On the Optimistic Capacity of Arbitrary Channels

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Abstract — A formula for the optimistic capacity of arbitrary channels is established. It is shown to equal the supremum, over all input processes, of the input-output zero-sup-information rate. A general expression for optimistic ε-capacity is also provided.

I. OVERVIEW

The conventional definition of channel capacity C [1] requires the existence of reliable block codes for all sufficiently large blocklengths. Alternatively, if it is required that reliable codes exist for infinitely many blocklengths, a new, more optimistic definition of capacity is obtained [1]. This concept of optimistic capacity (denoted by C) has recently been investigated by Verdú et al. for arbitrary single-user channels [1, 2]. More specifically, they provide an (additional) operational significance for the optimistic capacity by demonstrating that for a given channel, the classical statement of the source-channel separation theorem holds for every source if and only if C = C [2]. They also conjecture that a simple expression for C does not exist.

In this paper, we answer the latter point by demonstrating that C does indeed have a general formula. The key to this result is the application of the generalized super-information rate introduced in [3] to the existing proofs by Verdú and Han [1] of the direct and converse parts of the conventional coding theorem. A general expression for the optimistic ε-capacity is also established.

II. ε-INF/SUP-INFORMATION RATES

Consider an input process X = (X(t))_{t=1}^{∞} [1]. Denote by Y = (Y(t))_{t=1}^{∞} the corresponding output process induced by X via the channel W = (W(t) = P_{X(t)}: X(t) → Y(t))_{t=1}^{∞}. In [4, 1], Han and Verdú introduce the notions of inf/sup-information/entropy rates and illustrate the role these measures play in proving general standard source/channel coding theorems. The inf-information rate I(X; Y) (resp. sup-information rate I̅(X; Y)) between processes X and Y is defined in [4] as the liminf in probability (resp. limsup in prob.) of the sequence of normalized information densities \( \frac{1}{n} \log P_{X,Y}^{(n)} \). A simple example of a channel for which C > C is as follows. Consider a nonstationary channel W such that at odd time instances n = 1, 3, ..., Wn is the transition distribution of a BSC with crossover probability 1/2; and at even time instances n = 2, 4, 6, ..., Wn is the transition distribution of a BSC with crossover probability 1/4. Then C = 0 and C = 1 - \( \log_2 (1/4) \) > 0.

In [5], we further illustrate the application of the generalized information measures of [3] by providing an optimistic general source coding theorem.

REFERENCES