Likelihood Ratio Partitions for Distributed Signal Detection in Correlated Gaussian Noise

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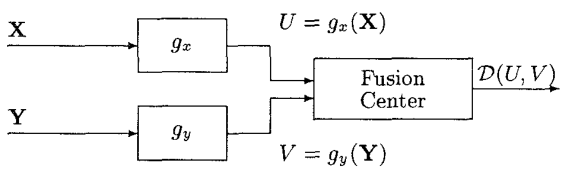
I. Introduction

A distributed detection system is considered in which two sensors and a fusion center jointly process the output of a random data source (see figure). It is assumed that the null and alternative distributions are spatially correlated Gaussian, differing in the mean; thus the random source is either noise only or a deterministic signal plus noise.



In the presence of spatial dependence, the joint optimization of local quantizers g_x, g_y, and global decision rule $\mathcal{D}$ may yield solutions in which g_x and g_y are *not* based on marginal likelihood ratio tests. This is one instance where distributed detection departs from the traditional statistical framework where likelihood ratios are sufficient for most purposes. This departure was first noted in [1], and was corroborated specifically for the additive Gaussian noise model by means of a counterexample [2] involving two-dimensional vectors $\mathbf{X}$ and $\mathbf{Y}$.

This work is an attempt to characterize noise models for which the optimal system employs marginal likelihood ratio tests. In the setup where each sensor draws one local observation (i.e., $\mathbf{X}$ and $\mathbf{Y}$ are scalars X and Y, respectively), we succeed in obtaining a sufficient condition on the noise mean and covariance under which the optimal binary quantizers are contiguous partitions of the marginal observation space. Since the marginal likelihood ratio is a linear function of the local observation (X or Y), this result implies that g_x and g_y are threshold-type functions of the marginal likelihood ratio. It also reduces the optimization to identifying break points (thresholds) in the marginal observation space.

We also examine whether the sufficient condition discussed previously is also necessary, and find that violation of this condition may in certain—but not all—cases render the contiguous marginal likelihood ratio partition suboptimal. We reach this conclusion by examining the special case where the noise marginals are the same for both sensors; the sufficient condition is then equivalent to positive correlation between X and Y. We find that for values of the correlation coefficient $\rho(X,Y)$ close to -1, local quantizers based on non-contiguous likelihood ratio partitions outperform those based on contiguous likelihood ratio partitions. We were not able to establish the same for $\rho(X,Y)$ close to 0^-.

Finally, we consider the following question. Assuming that the sufficient condition discussed previously is satisfied, does symmetry in the signal and noise models (same marginal for both sensors) imply symmetry in the optimal solution, with g_x and g_y being identical contiguous partitions of the real line? We find that this is indeed true, and in such cases, optimal design is further simplified.

II. Statement of Results

The observation statistics are denoted by

$$\begin{aligned} H_0 : P_{xy} &\sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}\right) \\ H_1 : Q_{xy} &\sim \mathcal{N}\left(\begin{pmatrix} \mu \\ \eta \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}\right). \end{aligned}$$

A Bayesian setting is assumed, in which H_0 and H_1 are assigned prior probabilities. Also, quantizers are binary throughout, i.e., $\|g_x\| = \|g_y\| = 2$.

Theorem 1 *If*

$$\sigma_{xy}(\eta\sigma_x^2 - \mu\sigma_{xy})(\mu\sigma_y^2 - \eta\sigma_{xy}) \ \geq \ 0, \qquad (1)$$

then there exist optimal quantizers of X and Y which are contiguous partitions of the real line.

Counterexample Assume a uniform prior. Let $\sigma_x^2 = \sigma_y^2 = \mu = \eta = 1$ and $\sigma_{xy} = -1$, so that (X,Y) lies on a straight line with probability 1 under each hypothesis. It can be shown that every contiguous binary partition of the real line is outperformed by noncontiguous one.

Remark The above counterexample clearly represents an extreme case where either of the local observations is a sufficient statistic for centralized testing. The same effect, however, can be obtained by choosing $\sigma_{xy} \approx -1$ and applying a continuity argument. A nondegenerate counterexample can then be constructed.

Theorem 2 *Let $\mu = \eta$ and $\sigma_x^2 = \sigma_y^2$. If the local quantizers are constrained to be binary contiguous partitions of the real line, then the optimal quantizer pair employs the same threshold in both quantizers.*

In conjunction with Theorem 1, the above theorem implies the following corollary.

Corollary 1 *Let the signal and noise models be symmetric. If $\sigma_{xy} \geq 0$, then an optimal solution exists in which both quantizers use the same contiguous partition of the observation space.*

References

[1] R. R. Tenney and N. R. Sandell, Jr. "Detection with distributed sensors." *IEEE Trans. Aerospace Electron. Syst.*, AES-17(4):501–510, July 1981.

[2] Po-Ning Chen. "Large deviations approaches to performance analysis of distributed detection systems." *Ph.D. Dissertation*, University of Maryland at College Park, 1994.

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