A Low-Complexity Channel Estimation Algorithm for OFDM Systems based on Parametric Channel Modeling

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Outline

- Motivation and Contribution
- System Model
- Proposed Channel Estimation Scheme
- Simulation Results
- Conclusion

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Motivation and Contribution

- Orthogonal frequency division multiplexing (OFDM) has been a well-known technique capable of supporting high-rate transmissions over multi-path fading channels.
- An OFDM modulator can convert a frequency-selective channel into multiple parallel flat fading links in the frequency domain.
- Toward coherent tone-by-tone symbol detection, channel state information (CSI) is needed at the receiver.
- Channel estimation for OFDM systems has received considerable attention in the past decade.

Motivation and Contribution

- Parametric-based channel estimation schemes [YLCC 01][1], [RBG 05][2]
  - Explicitly exploit multi-path characteristic of wireless channels for performance enhancement.
  - Estimation of path delays formulated in angle of direction-of-arrival DOA estimation setup.
  - Resort to the computationally intensive ESPRIT algorithm for channel parameters.

[1]B. Yang, K. B. Letaief, R. S. Clieng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, pp. 467-478, Mar. 2001.

[2]M. R. Raghavendra, S. Bhashyam, and K. Giridhar, "Exploiting hopping pilots for parametric channel estimation in OFDM systems," *IEEE Signal Processing Lett.*, vol. 12, pp. 737-740, Nov. 2005.

Motivation and Contribution

- In this thesis, we propose a low-complexity parametric-based OFDM channel estimatin scheme.
  - Exploit algebraic structure of the autocorrelation matrix of pilot data.
  - Certain submatrices contain sufficient information for channel parameter identification.
  - Advantage : processing with matrices with reduced dimensions $\Rightarrow$ Low algorithmic complexity compared with ESPRIT based solution.
  - Good performance
    $\Rightarrow$ Low channel mean square errors and low bit error rate.

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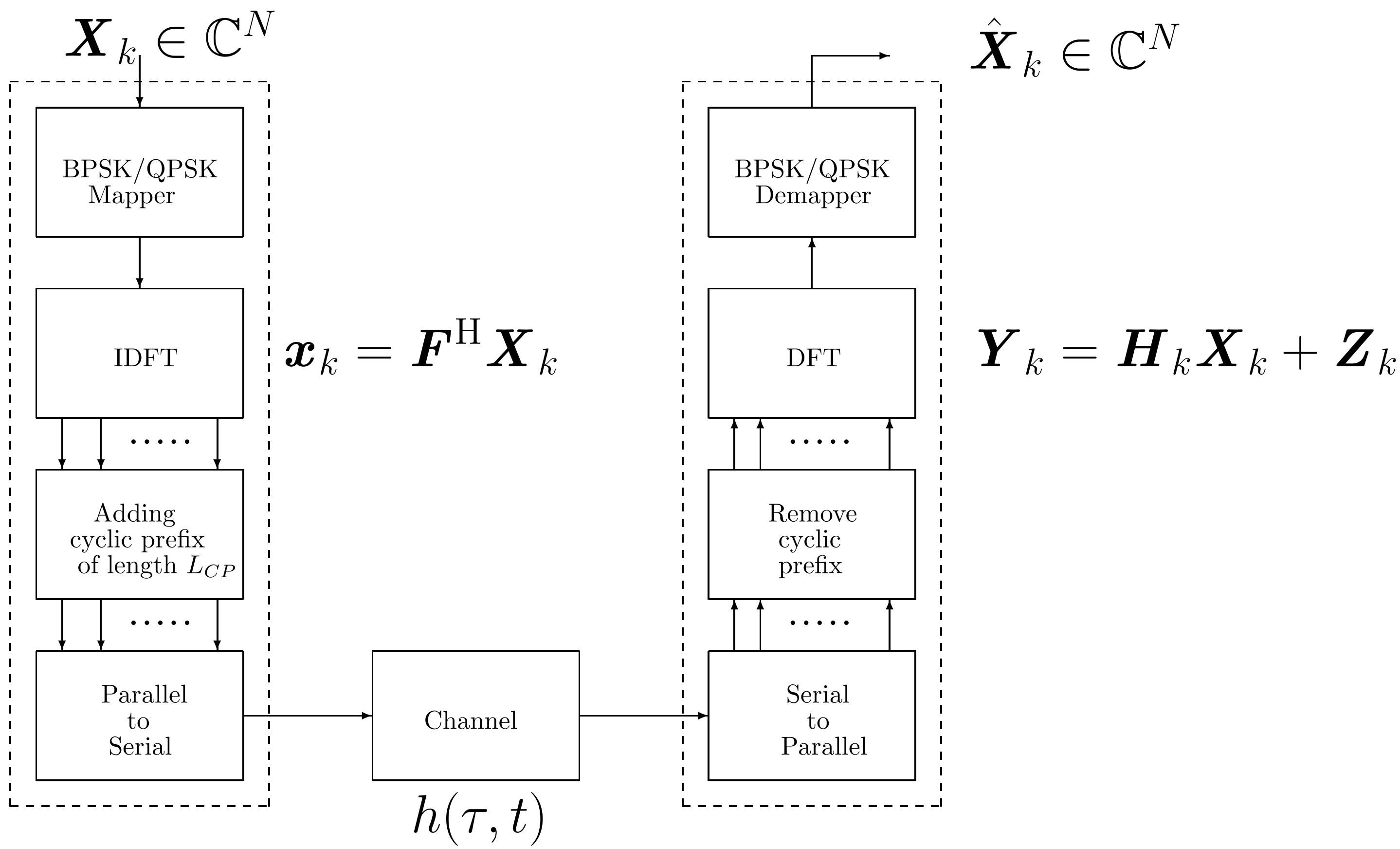
An OFDM System Structure Figure



Transmit Signal

- Let the kth OFDM symbol

$$\boldsymbol{X}_k = [X_{0,k}, X_{1,k}, ..., X_{N-1,k}]^T \in \mathbb{C}^N \tag{1}$$

$$\boldsymbol{x}_k = \boldsymbol{F}^{\mathrm{H}} \boldsymbol{X}_k \in \mathbb{C}^N \tag{2}$$

where

$$[\boldsymbol{F}^{\mathrm{H}}]_{m,n} = \frac{1}{\sqrt{N}}[e^{-j2\pi mn/N}] \in \mathbb{C}^{N \times N} \tag{3}$$

- The baseband signal can be expressed as

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{n}{T_s} t} \quad \text{for } t \in [0, T_s), \tag{4}$$

where T_s denotes the symbol duration.

- The signal $s_k(t)$ is then transmitted over an L-path wireless channel with impulse response

$$h(\tau, t) = \sum_{l=0}^{L-1} h_l(t)\delta(\tau - \tau_l), \tag{5}$$

where

- ◇ L denotes the number of paths.
- ◇ $\delta(\cdot)$ denotes Dirac delta function.
- ◇ h_l is the channel gain of the lth path.
- ◇ τ_l is the delay of the lth path.

Received Signal

- The received signal can expressed as

$$\begin{aligned} y_k(t) &= s_k(t) * h(\tau, t) + v_k(t) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{n}{T_s} t} \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi \frac{n}{T_s} \tau} d\tau + v_k(t) \quad (6) \end{aligned}$$

where $v(t)$ is the measurement noise.

Received Signal

- By sampling $y_k(t)$ in (6) at rate T_b, we get

$$\begin{aligned} y_{m,k} &= y_k(mT_b) + v_k(mT_b) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{nm}{N}} H_{n,k} + v_{m,k} \end{aligned} \tag{7}$$

- $H_{n,k}$ is the channel frequency response at the nth tone.

$$H_{n,k} = \sum_{l=0}^{L-1} h_{l,k} e^{-j2\pi \frac{n}{T_s} \tau_l} \tag{8}$$

- In matrix form

$$\boldsymbol{H}_k = [H_{1,k}, H_{2,k}, ..., H_{N,k}]^{\mathrm{T}} = \boldsymbol{F}_h \boldsymbol{h}_k \tag{9}$$

where

$$[\boldsymbol{F}_h]_{m,n} = e^{\frac{-j2\pi m \tau_n}{T_s}} \in \mathbb{C}^{N \times L} \tag{10}$$

$$\boldsymbol{h}_k = [h_{1,k}, h_{2,k}, ..., h_{L,k}]^T \in \mathbb{C}^L \tag{11}$$

Received Signal

- The received signal vector

$$\boldsymbol{y}_k = [y_{0,k}, y_{1,k}, ..., y_{N-1,k}]^T \in \mathbb{C}^N \tag{12}$$

- The received OFDM symbol in the frequency domain is

$$\begin{aligned}\boldsymbol{Y}_k &= [Y_{0,k}, Y_{1,k}, ..., Y_{N-1,k}]^T \\ &= \boldsymbol{F}\boldsymbol{y}_k + \boldsymbol{F}\boldsymbol{v}_k \\ &= \begin{bmatrix} H_{0,k} & 0 & \cdots & 0 \\ 0 & H_{1,k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{N-1,k} \end{bmatrix} \begin{bmatrix} X_{0,k} \\ X_{1,k} \\ \vdots \\ X_{N-1,k} \end{bmatrix} + \boldsymbol{F}\boldsymbol{v}_k, \end{aligned} \tag{13}$$

.

- Toward coherent detection, knowledge of the channel frequency response $H_{n,k}$'s at the receiver is needed.
  - ◇ Need to estimate $\{\tau_l\}$ and $\{h_{l,k}\}$.

Assumptions

The following assumptions are made in the sequel

- The path delay is **not necessary** an integer multiple of the sampling interval.
- The number of channel paths $L \leq L_{max}$, where L_{max} is a known an upper bound of L.
- The number of pilots P inserted in each OFDM symbol satisfies $P \geq L_{max} + 1$.
- The channel impulse response taps satisfy $E\{h_{l,k}(t_1)h^*_{m,k}(t_2)\} = \sigma_l^2 \delta(l-m)\delta(t_1 - t_2)$.
- The addtive noise samples $\{v_{m,k}\}$ are zero mean white Gaussian with variance σ^2.
- The channel gains $\{h_{l,k}\}$ are uncorrelated with the noise $\{v_{m,k}\}$.

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Channel Frequency Response on Pilot-tone

- [OG 04][3] has shown that MSE in the channel estimation achieves minimum if the pilot are equal-spaced and equal-powered on the frequency grid.
- The received data on the pilot subcarriers is

$$Y_{p_i,k} = X_{p_i,k} H_{p_i,k} + Z_{p_i,k}, \; 1 \leq i \leq P \tag{14}$$

where p_i's are indices of pilot tone and $Z_{p_i,k}$ is the p_ith entry of $\boldsymbol{F}\boldsymbol{v}_k$.

- The least squares (LS) estimates of the channel frequency responses are immediately obtained as

$$\hat{H}_{p_i,k} = \frac{Y_{p_i,k}}{X_{p_i,k}} = H_{p_i,k} + \nu_{p_i,k}, \; 1 \leq i \leq P \tag{15}$$

where $\nu_{p_i,k} = Z_{p_i,k}/X_{p_i,k}$.

[3] S.Ohno and G.B.Giannakis, "Capacity maximizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. Inform. Theory*, vol. 50, no. p, pp. 2138-2145, Sep, 2004.

Channel Frequency Response on Pilot-tone

- In a matrix form

$$\hat{\boldsymbol{H}}_{P,k} = \boldsymbol{H}_{P,k} + \boldsymbol{v}_{P,k} = \boldsymbol{F}_P \boldsymbol{h}_k + \boldsymbol{v}_{P,k} \quad (16)$$

where

$$\hat{\boldsymbol{H}}_{P,k} = [\hat{H}_{p_1,k}, \hat{H}_{p_2,k}, ..., \hat{H}_{p_P,k}]^T \in \mathbb{C}^P \quad (17)$$

$$\boldsymbol{H}_{P,k} = [H_{p_1,k}, H_{p_2,k}, ..., H_{p_P,k}]^T \in \mathbb{C}^P \quad (18)$$

$$\boldsymbol{v}_{P,k} = [\nu_{p_1,k}, \nu_{p_2,k}, ..., \nu_{p_P,k}]^T \in \mathbb{C}^P \quad (19)$$

$$\boldsymbol{F}_P = \begin{bmatrix} e^{\frac{-j2\pi\tau_1 0}{T_s}} & e^{\frac{-j2\pi\tau_2 0}{T_s}} & \cdots & e^{\frac{-j2\pi\tau_L 0}{T_s}} \\ e^{\frac{-j2\pi\tau_1 D}{T_s}} & e^{\frac{-j2\pi\tau_2 D}{T_s}} & \cdots & e^{\frac{-j2\pi\tau_L D}{T_s}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{\frac{-j2\pi\tau_1 (P-1)D}{T_s}} & e^{\frac{-j2\pi\tau_2 (P-1)D}{T_s}} & \cdots & e^{\frac{-j2\pi\tau_L (P-1)D}{T_s}} \end{bmatrix} \quad (20)$$

$\boldsymbol{F}_P \in \mathbb{C}^{P \times L}$ and $D = N/P$ denotes spacing between two adjacent pilot subcarriers.

Channel Estimation Scheme

- We propose a two-stage channel estimation scheme, in which the number of paths L and the path delays τ_l's are estimated first.
- The channel impulse response taps $h_{l,k}$ are then determined.
- In the following discussions, let us assume for the moment that the channel noise is absent.

Estimation of the Number of Channel Paths

- The autocorrelation matrix of the channel frequency response can be obtained as

$$\begin{aligned} \boldsymbol{R} &= E\left\{\boldsymbol{H}_{P,k}\boldsymbol{H}_{P,k}^{\mathrm{H}}\right\} \\ &= \boldsymbol{F}_P \Sigma \boldsymbol{F}_P^{\mathrm{H}} \end{aligned} \tag{21}$$

where $\Sigma = \mathrm{diag}\{\sigma_1^2, \sigma_2^2, ..., \sigma_L^2\} \in \mathbb{R}^{L \times L}$.

- Since $\mathrm{rank}(\boldsymbol{R}) = \mathrm{rank}\ (\Sigma) = L$, the number of channel paths can be estimated by the number of nonzero eigenvalues of $P \times P$ matrix $\boldsymbol{R}$.

Estimation of Channel Path Delays τ_l

- Let $\tilde{\boldsymbol{R}}$ be the first $(L+1) \times (L+1)$ principal submatrix of $\boldsymbol{R}$, thus

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{F}}_P \Sigma \tilde{\boldsymbol{F}}_P^{\mathrm{H}} \tag{22}$$

where $\tilde{\boldsymbol{F}}_P \in \mathbb{C}^{(L+1)\times L}$ consists of the first $(L+1)$ rows of $\boldsymbol{F}_P$.

- Let $\tilde{\boldsymbol{R}}_{(m,n)}$ be the matrix obtained by deleting the mth row and nth column of $\tilde{\boldsymbol{R}}$.

$$\tilde{\boldsymbol{R}}_{(m,n)} = \tilde{\boldsymbol{F}}_{P,(\backslash m)} \Sigma \tilde{\boldsymbol{F}}_{P,(\backslash n)}^{\mathrm{H}} \quad \text{for} \ \ 1 \le m, n \le L+1 \tag{23}$$

where $\tilde{\boldsymbol{F}}_{P,(\backslash m)}$ is obtained by removing the mth row of $\tilde{\boldsymbol{F}}_P$.

Estimation of Channel Path Delays τ_l

- **Key observation**

$$\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P,(\backslash 1)} = \Phi \tilde{\boldsymbol{F}}^{\mathrm{H}}_{P,(\backslash L+1)} \tag{24}$$

$$\Phi = \mathrm{diag}\{e^{-j2\pi\tau_1 D/T_s}, e^{-j2\pi\tau_2 D/T_s}, \ldots, e^{-j2\pi\tau_L D/T_s}\} \in \mathbb{C}^{L\times L} \tag{25}$$

- Combining (23) and (24), we get

$$\begin{aligned} \tilde{\boldsymbol{R}}_{(m,1)} &= \tilde{\boldsymbol{F}}_{P,(\backslash m)} \Sigma \tilde{\boldsymbol{F}}^{\mathrm{H}}_{P,(\backslash 1)} \\ &= \tilde{\boldsymbol{F}}_{P,(\backslash m)} \Sigma \Phi \tilde{\boldsymbol{F}}^{\mathrm{H}}_{P,(\backslash L+1)} \end{aligned} \tag{26}$$

and

$$\tilde{\boldsymbol{R}}_{(m,L+1)} = \tilde{\boldsymbol{F}}_{P,(\backslash m)} \Sigma \tilde{\boldsymbol{F}}^{\mathrm{H}}_{P,(\backslash L+1)} \tag{27}$$

Estimation of Channel Path Delays τ_l

- From (26) and (27)

$$\begin{aligned} \boldsymbol{U} &= (\tilde{\boldsymbol{R}}_{(m,L+1)}^{-1} \tilde{\boldsymbol{R}}_{(m,1)})^{\mathrm{H}} \\ &= \tilde{\boldsymbol{F}}_{P,(\backslash L+1)} \Phi^{\mathrm{H}} \tilde{\boldsymbol{F}}_{P,(\backslash L+1)}^{-1} \end{aligned} \tag{28}$$

- The eigenvalues λ_l's of $\boldsymbol{U}$ are precisely the diagonal entries of Φ^H, i.e, $\lambda_l = e^{-j2\pi\tau_l D/T_s}$.

- The path delays τ_i's can be estimated as

$$\hat{\tau}_i = \frac{arg\{\lambda_i^*\} T_s}{2\pi D} \text{ for } i = 0, 1, ..., \hat{L} - 1 \tag{29}$$

where $arg\{\lambda_i^*\}$ denotes the phase angle of λ_i^*.

Estimation of Channel Path Delays τ_l

- Once L and τ_l's are available, we can form the following two matrices

  ◇

$$\hat{\boldsymbol{F}}_P = \begin{bmatrix} e^{\frac{-j2\pi\hat{\tau}_1 0}{T_s}} & e^{\frac{-j2\pi\hat{\tau}_2 0}{T_s}} & \cdots & e^{\frac{-j2\pi\hat{\tau}_L 0}{T_s}} \\ e^{\frac{-j2\pi\hat{\tau}_1 D}{T_s}} & e^{\frac{-j2\pi\hat{\tau}_2 D}{T_s}} & \cdots & e^{\frac{-j2\pi\hat{\tau}_L D}{T_s}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{\frac{-j2\pi\hat{\tau}_1 (P-1)D}{T_s}} & e^{\frac{-j2\pi\hat{\tau}_2 (P-1)D}{T_s}} & \cdots & e^{\frac{-j2\pi\hat{\tau}_L (P-1)D}{T_s}} \end{bmatrix} \quad (30)$$

  where $\hat{\boldsymbol{F}}_P \in \mathbb{C}^{P\times\hat{L}}$

  ◇

$$[\hat{\boldsymbol{F}}_h]_{m,n} = e^{\frac{-j2\pi m\hat{\tau}_n}{T_s}} \in \mathbb{C}^{N\times\hat{L}} \quad (31)$$

Estimation of Channel Impulse Responses

- The channel impulse response $\boldsymbol{h}_k$ can then be computed as

$$\hat{\boldsymbol{h}}_k = \hat{\boldsymbol{F}}_P^{\dagger} \hat{\boldsymbol{H}}_{P,k} \tag{32}$$

  where $\hat{\boldsymbol{F}}_P^{\dagger}$ is the pseudo-inverse of $\hat{\boldsymbol{F}}_P$

- The channel frequency response across all subcarriers are then given by

$$\hat{\boldsymbol{H}}_k = \hat{\boldsymbol{F}}_h \hat{\boldsymbol{h}}_k \tag{33}$$

- When noise is present, the autocorrelation matrix of $\hat{\boldsymbol{H}}_{P,k}$ becomes

$$\begin{aligned}\boldsymbol{R} &= E\left\{\boldsymbol{H}_{P,k}\boldsymbol{H}_{P,k}^{\mathrm{H}}\right\} + E\left\{\boldsymbol{v}_{P,k}\boldsymbol{v}_{P,k}^{\mathrm{H}}\right\} \\ &= \boldsymbol{F}_P \Sigma \boldsymbol{F}_P^{\mathrm{H}} + \sigma^2 \boldsymbol{I}_P \qquad (34)\end{aligned}$$

where $\boldsymbol{I}_P$ is $P \times P$ identity matrix.

-

$$\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{k=0}^{K-1} \hat{\boldsymbol{H}}_{P,k}\hat{\boldsymbol{H}}_{P,k}^{\mathrm{H}} \qquad (35)$$

Based on $\hat{\boldsymbol{R}}$ in (35), one can further resort to existing model-order selection schemes, such as minimum description length (MDL) [WK 1985][4] and Akaike's information criterion (AIC) [ZWYR 1989][5], to obtain an estimate of L.

[4] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp. 387-392, Apr. 1985.

[5] Q. Zhang, K. M. Wong, P. C. Yip, and J. P. Reilly, "Statistical analysis of the performance of information theoretic criteria in the detection of the number of signals in array processing," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 1557-1567, Apr. 1989.

- In case that the noise power σ^2 is known [YLCC 01][6] [RBG 05][7] [LWZL 08][8], one can come up with an $\hat{\boldsymbol{R}}_0$ free from noise corruption as

$$\hat{\boldsymbol{R}}_0 = \hat{\boldsymbol{R}} - \sigma^2 \boldsymbol{I}_P \tag{36}$$

Such a technique has been employed in the study of blind channel estimation [KZ 00] [9] [Z 1997] [10] for improving the robustness against noise.

[6] B. Yang, K. B. Letaief, R. S. Clieng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, pp. 467-478, Mar. 2001.

[7] M. R. Raghavendra, S. Bhashyam, and K. Giridhar, "Exploiting hopping pilots for parametric channel estimation in OFDM systems," *IEEE Signal Processing Lett.*, vol. 12, pp. 737-740, Nov. 2005.

[8] Siyang Liu, Student Member, *IEEE*, Feifei Wang, Ranran Zhang, and Yuanan Liu, Member, *IEEE*. "A Simplified Parametric Channel Estimation Scheme for OFDM Systems," in *IEEE Trans. Wireless Commun.*, vol. 7, No. 12, Dec. 2008.

[9] Thomas P. Krauss, Student Member, IEEE, and Michael D. Zoltowski, Fellow, IEEE, "Bilinear approach to multiuser second-order statistics-based blind channel estimation," *IEEE Trans. Signal Processing*, vol. 48, No. 9. 2000.

[10] Zhi Ding, Senior Member, IEEE, "Matrix outer-product decomposition method for blind multiple channel identification," *IEEE Trans. Signal Processing*, vol. 45, Dec. 1997.

Proposed Channel Estimation Algorithm

1. Estimate the frequency responses on the pilot subcarriers using $\hat{H}_{p_i,k} = H_{p_i,k} + \nu_{p_i,k}$.

2. Calculate the autocorrelation matrix $\hat{\boldsymbol{R}}$ using $\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{k=0}^{K-1} \hat{\boldsymbol{H}}_{P,k}\hat{\boldsymbol{H}}^{\mathrm{H}}_{P,k}$.

3. EVD of $\hat{\boldsymbol{R}}$ for estimating the number of paths L.

4. Calculate the matrix $\boldsymbol{U}$ using $\boldsymbol{U} = (\tilde{\boldsymbol{R}}^{-1}_{(m,L+1)}\tilde{\boldsymbol{R}}_{(m,1)})^{\mathrm{H}}$.

5. An EVD of $\boldsymbol{U}$ for delay estimation using $\hat{\tau}_i = \frac{arg\{\lambda_i^*\}T_s}{2\pi D}$.

6. Estimate the channel impulse response using $\hat{\boldsymbol{h}}_k = \hat{\boldsymbol{F}}^{\dagger}_{P}\hat{\boldsymbol{H}}_{P,k}$ and channel frequency response using $\hat{\boldsymbol{H}}_k = \hat{\boldsymbol{F}}_h\hat{\boldsymbol{h}}_k$.

Complexity Comparison[LWZL 08]

- **[LWZL 08]**[11]:
  $16P^3 + 8KP^2 + 7P^2 + 8(P+N)LK - 2(L+N)K + 6PK + 10P.$
- **[YLCC 01]** [12]:
  $(48 - 2K)P^3 + (6KP + 3)P^2 + (24L^2 - 2L + 12)P + 10PK + 104L^3 - 24L^2 + 2L + 8(P + N)LK - 2(L + N)K.$
- **[RBG 05]**[13]:
  $(48 - 2K)P^3 + (6KP + 3)P^2 + (24L^2 - 2L + 12)P + 10PK + 104L^3 - 24L^2 + 2L + 8(P + N)LK - 2(L + N)K.$
- **Proposed scheme**:
  $48P^3 + 4KP^2 + 8(P + N)LK - 2(L + N)K + 4PK + 2P^2 + 2P + 112L^3 + 2L^2.$

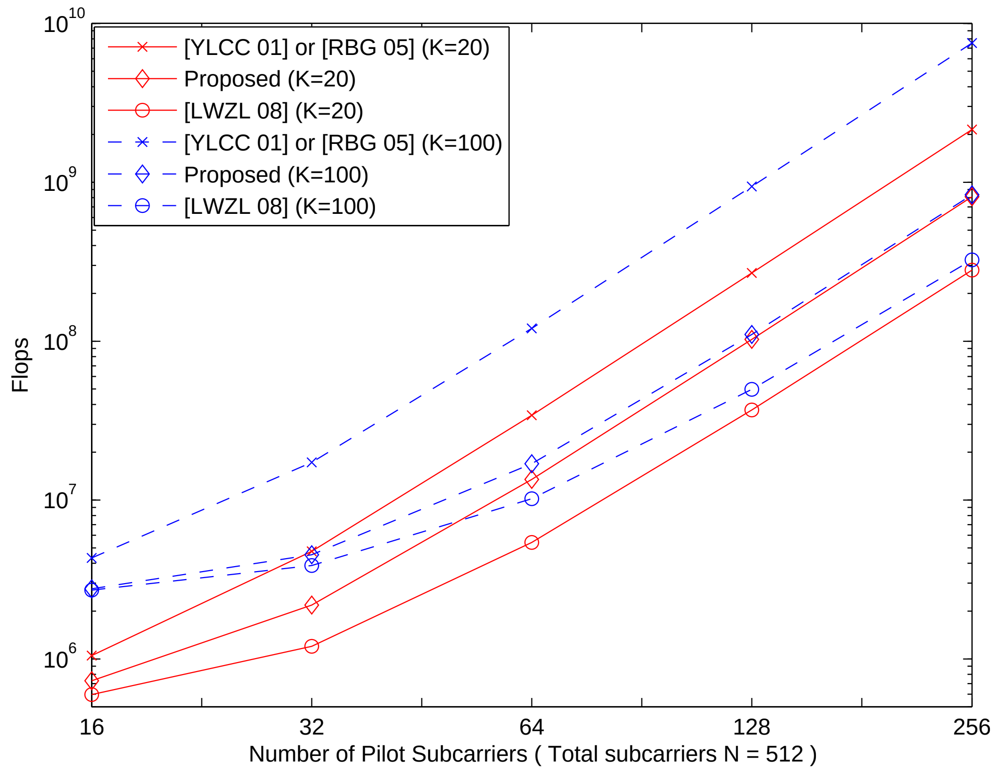
[11] Siyang Liu, Student Member, *IEEE*, Feifei Wang, Ranran Zhang, and Yuanan Liu, Member, *IEEE*. "A Simplified Parametric Channel Estimation Scheme for OFDM Systems," in *IEEE Trans. Wireless Commun.*, vol. 7, No. 12, Dec. 2008.

[12] B. Yang, K. B. Letaief, R. S. Clieng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, pp. 467-478, Mar. 2001.

[13] M. R. Raghavendra, S. Bhashyam, and K. Giridhar, "Exploiting hopping pilots for parametric channel estimation in OFDM systems," *IEEE Signal Processing Lett.*, vol. 12, pp. 737-740, Nov. 2005.

Complexity Comparison

- For $L = 3$ and $N = 512$, the results are ploted in Figure 1.



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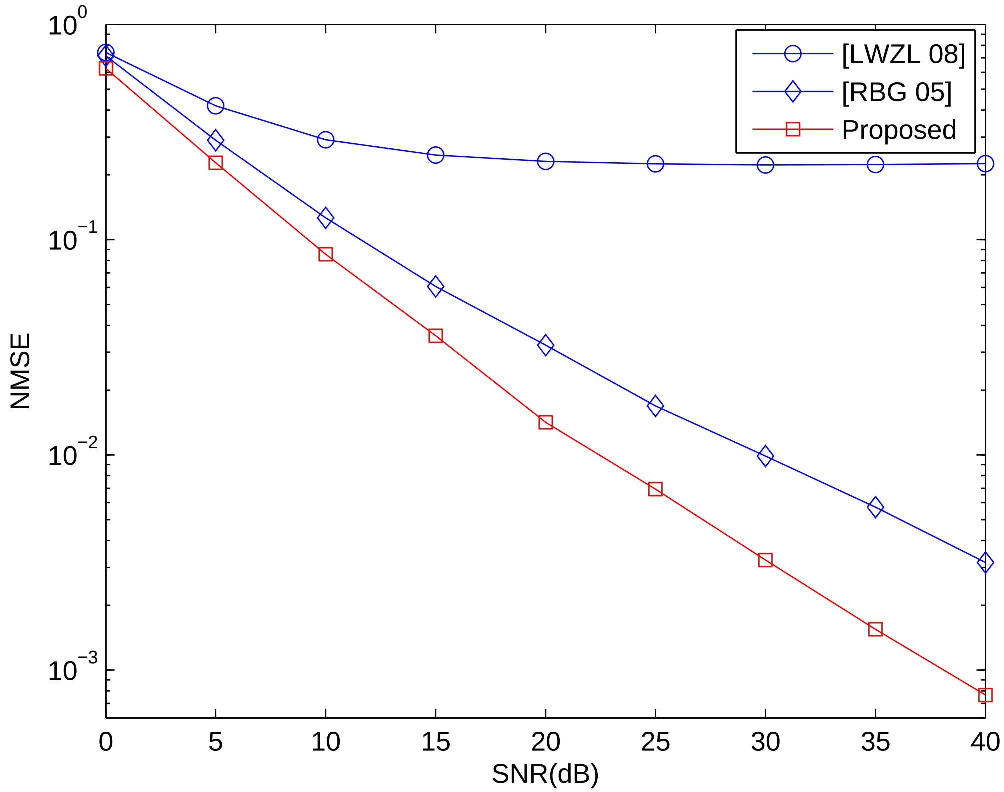
Simulation Results

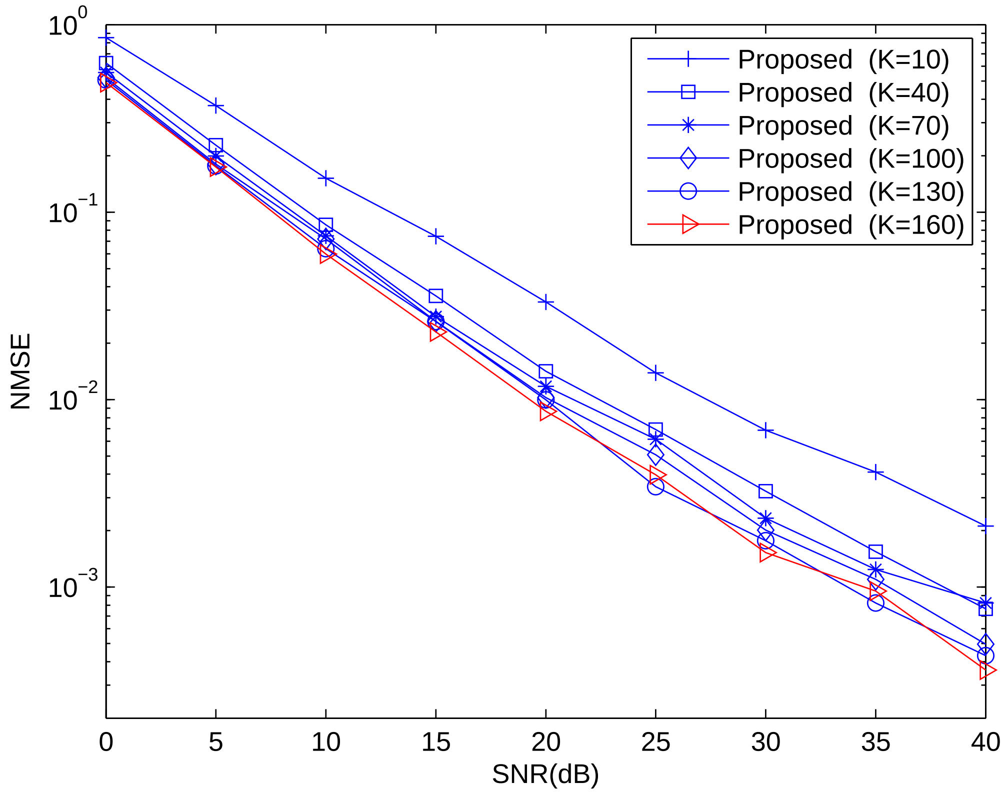
- The system parameters used in the simulations follow the setup in [RBG 05].[14].
    - ◇ The number of subcarriers is $N = 128$ and the number of pilots is $P = 8$.
    - ◇ $L = 3$ with multipath delays uniformly distributed over $[0, 4.5\,\mu\text{s}]$, and the power delay profile of the channel gain obeys the exponential distribution with pdf $f(\tau) \sim e^{-\beta\tau}$ with $\beta = 1/4$.
- The normalized mean square error (NMSE) is defined to be $E[\|\boldsymbol{H}_k - \hat{\boldsymbol{H}}_k\|^2])/E[\|\boldsymbol{H}_k\|^2]$ [RBG 05].

[14] M. R. Raghavendra, S. Bhashyam, and K. Giridhar, "Exploiting hopping pilots for parametric channel estimation in OFDM systems," *IEEE Signal Processing Lett.*, vol. 12, pp. 737-740, Nov. 2005.

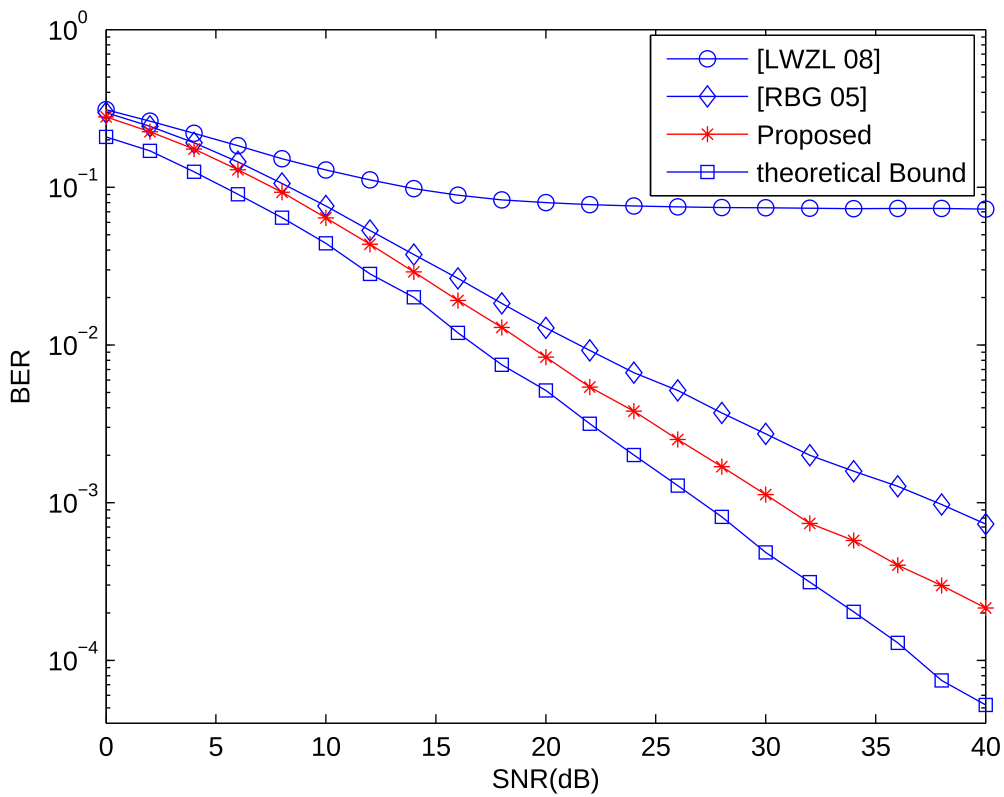
Simulation Results



Simulation Results



Simulation Results



Outline

Conclusion

- We propose a low-complexity parametric-based channel estimation scheme for OFDM.
- Algebraic structure of certain submatrices of the receive autocorrelation matrix is exploited for unique identification of channel parameters.
- Main computations involved : inversion of submatrices followed by an EVD.
- Compared with the existing ESPRIT based algorithm, the proposed approach benefits from reduced-dimension processing and thus results in lower algorithmic complexity.
- The proposed algorithm can achieve better performance in NMSE and BER .

Thank You for Your Attention

Appendix—ESPRIT Algorithm

- The ESPRIT algorithm can be regarded as a "Signal Subspace Method". Putting it into the current formulation for OFDM channel estimation, we stack the pilot tone channel frequency response at the kth OFDM symbol into a vector $\boldsymbol{H}_{P,k}$.
- Then we have the following linear model.

$$\boldsymbol{H}_{P,k} = \boldsymbol{F}_P \boldsymbol{h}_k$$

where the rows of $\boldsymbol{F}_P$ are those rows of the DFT matrix $\boldsymbol{F}$ corresponding to the indices of pilot subcarriers.

Appendix—ESPRIT Algorithm

- By rearranging entries of $\boldsymbol{H}_{P,k}$ into two groups, one for the ones with odd subcarrier indices while the other for those with even subcarrier indices, we obtain

$$\boldsymbol{H'}_{P,k} = \begin{bmatrix} \boldsymbol{H}_{P,k}^{(o)} \\ \boldsymbol{H}_{P,k}^{(e)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{P}^{(o)} \\ \boldsymbol{F}_{P}^{(e)} \end{bmatrix} \boldsymbol{h}_k$$

where $\boldsymbol{F}_P^{(o)}$ and $\boldsymbol{F}_P^{(e)}$, respectively, certain rows of $\boldsymbol{F}_P$ with odd and even indices.

- Such a rearrangement then introduces the signal structure

$$\boxed{\boldsymbol{F}_P^{(e)} = \boldsymbol{F}_P^{(o)}\Phi, \text{ where } \Phi{=}diag \left[\exp\{\tfrac{-j2\pi D\tau_1}{NT}\} \ \cdots \ \exp\{\tfrac{-j2\pi D\tau_L}{NT}\} \right]}$$

which is crucial for the **ESPRIT** algorithm.

Appendix—ESPRIT Algorithm

- The (ideal) autocorrelation matrix associated with $\boldsymbol{H'}_{P,k}$ is obtained as

$$\boldsymbol{R} \equiv E\{\boldsymbol{H'}_{P,k}\boldsymbol{H'}^{\mathrm{H}}_{P,k}\} = \begin{bmatrix} \boldsymbol{F}_P^{(o)} \\ \boldsymbol{F}_P^{(e)} \end{bmatrix} \underbrace{E\{\mathbf{h}_k\mathbf{h}_k^H\}}_{\Sigma} \begin{bmatrix} \boldsymbol{F}_P^{(o)} \\ \boldsymbol{F}_P^{(e)} \end{bmatrix}^H$$

- For a L-order channel and under rich scattering assumption, we have $\mathrm{rank}(\Sigma) = L$. Hence, the matrix $\boldsymbol{R}$ has L nonzero dominant eigenvectors. Then collect these eigenvectors to form $\boldsymbol{U} = \begin{bmatrix} \boldsymbol{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_L \end{bmatrix}$

- Since

$$\begin{aligned} Span\left\{ \boldsymbol{u}_1 \;\; \mathbf{u}_2 \;\; \cdots \;\; \mathbf{u}_L \right\} &= Column\ Space\ of\ \boldsymbol{R} \\ &= Column\ Space\ of \begin{bmatrix} \boldsymbol{F}_P^{(o)} \\ \boldsymbol{F}_P^{(e)} \end{bmatrix} \end{aligned}$$

Appendix—ESPRIT Algorithm

- There exists a nonsingular matrix $\mathbf{B}$ such that

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{F}_P^{(o)} \\ \boldsymbol{F}_P^{(e)} \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} \boldsymbol{F}_P^{(o)} \boldsymbol{B} \\ \boldsymbol{F}_P^{(o)} \Phi \boldsymbol{B} \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{U}_2 \end{bmatrix}$$

- Combing $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$, we have

$$(\boldsymbol{U}_1)^{\dagger} \boldsymbol{U}_2 = \boldsymbol{B}^{-1} \Phi \boldsymbol{B}$$

  where $(.)^{\dagger}$ denotes the pseudo-inverse.

- Hence the matrix $(\boldsymbol{U}_1)^{\dagger} \boldsymbol{U}_2$ is similar to Φ, and the delays can be identified by via an eigen-decomposition of $(\boldsymbol{U}_1)^{\dagger} \boldsymbol{U}_2$

Bibliography

[YLCC 01] B. Yang, K. B. Letaief, R. S. Clieng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, pp. 467-478, Mar. 2001.

[RBG 05] M. R. Raghavendra, S. Bhashyam, and K. Giridhar, "Exploiting hopping pilots for parametric channel estimation in OFDM systems," *IEEE Signal Processing Lett.*, vol. 12, pp. 737-740, Nov. 2005.

[OG 04] S.Ohno and G.B.Giannakis, "Capacity maximizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. Inform. Theory.*, vol. 50, no. p, pp. 2138-2145, Sep, 2004.

[LWZL 08] Siyang Liu, Student Member, *IEEE*, Feifei Wang, Ranran Zhang, and Yuanan Liu, Member, *IEEE*. "A Simplified Parametric Channel Estimation Scheme for OFDM Systems," in *IEEE Trans. Wireless Commun.*, vol. 7, No. 12, Dec. 2008.

[WK 1985] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp. 387-392, Apr. 1985.

[ZWYR 1989] Q. Zhang, K. M. Wong, P. C. Yip, and J. P. Reilly, "Statistical analysis of the performance of information theoretic criteria in the detection of the number of signals in array processing," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 1557-1567, Apr. 1989.

[KZ 00] Thomas P. Krauss, Student Member, IEEE, and Michael D. Zoltowski, Fellow, IEEE, "Bilinear approach to multiuser second-order statistics-based blind channel estimation," *IEEE Trans. Signal Processing*, vol. 48, No. 9. 2000.

[Z 1997] Zhi Ding, Senior Member, IEEE, "Matrix outer-product decomposition method for blind multiple channel identification," *IEEE Trans. Signal Processing*, vol. 45, Dec. 1997.