is then given by either the remaining one or the average of the remaining two, which is exactly the median of those values involved in BI and SCB interpolations. The resulting interpolation methods are referred as EP-BI and EP-SCB, where EP stands for edge-perserving (EP) modification due to its capability in preserving the edge patterns of high spatial frequency nature.

Clearly, the proposed EP modification refines, for example, the first sub-equation in (6) to:

\[
\hat{G}_7 = R_7 + \frac{1}{2} \left( K_r 3 + K_r 6 + K_r 8 + K_r 11 - \overline{K}_r - \underline{K}_r \right),
\]

where

\[
\overline{K}_r \triangleq \max\{K_r 3, K_r 6, K_r 8, K_r 11\}
\]

and

\[
\underline{K}_r \triangleq \min\{K_r 3, K_r 6, K_r 8, K_r 11\}.
\]
Eq. (7) stays unchanged because it averages only two numbers. Since the same EP modification applies for all other formulas in SCB interpolation, we omit their presentation.

The anticipated edge-preserving capability of EP modification can be elucidated as follows. For an "edge" pattern in a $3 \times 3$ window, the neighbor pixel values appear to lie mostly in two categories, which are the high value category and the low value category. In a certain pattern such as the upper-left $3 \times 3$ window in Figure 2, when three of the four red color neighbors (i.e., $R_a$, $R_1$, $R_5$ and $R_7$) lie in the high (respectively, low) value category but the value of the remaining one is low (respectively, high), the center pixel should be interpolated as a high (respectively, low) value. This is exactly what EP modification does. In other cases, where the center pixel is surrounded by either four high values or four low values or two high values plus two low values, the proposed EP modification is expected to result in an interpolated center value similar to that obtained by the original average operation. This indicates that improvement in CPSNR by EP modification can be acquired at edge regions. Our experiments in the next section will confirm such an anticipation in a quantitative manner.

4. Experimental results on edge-preserving (EP) modification

In this section, experimental results for EP modification are illustrated. The images used in the experiments are CZP and SZP images of size $512 \times 512$ pixels as well as five real images selected from 24 Kodak images of size $512 \times 768$ pixels [24].

For a pair of source image $f(i,j)$ and reconstructed image $\hat{f}(i,j)$, where $1 \leq i \leq M$ and $1 \leq j \leq N$, the mean squared error (MSE) is given by:

$$\text{MSE} = \frac{\sum_{i=11}^{M-10} \sum_{j=11}^{N-10} [f(i,j) - \hat{f}(i,j)]^2}{(M-20)(N-20)},$$

where 10 border pixels are excluded in the MSE computation since the interpolated masks in these positions cover some non-existing pixels and hence the resulting interpolated color values of border pixels are by no means accurate [9, 14]. The composite mean squared error (CMSE) is then defined by:

$$\text{CMSE} \triangleq \frac{1}{3} (\text{MSE}_R + \text{MSE}_G + \text{MSE}_B),$$

where $\text{MSE}_R$, $\text{MSE}_G$ and $\text{MSE}_B$ respectively stand for the MSEs in red, green and blue channels. The key performance index adopted in this paper, i.e., CPSNR [28], can be calculated as:

$$\text{CPSNR} = 10 \log_{10} \left( \frac{255^2}{\text{CMSE}} \right),$$

where the number 255 in the numerator is the largest possible color value at a pixel. Based on these settings, we proceed to compare the performances of BI and SCB with EP-BI and EP-SCB interpolations.

4.1. Circular zone plate (CZP) image test

In this test, a CZP image of size $N \times N$ is generated according to

$$f_{CZP}(i,j) = \frac{255}{2} \cos \left( \frac{f_{\text{max}}}{N} \left( \left( i - \frac{N+1}{2} \right)^2 + \left( j - \frac{N+1}{2} \right)^2 \right) \right) + \frac{255}{2},$$

7
Figure 3. Results for CZP image test. (a) Original CZP image. (b) & (c) Zoomed image details corresponding to the small box area marked on the original image respectively for two different interpolation methods. It can be observed that the edge becomes zigzagged like saw teeth after being SCB interpolated, while EP-SCB preserves a clearer shape of arcs due to the removing of outliners in the average operations.

where we set \( N = 512 \) and \( f_{\text{max}} = \pi/5 \), and both \( i \) and \( j \) range from 1 to \( N \). This \( f_{\text{CZP}}(i, j) \) value is used for all three color channels, as depicted in Figure 3(a), from which it can be observed that the farther the position is from the center, the higher the spatial frequency. Aliasing is then expected to occur more seriously at higher spatial frequency pixels.

Figures 3(b) and 3(c) display the resultant interpolated images corresponding to the small box area for SCB and EP-SCB interpolations, respectively. It can be observed that EP modification helps preserving a better shape of arcs due to the removing of outliners in the average operations [22]. In particular, the resulting CPSNR increases from 36.65 to 37.88 when EP modification is applied to SCB interpolation.

4.2. Square zone plate (SZP) based mean-absolute-value (MAE) analysis

Based on a similar concept to (10), an SZP image can be defined via

\[
f_{\text{SZP}}(i, j) = \frac{255}{2} \cos \left( \frac{f_{\text{max}}}{N} \left[ \left( i - \frac{N + 1}{2} \right)^2 \vee \left( j - \frac{N + 1}{2} \right)^2 \right] \right) + \frac{255}{2}
\]

where "\( \vee \)" denotes the maximum operator. A sample SZP image is depicted in Figure 4. Notably, the resulting CPSNRs of CZP and SZP images for the same interpolation approaches are close to each other, confirming that both types of images vary their frequency components in a similar manner.
Figure 4. A sample SZP image with $N = 512$ and $f_{\text{max}} = \pi/5$.

Figure 5. 1/8 part of an SZP image with $i = N/2 + 1, \ldots, N$ (from top to bottom) and $j = N/2 + 1, \ldots, N$ (from left to right).

For MAE analysis of SZP images, it suffices to consider only 1/8 part of the image as shown in Figure 5, in which area the SZP formula is simplified to:

$$f_{\text{SZP}}(i, j) = \alpha \cos \left( \beta (j - j_0)^2 \right) + \alpha,$$

(11)

where $\alpha = 255/2$, $\beta = f_{\text{max}}/N$, and $j_0 = (N + 1)/2$. Since the MAE difference between BI and EP-BI interpolation outputs is mainly contributed by the green channel, we will focus only on the analysis of this channel. Analytical results for other channels can be likewise obtained.

For the four neighboring positions of green color, illustrated in Figure 6, it can be obtained from (11) that

$$\begin{align*}
    f_{\text{SZP}}(i, j - 1) &= \alpha \cos \left( \beta (j - j_0 - 1)^2 \right) + \alpha; \\
    f_{\text{SZP}}(i, j + 1) &= \alpha \cos \left( \beta (j - j_0 + 1)^2 \right) + \alpha; \\
    f_{\text{SZP}}(i - 1, j) &= \alpha \cos \left( \beta (j - j_0)^2 \right) + \alpha; \\
    f_{\text{SZP}}(i + 1, j) &= \alpha \cos \left( \beta (j - j_0)^2 \right) + \alpha,
\end{align*}$$
The resulting MAE is equal to:

$$\text{MAE}_{\text{BI}}(i, j) = |f_{\text{SZP}}(i, j) - \hat{f}_{\text{BI}}(i, j)|$$

$$= \frac{\alpha}{4} \left| \cos \left[ \beta(j - j_0 - 1)^2 \right] + \cos \left[ \beta(j - j_0 + 1)^2 \right] + 2 \cos \left[ \beta(j - j_0)^2 \right] \right| + \alpha.$$ 

Similarly, the output of EP-BI interpolation for the green channel is given by:

$$\hat{f}_{\text{EP-BI}}(i, j) = \begin{cases} 
\alpha \cos[\beta(j - j_0)^2] + \alpha, & \text{for Case I;} \\
\frac{\alpha}{2} \left( \cos \left[ \beta(j - j_0 + 1)^2 \right] + \cos \left[ \beta(j - j_0)^2 \right] \right) + \alpha, & \text{for Case II;} \\
\frac{\alpha}{2} \left( \cos \left[ \beta(j - j_0 - 1)^2 \right] + \cos \left[ \beta(j - j_0)^2 \right] \right) + \alpha, & \text{for Case III,}
\end{cases}$$

where Cases I, II and III respectively correspond to whether \( f_{\text{SZP}}(i \pm 1, j), f_{\text{SZP}}(i, j + 1), \) or \( f_{\text{SZP}}(i, j - 1) \) is the middle number of these three SZP image values. Thus, the MAE due to EP-BI interpolation is given by:

$$\text{MAE}_{\text{EP-BI}}(i, j) = \left| f_{\text{SZP}}(i, j) - \hat{f}_{\text{EP-BI}}(i, j) \right|$$

$$= \begin{cases} 
0, & \text{for Case I;} \\
\frac{\alpha}{2} \cos \left[ \beta(j - j_0 + 1)^2 \right] - \cos \left[ \beta(j - j_0)^2 \right], & \text{for Case II;} \\
\frac{\alpha}{2} \cos \left[ \beta(j - j_0 - 1)^2 \right] - \cos \left[ \beta(j - j_0)^2 \right], & \text{for Case III.}
\end{cases}$$

As a result, the MAE difference between BI and EP-BI can be calculated as:

$$\text{MAE}_{\text{EP}}(i, j) - \text{MAE}_{\text{EP-BI}}(i, j)$$

$$= \begin{cases} 
\frac{\alpha}{4} \cos \left[ \beta(j - j_0 - 1)^2 \right] + \cos \left[ \beta(j - j_0 + 1)^2 \right] \\
-2 \cos \left[ \beta(j - j_0)^2 \right], & \text{if Condition } \mathcal{M} \text{ holds;}
\end{cases}$$

$$\frac{\alpha}{4} \cos \left[ \beta(j - j_0 - 1)^2 \right] - \cos \left[ \beta(j - j_0 + 1)^2 \right], \text{ otherwise}$$

where Condition \( \mathcal{M} \) (equivalently, Case I) dictates that \( f_{\text{SZP}}(i \pm 1, j) \) lies between \( f_{\text{SZP}}(i, j - 1) \) and \( f_{\text{SZP}}(i, j + 1) \). Similar result can be obtained for the remaining 7/8 parts of the SZP image. The derivation indicates that for all \( 1 \leq i \leq N \) and \( 1 \leq j \leq N \),

$$\text{MAE}_{\text{EP}}(i, j) \geq \text{MAE}_{\text{EP-BI}}(i, j),$$

and that EP-BI improving BI in MAE is not limited to those anticipated edge patterns but actually is guaranteed over the entire SZP image. We conclude this analysis by pointing out that Condition
Figure 6. Four green-channel-neighboring positions used in BI interpolation for the center pixel $(i, j)$. The four corner positions are red channels, the center position is blue channel, and the remaining four neighboring positions are green channels, of which the indices are indicated at the bottom.

Figure 7. MAE ratios of EP-BI against BI as a function of $f_{\text{max}}$. The best-fit lines in least square sense are plotted in dotted black. The formulas of the best-fit lines for MAE ratios are $y = 0.0010x + 0.4904$ and $y = 0.0013x - 0.0180$ for CZP and SZP images, respectively. Also depicted is the percentage of occurrence of Condition $\mathcal{M}$ for SZP images.

$\mathcal{M}$, in which case EP-BI makes perfect reconstruction, occurs 5.6 times more often than the otherwise case (i.e., Cases II and III) in the SZP image parameterized with $N = 512$ and $f_{\text{max}} = \pi/5$; hence, a big improvement in the overall MAE is obtained, resulting an MAE ratio of:

$$\frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \text{MAE}_{\text{EP-BI}}(i, j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} \text{MAE}_{\text{BI}}(i, j)} \approx 0.0979.$$  

An interesting empirical observation is that the MAE ratio of EP-BI against BI for SZP images appears to be a near-linear function with respect to $f_{\text{max}}$. More strikingly, such MAE ratio becomes an exact linear function of $f_{\text{max}}$ for CZP images, as indicated in Figure 7. This seems somehow contrary to what we have anticipated that EP-BI improves BI more for images with more high-spatial-frequency contents. The explanation of this anti-intuition phenomenon is that Condition $\mathcal{M}$ occurs less frequent when $f_{\text{max}}$ grows (See the percentage of occurrence of condition $\mathcal{M}$ for SZP images in Figure 7). As a consequence, even though EP-BI improves BI in MAE in every pixel of SZP images, the overall quantitative improvement attainable is still case-sensitive and depends on the image compositions of different spatial frequencies.
4.3. Filtering test for real images

Clearly, the above MAE analysis is hard to be applied to real images, nor can it be extended to MSE. In addition, examining the MAEs empirically for real images can only confirm that EP-BI improves BI as a whole, but could not verify the claim that EP-BI improves the edge-preserving capability of BI and is thus so named.

Along with the MAE analysis, we herein propose a simple way applicable to real images to compare the capabilities of BI and EP-BI in restoring image contents of different spatial frequencies. Specifically, we pass a square image of size $N \times N$ through a two-dimensional ideal lowpass filter with cutoff frequency $f_c$, varying from 0, $\pi/N$, $2\pi/N$, ..., up to $\pi$. At the filter output, the image is normalized by a multiplicative factor of $\pi^2/(\pi - f_c)^2$ to maintain a constant energy in the sense of $\sum_{i=1}^{N} \sum_{j=1}^{N} |f(i,j)|^2$. The Bayer CFA counterparts of these filtered images are then generated for MAE test of BI and EP-BI.

Experimenting the proposed filtering test to CZP images with $N = 512$ and $f_{\text{max}} = \pi/5$ confirms the anticipation that when the cutoff frequency increases, the MAE ratio of EP-BI against BI visually decreases despite of some momentary fluctuation, as summarized in Figure 8, and this ratio converges to its ultimate value in Figure 7 when $f_c$ is beyond 0.225$\pi$. A similar trend to CZP images can be observed from Figure 8 for five real images selected from Kodak [24] and IMAX [25, 26]. These experiments support that EP modification can help improving the edge-preserving capability of a linear interpolation approach.

**Figure 8. MAE ratios of EP-BI against BI for CZP and five real images from Kodak and IMAX**
5. Improving an existing Interpolation approach by edge-preserving signal-correlation-based (EP-SCB) post-processing

Continued from the previous section, we proceed to examine how EP-SCB improves SCB in terms of CMSE performance index. Among 24 Kodak images of size $512 \times 768$ [24] and 18 IMAX images of size $500 \times 500$ [25, 26], EP-SCB gives a better CMSE than SCB in 41 out of 42 tested images as can be observed from the CMSE ratios of EPSCB against SCB in Figure 10. A closer look at this figure even reveals that for most images, a large quantitative improvement in CMSE can be obtained by applying EP modification onto SCB, yet for the single image that degrades in quality, only a slight decrease in CMSE is resulted. This experiment demonstrates that EP-SCB can result in a much better reconstructed image than SCB.

When further investigating EP-SCB interpolation for possible improvement, we found that the CPSNRs of the tested images can be increased more if the color differences $K_r$ and $K_b$ are more accurately estimated, particularly for those images fulfilling the premise of color difference model. In the extreme case, if the ideal $K_r$ and $K_b$ values (in the sense that they are obtained from the original source image) are used, EP-SCB can reconstruct exactly the CZP and SZP images with zero CMSEs.

This finding motivates our proposal of adopting EP-SCB as a post-processing for an existing interpolation method (due to which better $K_r$ and $K_b$ values can possibly be provided) in order to further improve the image equality. Since there do exist certain real images that seemingly violate the premise of color difference model, an additional variance ratio test is elaborately designed to decide whether the EP-SCB post-processed image should overwrite the interpolated image without EP-SCB post-processing.

5.1. Edge-preserving signal-correlation-based (EP-SCB) post-processing

The EP-SCB post-processing scheme we propose consists of two operational phases. In the first phase, an initial estimation of the missing RGB colors is performed by using an existing interpolation method. Four state-of-the-art interpolation methods are tested, which are HA [27], LPA-ICI [14], MLRI [17] and ICC [19]. The second phase applies EP-SCB post-processing with Eqs. (1)–(4) replacing by

$$
\begin{align*}
K_{r2} &= \text{unused} \\
K_{b2} &= \hat{G}2 - B2 \\
K_{r3} &= G3 - \hat{R}3 \\
K_{b3} &= G3 - \hat{B}3
\end{align*}
$$
Figure 10. CMSE ratios of EP-SCB against SCB for real images. Indices 1 to 24 correspond to images Kodak01 to Kodak24 [24], and indices 25 to 42 number images IMAX01 to IMAX18 [25, 26].

\[
\begin{align*}
K_r6 &= G6 - \hat{R}6 \\
K_b6 &= G6 - \hat{B}6 \\
K_r7 &= \hat{G}7 - R7 \\
K_b7 &= \text{unused}
\end{align*}
\]

where \(\hat{G}2, \hat{R}3, \hat{B}3, \hat{R}6, \hat{B}6\) and \(\hat{G}7\) are the interpolated color values obtained from the first phase.

5.2. Local variance ratio test

The SCB interpolation is designed based on the color difference model. For images that do not fulfill the premise of such model, the performance of its interpolation may degrade. At this background, our EP-SCB post-processing might also result in a rebuilt image of reduced quality for images with high color-difference variations in local areas.

Since the true RGB values of the source image are unknown to the interpolation process, only the local variances of color differences of the interpolated images before and after EP-SCB post-processing can be obtained. Through experiments, we sense that improvement of the image quality in terms of CPSNR by EP-SCB post-processing often comes with a decrease of the local variances of color differences. In other words, SCB-type manipulation tends to converge the images towards its assumed color difference model. Therefore, when the local variances of color differences of the interpolated image before EP-SCB post-processing are already small, the color-difference variances of the EP-SCB post-processed images are inclined to increase rather than decrease, by which CPSNR improvement usually becomes less likely.

We thus devise a variance ratio test to decide which of the two reconstructed images, obtained before and after EP-SCB post-processing, respectively, should be retained. Specifically, we first compute the color differences of the entire reconstructed image:

\[
\begin{align*}
D_r(i, j) &= G(i, j) - R(i, j) \\
D_b(i, j) &= G(i, j) - B(i, j)
\end{align*}
\]

where \(R(i, j), G(i, j)\) and \(B(i, j)\) stand for red, green and blue channel values at position \((i, j)\).
of the image, respectively. The local color-difference variance of a pixel at position \((i, j)\) for \(1 < i < M\) and \(1 < j < N\) is then defined as:

\[
\begin{align*}
\sigma_r^2(i, j) &= \text{Variance}\{D_r(x, y)\} \\
\sigma_b^2(i, j) &= \text{Variance}\{D_b(x, y)\}
\end{align*}
\]

where in the above evaluation of variances, the ranges of \(x\) and \(y\) follow \(i - 1 \leq x \leq i + 1\) and \(j - 1 \leq y \leq j + 1\), i.e., \(\sigma_r^2(i, j)\) and \(\sigma_b^2(i, j)\) are the variances of color differences in a local \(3 \times 3\) window. The mean local variance of the entire reconstructed image of size \(M \times N\) is:

\[
\sigma^2 = \frac{1}{(M - 2)(N - 2)} \sum_{i=2}^{M-1} \sum_{j=2}^{N-1} \frac{\sigma_r^2(i, j) + \sigma_b^2(i, j)}{2}.
\]

Note that we exclude the border pixels in evaluating \(\sigma^2\) since computations of their local variances involve non-existing color-difference values.

After obtaining the mean local variances of the reconstructed images before and after EP-SCB post-processing, a threshold test on mean local variance ratio is used to decide which image should be chosen, for which the block diagram in the form of a flowchart is illustrated in Figure 9. It remains to decide the threshold \(\lambda\) for the variance ratio test, which in this work is chosen to be the minimizer of misclassification error from empirical hypothesis distributions based on a set of training images. In its practice, we divide the local variance ratios of training images into two disjoint subsets: One subset \(\Omega_0\) consists of those variance ratios whose corresponding images have a better CPSNR after EP-SCB post-processing, and the other subset \(\Omega_1\) contains the remaining. We then plot the empirical cumulative distribution function (CDF) of \(\Omega_0\) and the empirical complementary cumulative distribution function (CCDF) of \(\Omega_1\), which can be formulated as:

\[
F_0(x) = \frac{1}{|\Omega_0|} \sum_{r \in \Omega_0} I(r \leq x)
\]

and

\[
\bar{F}_1(x) = 1 - F_1(x) = \frac{1}{|\Omega_1|} \sum_{r \in \Omega_1} I(r > x),
\]

where \(I(\cdot)\) is the set indicator function. The \(\lambda\) value that satisfies \(F_0(\lambda) = \bar{F}_1(\lambda)\) then minimizes the empirical misclassification error under equal prior probability.


As mentioned previously, four state-of-the-art interpolation methods will be used as initial interpolation methods in the experiments, which are HA [27], LPA-ICI [14], MLRI [17] and ICC [19]. EP-SCB itself will also be used as the fifth one. Thus with 24 Kodak images and 18 IMAX images, there are \(5 \times (24 + 18) = 210\) combinations. We then choose the first 12 Kodak images and the first 9 IMAX images to be the training images. Among \(5 \times (12 + 9) = 105\) training runs, EP-SCB post-processing improves CPSNRs in 59 of them; hence, \(|\Omega_0| = 59\) and \(|\Omega_1| = 46\). CDF \(F_0(x)\) and CCDF \(\bar{F}_1(x)\) are accordingly evaluated and plotted in Figure 11. The two curves intersect at point \((1.0456, 6/46) = (1.0456, 0.1304)\). As a result, the threshold that reaches the minimum
Figure 11. Training runs to determine the variance-ratio threshold $\lambda = 1.0456$. Here, subset $\Omega_0$ consists of those variance ratios whose corresponding images have a better CPSNR after EP-SCB post-processing, and subset $\Omega_1$ contains the remaining. The integer numbers in the four table entries denote the numbers of tested cases that lie in the respective subset on top and also satisfy the variance ratio inequality on left, where the corresponding percentages with respect to the total number of tested cases are given in the parentheses.
Table 1. Testing runs with variance-ratio threshold $\lambda = 1.0456$. Here, $H_0$ denotes the cases that the CPSNR of the EP-SCB post-processed image improves that of the reconstructed image before performing EP-SCB post-processing, and $H_1$ is the hypothesis opposite to $H_0$. The integer numbers in the four table entries denote the numbers of tested cases that validate the respective hypothesis on top and also satisfy the variance ratio inequality on left, where the corresponding percentages with respect to the total number of tested cases are given in the parentheses.

<table>
<thead>
<tr>
<th></th>
<th>$H_0$</th>
<th>$H_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance ratios $\geq \lambda$</td>
<td>40 (38%)</td>
<td>6 (6%)</td>
</tr>
<tr>
<td>Variance ratios $&lt; \lambda$</td>
<td>11 (10%)</td>
<td>48 (46%)</td>
</tr>
</tbody>
</table>

Table 2. Average CPSNRs of all 42 tested images under ten different interpolation methods, where “PP” stands for “EP-SCB Post-Processing.” To facilitate their identifications, the larger CPSNR between those of the images before and after EP-SCB post-processing is boldfaced.

<table>
<thead>
<tr>
<th>Method</th>
<th>Kodak</th>
<th>IMAX</th>
<th>Kodak + IMAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA</td>
<td>35.34</td>
<td>36.81</td>
<td>35.03</td>
</tr>
<tr>
<td>HA+PP</td>
<td>36.91</td>
<td>39.28</td>
<td>36.63</td>
</tr>
<tr>
<td>EP-SCB</td>
<td>36.52</td>
<td>38.28</td>
<td>36.17</td>
</tr>
<tr>
<td>EP-SCB+PP</td>
<td>37.30</td>
<td>39.89</td>
<td>37.02</td>
</tr>
<tr>
<td>LPA</td>
<td>39.25</td>
<td>42.16</td>
<td>38.57</td>
</tr>
<tr>
<td>LPA+PP</td>
<td>39.20</td>
<td>42.32</td>
<td>38.57</td>
</tr>
<tr>
<td>MLRI</td>
<td>38.18</td>
<td>40.55</td>
<td>37.52</td>
</tr>
<tr>
<td>MLRI+PP</td>
<td>38.54</td>
<td>41.71</td>
<td>37.98</td>
</tr>
<tr>
<td>ICC</td>
<td>37.25</td>
<td>39.64</td>
<td>36.66</td>
</tr>
<tr>
<td>ICC+PP</td>
<td>38.41</td>
<td>41.32</td>
<td>37.87</td>
</tr>
</tbody>
</table>

misclassification error $(6 + 7)/105 = 0.1238$ is $\lambda = 1.0456$. We proceed to use this threshold to test the remaining 21 images. The classification results are summarized in Table 1, where the misclassification rate is shown to be $(6 + 11)/105 = 0.1619$.

The average CPSNRs, as well as average PSNRs for each color, of all 42 tested images under 10 different interpolation methods are listed extensively in Table 2. To facilitate their identifications, the larger CPSNR between those of the images before and after EP-SCB post-processing is boldfaced.

Several observations can be made in this experiment. First, the average CPSNR of the chosen group of images after performing EP-SCB post-processing with variance ratio test is worse than that of the images rebuilt only by the existing interpolation method in none of the 15 CPSNRs shown in Table 2, among which four remains the same. In particular, LPA gives the least average CPSNR improvement among the five interpolation methods under test. We wish to point out that among $5 \times 9 = 45$ PSNRs shown in Table 2, only three decrease their numbers by EP-SCB post-processing, which are average PSNR$_R$ for LPA applying to Kodak images, average PSNR$_R$ for ICC applying to IMAX images, and average PSNR$_R$ for LPA applying to all 42 images. It is interesting that all three degradations in PSNR occur at the red channel, which suggests that further improvement in EP-SCB post-processing may need to unequally treat red and blue channels. Second, the assumption of constant color difference in local areas seem to be less supported by IMAX images, and hence a more elaborate signal correlation model may need to be developed for these images.
Lastly, the proposed EP-SCB post-processing with variance ratio test generally perform well for Kodak images. The gain in CPSNR can be as large as 1.79 for HA and is at least 0.01 for LPA.

7. Conclusion

In this work, we revisited the EP modification on SCB interpolation and proposed a novel EP-SCB post-processing with variance ratio test to existing interpolation methods. We showed by analysis and extensive experiments on CZP images, SZP images, 24 Kodak images and 18 IMAX images that our proposal is effective in enhancing the image quality, especially for those that fit well to the color difference model.

As a summary, the EP modification was devised by replacing the average operation in SCB interpolation with the median operation. Despite its simplicity in implementation, our experiments for 42 real images from Kodak and IMAX indicated that the reconstructed images from EP-SCB interpolation do exhibit visible improvements in CMSE over the reconstructed images from SCB. In particular, 30 images under test can reach a 10% improvement in the CMSE ratio of EP-SCB against SCB, and over 40% CMSE-ratio improvement was even observed for two tested images. An MAE analysis based on SZP images was also provided to complement why EP modification is so effective in preserving the edge patterns of high spatial frequency contents, and a situation decided by nearly pixels, termed as Condition $\mathcal{M}$, was particular singled out to uphold the linearly growing trend of EP-BI/BI MAE ratios with respect to the maximum frequencies of CZP and SZP images.

After confirming the effectiveness of EP-SCB, we attempt to seek for further improvement. Surprisingly, we found that using EP-SCB interpolation as a post-processing to an existing interpolation approach can result in a reconstructed image of improving quality. Since some images may not satisfy the premise of color difference model assumed by SCB, a variance ratio test is devised to effectively exclude them. Experiments by applying the EP-SCB post-processing to five existing interpolations, which are HA [27], LPA-ICI [14], MLRI [17], ICC [19] and EP-SCB itself, confirmed that the averaged CPSNR was never decreased by the EP-SCB post-processing with variance ratio test, and only four among the 15 averaged CPSNRs under test remain the same. This coincides with what has been shown in Table 1 that the chance of keeping the EP-SCB post-processed images (respect to Variance ratios $\geq \lambda$) with a worse CPSNR than the image reconstructed without EP-SCB post-processing (respect to hypothesis $H_1$) is as low as 6%.

A future work of interest could be to take into consideration the local variances of several subdivided portions of images to further reduce the misclassification rate between images that support EP-SCB post-processing and images that do not. Investigation of the robustness of EP-SCB post-processing with variance ratio test against noise might potentially add the practical value of the approach.

8. References


