國 立 清 華 大 學

碩 士 論 文

用於正交分頻多工系統基於參數通道模型

之低複雜度通道估測

A Low-Complexity Channel Estimation Algorithm for OFDM

Systems based on Parametric Channel Modeling

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Abstract

In this thesis, a low-complexity OFDM channel estimation scheme is proposed based on a practical assumption that the propagation delays of multipath components of the real channel are not necessarily integer multiples of the sampling period. By exploiting the algebraic structure of the autocorrelation matrix of pilot frequency responses, it is observed that certain sub-matrices already contain sufficient information for unique identification of the channel parameters; hence, the computations involved in the proposed method are only the inversion and eigen-decompositions of selective sub-matrices with reduced dimensions. In comparison with existing solutions using the original autocorrelation matrix and developed under the same assumption, the proposed method is more suitable for a low-complexity algorithmic implementation. Particularly and perhaps surprisingly, simulations show that less computations also improve the estimation mean square error due to less accumulation of numerical errors.

摘要

在最近幾年來，正交分頻多工系統是一種非常受到注目的技術，因爲將它應用在無線通訊系統上，此調變技本身具有一些特性，例如 它可以提高傳輸的位元速率、使接收端的設計簡單化並透過這樣的技術可以對抗多重路徑衰退通道（multipath fading channel）處於 frequency selective 的情形下大大降低對信號產生的影響。

在這篇論文中，我們採用正交分頻多工系統去實現通道估測，一般而言，通道估測，主要可以被區分爲二大部分，它們分別爲通道參數（Channel Parametres）的估測及通道係數（Channel Coefficients）的估測。在傳統研究通道估測問題時，通道的模型大致可分爲非參數型（Non-Parameteric）及參數型（Parameteric）二種類別。很多的研究指出，在使用非參數型（Non-Parameteric）的通道模型時，由於沒有好好利用通道多路徑的特性，造成效能的表現不如參數型（Parameteric）的通道模型。

當我們使用協調的正交多頻分工做通道估測並且假設通道模型爲參數型（Parameteric Channel Modeling）時，一般大部份的估測方法都是基於假設其通道路徑的延遲 （Chennel Delay）爲接收端取樣週期的整數倍之情況下去做分析。然而，在實際傳輸通道，其通道路徑的延遲，往往非接收端取樣週期的整數倍，導致估測效能不如預期。針對這樣的情況，Yang 首先提出了使用 ESTRIT 演算法來解決此問題。但由於ESTRIT演算法的複雜度太高，使得實現上非常的困難。故而本論文，提出另一演算法，主要目的是致力於降低估測複雜度並且改善其估測效能。

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目錄

Contents

List of Figures

Chapter 1

Introduction

Orthogonal frequency division multiplexing (OFDM) has been a well-known technique capable of supporting high-rate transmissions over multi-path fading channels. By resorting to transmit redundancy in the form of cyclic prefix (CP), an OFDM modulator can convert a frequency-selective channel into multiple parallel flat fading links in the frequency domain. Toward coherent tone-by-tone symbol detection, channel state information (CSI) is necessary at the receiver. This call for the need of channel estimation for an OFDM system.

The problem of channel estimation for OFDM has received considerable attention in the past decade [1], [2]. Among the existing channel estimation schemes, the model-based approaches e.q., [3], [4], [5] have been evidenced as an effective means since the multi-path characteristic of wireless channels is explicitly exploited in the development of the algorithm for further enhancing the channel estimation performance [4]. In particular, each multi-path component is parameterized by the corresponding propagation delay and attenuation gain; the delays across all paths are estimated first, based on which the channel gains are then determined. A common assumption made in most of the model-based methods is that the each path delay is an integer multiple of the sampling period. In [4], [5] such an assumption was removed; the problem of delay acquisition therein was formulated in an array signal processing setup, and an ESPRIT [6] based channel estimation algorithm was proposed.

Under the general assumption that each path delay is not necessarily an integer multiple of the sampling interval, this thesis proposes a low-complexity model-based OFDM channel estimation scheme. By exploiting the algebraic structure inherent in the autocorrelation matrix of the estimated pilot frequency responses, it is shown that certain principle submatrices already contain sufficient information for unique identification of path delays. More specifically, by judiciously removing some columns and rows of the autocorrelation matrix, the signal subspace defined by the resultant sub-matrices still enjoys the rotational invariance characteristic [6]. This allows for unique identification of path delays through a simple matrix inversion operation followed by an eigenvalue decomposition (EVD). The distinctive feature of the proposed scheme is that the processing is done on the basis of submatrices of reduced dimensions. In contrast with the ESPRIT-based algorithm [4], [5], wherein the EVD of the full autocorrelation matrix are needed, the proposed method benefits from reduced-dimensional processing and thus results in a lower algorithmic complexity. In addition, our method requires one less SVD operation in the delay estimation stage; this can further reduce arithmetic error accumulation caused by imperfect covariance matrix estimation. Through simulations, our method can be compared favorably with the ESPRIT-based solution in terms of both the channel estimation performance and the symbol error probability.

Throughout the paper, superscripts "T" and "H" are used to denote matrix transpose and Hermitian transpose operations. The symbols $\mathbb{C}$, $\mathbb{C}^m$, and $\mathbb{C}^{m\times n}$ denote, respectively, the set of complex numbers, the set of m-dimensional vectors over the complex field, and the set of all $m \times n$ complex matrices. $\mathbb{R}^{m\times n}$ stands for the set of all $m \times n$ real matrices. For a matrix $\boldsymbol{M}$, $[\boldsymbol{M}]_{m,n}$ denotes the entry located at the mth row and the nth column. $\boldsymbol{F}$ denotes the FFT matrix of a suitable size; if the dimension of $\boldsymbol{F}$ is N, then $[\boldsymbol{F}]_{m,n} = e^{-j2\pi mn/N}/\sqrt{N}$. The notation $\text{diag}\{a_1, a_2, ..., a_m\}$ denotes the $m \times n$ diagonal matrix with $a_1, a_2, ..., a_m$ as the diagonal entries.

Chapter 2

Channel Model and System Model

We consider an OFDM transmission system over wireless multipath channels in this thesis.

At the transmitter, the kth OFDM symbol $\boldsymbol{X}_k = [X_{0,k}, X_{1,k}, ..., X_{N-1,k}]^{\mathrm{T}} \in \mathbb{C}^N$ is left-multiplied by the inverse FFT matrix $\boldsymbol{F}^{\mathrm{H}}$ to obtain the transmitted baseband signal $\boldsymbol{x}_k = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{X}_k \in \mathbb{C}^N$. After inserting the cyclic prefix (CP), the baseband data is pulse-shaped as

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{n}{T_s} t} \quad \text{for } t \in [kT_s, (k+1)T_s), \tag{2.1}$$

where T_s denotes the OFDM symbol duration. The signal $s_k(t)$ is then transmitted over an L-path wireless channel with impulse response

$$h(\tau, t) = \sum_{l=0}^{L-1} h_l(t)\delta(\tau - \tau_l), \tag{2.2}$$

where $h_l(t)$ and τ_l are, respectively, the channel gain and the delay of the lth path, and $\delta(\cdot)$ is the Dirac delta function. Assume that the channel impulse response $h_l(t)$ remains constant during an OFDM symbol interval, that is, for each integer $k \geq 0$,

$$h_l(kT_s) = h_l(kT_s + \Delta t), \quad 0 \leq \Delta t < T_s,\ 0 \leq l \leq L-1, \tag{2.3}$$

and can vary independently across different symbols. With (2.1) and (2.2), the received signal reads

$$\begin{aligned} y_k(t) &= \int_{-\infty}^{\infty} h(\tau, t) s_k(t-\tau) d\tau + v_k(t) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{n}{T_s} t} \int_{-\infty}^{\infty} \sum_{l=0}^{L-1} h_l(t)\delta(\tau - \tau_l) e^{-j2\pi \frac{n}{T_s}\tau} d\tau + v_k(t) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{n}{T_s} t} \sum_{l=0}^{L-1} h_l(t) e^{-j2\pi \frac{n}{T_s}\tau_l} + v_k(t), \end{aligned} \tag{2.4}$$

where $v_k(t)$ is the measurement noise. The sampled data (spaced per T_b =

T_s/N seconds) is then given by

$$\begin{aligned} y_{m,k} &= y_k(kT_s + mT_b) \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{nm}{N}} \sum_{l=0}^{L-1} h_l(kT_s + mT_b) e^{-j2\pi \frac{n\tau_l}{T_s}} + v_k(kT_s + mT_b) \\ &\overset{(a)}{=} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{nm}{N}} \sum_{l=0}^{L-1} h_l(kT_s) e^{-j2\pi \frac{n\tau_l}{T_s}} + v_k(kT_s + mT_b) \\ &\overset{(b)}{=} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi \frac{nm}{N}} H_{n,k} + v_{m,k}, \end{aligned} \tag{2.5}$$

where (a) follows from the assumption in (2.3) and (b) holds by defining $h_{l,k} = h_l(kT_s)$, $v_{m,k} = v_k(kT_s + mT_b)$ and

$$H_{n,k} = \sum_{l=0}^{L-1} h_{l,k} e^{-j2\pi \frac{n\tau_l}{T_s}}. \tag{2.6}$$

Notably, $H_{n,k}$ is the channel frequency response at the nth tone that we wish to estimate at the receiver. By stacking $y_{m,k}$, $0 \le m \le N-1$, into a vector $\boldsymbol{y}_k = [y_{0,k}, y_{1,k}, ..., y_{N-1,k}]^{\mathrm{T}} \in \mathbb{C}^N$, the received OFDM symbol in the frequency domain is

$$\begin{aligned} \boldsymbol{Y}_k &= [Y_{0,k}, Y_{1,k}, ..., Y_{N-1,k}]^{\mathrm{T}} \\ &= \boldsymbol{F}\boldsymbol{y}_k + \boldsymbol{F}\boldsymbol{v}_k \\ &= \begin{bmatrix} H_{0,k} & 0 & \cdots & 0 \\ 0 & H_{1,k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{N-1,k} \end{bmatrix} \begin{bmatrix} X_{0,k} \\ X_{1,k} \\ \vdots \\ X_{N-1,k} \end{bmatrix} + \boldsymbol{F}\boldsymbol{v}_k. \end{aligned} \tag{2.7}$$

Toward coherent detection, knowledge of the channel frequency response $\{H_{n,k}\}$ at the receiver is needed. From (2.6), it can be seen that $H_{n,k}$ is characterized by not only the channel impulse response taps $\{h_{l,k}\}_{l=0}^{L-1}$ but also the path delays $\{\tau_l\}_{l=0}^{L-1}$. Before the introduction of our proposed estimation scheme on $\{H_{n,k}\}$ in the next section, the following assumptions are made.

(a) The path delay τ_l, $0 \le l \le L-1$, is not necessary an integer multiple of the sampling interval T_b.

(b) An upper bound L_{bound} of the number of paths L is known.

(c) The number of pilot tones P inserted in each OFDM symbol satisfies $P \ge L_{\text{bound}} + 1$.

(d) The channel impulse response taps satisfy $E\{h_l(k_1 T_s) h_m^*(k_2 T_s)\} = E\{h_{l,k_1} h_{m,k_2}^*\} = \sigma_l^2 \delta_K(l-m)\delta_K(k_1-k_2)$, where $\delta_K(\cdot)$ is the Kronecker delta function.

(e) The noise samples $\{v_{m,k}\}$ are zero-mean white Gaussian distributed with variance σ^2.

(f) The channel gains $\{h_{l,k}\}$ are uncorrelated with the noise $\{v_{m,k}\}$.

Chapter 3

The Proposed Channel Estimation Scheme

In our scheme, we let the P pilot tones, indexed by p_i, $1 \leq i \leq P$, be equally spaced among the N frequency tones.[1] We then choose N to be an integer multiple of P and denote by $D = N/P$ the spacing between two adjacent pilots. From (2.7), the received data on these pilot subcarriers can be written as

$$Y_{p_i,k} = X_{p_i,k} H_{p_i,k} + Z_{p_i,k}, \ 1 \leq i \leq P, \tag{3.1}$$

[1] According to [8] and [9], the equally spaced pilot placement can minimize the mean square error of the channel frequency response estimates; thus, we follow their conclusion by using equally spaced pilot placement in this thesis.

where $Z_{p_i,k}$ is the p_ith entry of $\boldsymbol{F}\boldsymbol{v}_k$. Based on (3.1), the least squares (LS) [7] estimates of the channel frequency responses at the pilot tones can accordingly be obtained as

$$\hat{H}_{p_i,k} = \frac{Y_{p_i,k}}{X_{p_i,k}} = H_{p_i,k} + \nu_{p_i,k},\ 1 \leq i \leq P, \tag{3.2}$$

where $\nu_{p_i,k} = Z_{p_i,k}/X_{p_i,k}$. By denoting

$$\begin{aligned} \hat{\boldsymbol{H}}_{P,k} &= [\hat{H}_{p_1,k}, \hat{H}_{p_2,k}, ..., \hat{H}_{p_P,k}]^{\mathrm{T}}, \\ \boldsymbol{H}_{P,k} &= [H_{p_1,k}, H_{p_2,k}, ..., H_{p_P,k}]^{\mathrm{T}}, \\ \boldsymbol{v}_{P,k} &= [\nu_{p_1,k}, \nu_{p_2,k}, ..., \nu_{p_P,k}]^{\mathrm{T}}, \end{aligned} \tag{3.3}$$

(3.2) can be re-expressed as

$$\begin{aligned} \hat{\boldsymbol{H}}_{P,k} &= \boldsymbol{H}_{P,k} + \boldsymbol{v}_{P,k} \\ &= \boldsymbol{F}_P \boldsymbol{h}_k + \boldsymbol{v}_{P,k}, \end{aligned} \tag{3.4}$$

where $\boldsymbol{h}_k = [h_{0,k}, h_{2,k}, ..., h_{L-1,k}]^{\mathrm{T}}$ and

$$\boldsymbol{F}_P = \begin{bmatrix} W_1^0 & W_2^0 & \cdots & W_L^0 \\ W_1^D & W_2^D & \cdots & W_L^D \\ \vdots & \vdots & \ddots & \vdots \\ W_1^{(P-1)D} & W_2^{(P-1)D} & \cdots & W_L^{(P-1)D} \end{bmatrix} \in \mathbb{C}^{P \times L}, \tag{3.5}$$

as for notational convenience for $\boldsymbol{F}_P$, we let $W_i = e^{-j2\pi\tau_i/T_s}$. Based on (3.4), we then propose a two-stage channel estimation scheme, in which the number of paths L and the path delays $\{\tau_l\}$ are estimated first, followed by the determination of the channel impulse response taps $\{h_{l,k}\}$.

In order to facilitate the explanation of the idea behind our proposed approach in the following discussions, we assume for the moment that the channel noise is absent at first; the general case when noise is present will be discussed later. Based on the same reason, we also assume the *priori* known upper bound L_{bound} is equal to L (yet, the receiver is only certain that $L_{\text{bound}} \geq L$); the characteristic of the proposed approach in case $L_{\text{bound}} > L$ will be discussed at the end of the next section.

3.1 Estimation of the Number of Channel Paths, L

With the absence of the noise, i.e., $\boldsymbol{v}_{P,k} = \mathbf{0}$ in (3.4), and also with assumption (d) at the end of Section II, the autocorrelation matrix of the estimated channel frequency response can be obtained from

$$\boldsymbol{R} = E\left\{\hat{\boldsymbol{H}}_{P,k}\hat{\boldsymbol{H}}_{P,k}^{\text{H}}\right\} = E\left\{\boldsymbol{H}_{P,k}\boldsymbol{H}_{P,k}^{\text{H}}\right\} = \boldsymbol{F}_P \Sigma \boldsymbol{F}_P^{\text{H}}, \tag{3.6}$$

where $\Sigma = \text{diag}\{\sigma_1^2, \sigma_2^2, ..., \sigma_L^2\} \in \mathbb{R}^{L\times L}$. Since without the presence of the noise, rank$(\boldsymbol{R})$ = rank $(\Sigma) = L$, the number of channel paths can be exactly determined via the number of nonzero eigenvalues of $\boldsymbol{R}$.

Notably, in practice, $\hat{\boldsymbol{R}}$ defined in (3.24) will be used instead of this theoretical $\boldsymbol{R}$; so, an estimate $\hat{L}$ will be obtained via $\hat{\boldsymbol{R}}$. Discussions of the corresponding details will be given in the next section.

3.2 Estimation of Channel Path Delays, $\{\tau_l\}$

Unlike the ESPRIT-based algorithms [4], [5] that call for the EVD of the full autocorrelation matrix $\boldsymbol{R}$, we will show in the following that submatrices of $\boldsymbol{R}$ are sufficient to identify $\{\tau_l\}$. As anticipated, using properly selected submatrices can reduce considerately the algorithmic complexity. However to our surprise, the resultant estimation mean square error can also be compared favorably to the existing ESPRIT-based algorithms due to the accumulation of less numerical errors. Details are given below.

Let $\tilde{\boldsymbol{R}}$ be the first $(L+1)\times(L+1)$ principal submatrix of the $P\times P$ matrix $\boldsymbol{R}$; thus, we have

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{F}}_P \Sigma \tilde{\boldsymbol{F}}_P^H, \tag{3.7}$$

where $\tilde{\boldsymbol{F}}_P \in \mathbb{C}^{(L+1)\times L}$ consists of the first $(L+1)$ rows of the $P\times L$ matrix $\boldsymbol{F}_P$. Let $\tilde{\boldsymbol{R}}_{/(m,n)}$ be the matrix obtained by deleting the mth row and nth column of $\tilde{\boldsymbol{R}}$. From (3.7), $\tilde{\boldsymbol{R}}_{/(m,n)}$ can be expressed as

$$\tilde{\boldsymbol{R}}_{/(m,n)} = \tilde{\boldsymbol{F}}_{P/(m)} \Sigma \tilde{\boldsymbol{F}}_{P/(n)}^H \quad \text{for} \quad 1 \le m, n \le L+1, \tag{3.8}$$

where $\tilde{\boldsymbol{F}}_{P/(m)} \in \mathbb{C}^{L\times L}$ is obtained by removing the mth row of $\tilde{\boldsymbol{F}}_P$. Since the mth row of $\tilde{\boldsymbol{F}}_P$ is simply its nth row multiplied by the factor $e^{-j2\pi|m-n|D/T_s}$ (see (3.5) as a reference), it can be verified that

$$\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P/(1)} = \Phi\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P/(L+1)}, \tag{3.9}$$

where

$$\Phi = \mathrm{diag}\{e^{-j2\pi\tau_1 D/T_s}, e^{-j2\pi\tau_2 D/T_s}, ..., e^{-j2\pi\tau_L D/T_s}\} \in \mathbb{C}^{L\times L}. \tag{3.10}$$

Combining (3.8) and (3.9), we get

$$\begin{aligned}\tilde{\boldsymbol{R}}_{/(L+1,1)} &= \tilde{\boldsymbol{F}}_{P/(L+1)}\Sigma\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P/(1)} \\ &= \tilde{\boldsymbol{F}}_{P/(L+1)}\Sigma\Phi\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P/(L+1)}\end{aligned} \tag{3.11}$$

and

$$\tilde{\boldsymbol{R}}_{/(L+1,L+1)} = \tilde{\boldsymbol{F}}_{P/(L+1)}\Sigma\tilde{\boldsymbol{F}}^{\mathrm{H}}_{P/(L+1)}. \tag{3.12}$$

From (3.11) and (3.12), we reach the following key equation

$$\begin{aligned}\boldsymbol{U} &= (\tilde{\boldsymbol{R}}^{-1}_{/(L+1,L+1)}\tilde{\boldsymbol{R}}_{/(L+1,1)})^{\mathrm{H}} \\ &= \tilde{\boldsymbol{F}}_{P/(L+1)}\Phi^{\mathrm{H}}\tilde{\boldsymbol{F}}^{-1}_{P/(L+1)}.\end{aligned} \tag{3.13}$$

We conclude that the eigenvalues $\{\lambda_l\}$ of $\boldsymbol{U}$ are precisely the diagonal entries of Φ^{H}, i.e., $\lambda_l = e^{-j2\pi\tau_l D/T_s}$. Hence, the path delays $\{\tau_l\}$ can be determined through

$$\tau_i = \frac{\arg\{\lambda_i^*\}T_s}{2\pi D} \quad \text{for } i = 0, 1, ..., L-1, \tag{3.14}$$

where $\arg\{\lambda_i^*\}$ denotes the phase angle of λ_i^*.

Again, in practice, $\hat{\boldsymbol{R}}$ defined in (3.24) will be used instead of the theoretical $\boldsymbol{R}$ in the above procedure; so, what we obtain will be the path delay estimates $\{\hat{\tau}_i\}$ rather than the true $\{\tau_i\}$.

3.3 Estimation of Channel Frequency Response, $\boldsymbol{H}_k$

Once $\hat{L}$ and $\{\hat{\tau}_l\}$ are available, the estimate $\hat{\boldsymbol{F}}_P \in \mathbb{C}^{P\times L}$ can be obtained via (3.5) by replacing τ_l with $\hat{\tau}_l$, $1 \leq l \leq L$. By (3.4), the channel impulse response estimate $\hat{\boldsymbol{h}}_k$ can then be computed as

$$\hat{\boldsymbol{h}}_k = \hat{\boldsymbol{F}}_P^{\dagger} \hat{\boldsymbol{H}}_{P,k}, \tag{3.15}$$

where $\hat{\boldsymbol{F}}_P^{\dagger}$ is the pseudo-inverse of $\hat{\boldsymbol{F}}_P$, and $\hat{\boldsymbol{H}}_{P,k}$ has been obtained in (3.3). With (3.15), the channel frequency response across all subcarriers can be derived similar to (2.6) as

$$\hat{\boldsymbol{H}}_k = \hat{\boldsymbol{F}}_N \hat{\boldsymbol{h}}_k, \tag{3.16}$$

where $\hat{\boldsymbol{F}}_N \in \mathbb{C}^{N\times L}$ is defined as

$$[\hat{\boldsymbol{F}}_N]_{m,l} = e^{\frac{-j2\pi m\hat{\tau}_l}{T_s}} \quad \text{for } 0 \leq m \leq N-1, 0 \leq l \leq L-1. \tag{3.17}$$

3.4 Robustness of the proposed Method Against Overestimation of L

When $\hat{L} > L$, the truncated autocorrelation matrix $\tilde{\boldsymbol{R}}$ in (3.7), as well as the two corresponding submatrices $\tilde{\boldsymbol{R}}_{(m,1)}$ and $\tilde{\boldsymbol{R}}_{(m,\hat{L}+1)}$ in (3.11) and (3.12), are of dimension $\hat{L} \times \hat{L}$. Since $\hat{L} > L$, the matrices $\tilde{\boldsymbol{F}}_{P,(\backslash m)}$ and $\tilde{\boldsymbol{F}}_{P,(\backslash \hat{L}+1)}$, both of dimension $\hat{L} \times L$, are not square, and an EVD of $\boldsymbol{U} = (\tilde{\boldsymbol{R}}^{-1}_{(m,\hat{L}+1)} \tilde{\boldsymbol{R}}_{(m,1)})^{\mathrm{H}}$ does not admit the form (3.13). In this case, one immediate question we ask is to what extent the information about the delays τ_l's can be preserved in the eigen-system of $\boldsymbol{U}$. Specifically, we have the following result.

Proposition 1. Assume that $\hat{L} > L$, and let $\boldsymbol{U} = (\tilde{\boldsymbol{R}}^{-1}_{(m,\hat{L}+1)} \tilde{\boldsymbol{R}}_{(m,1)})^{\mathrm{H}} \in \mathbb{C}^{\hat{L} \times \hat{L}}$. Then

$$\{e^{-j2\pi\tau_1 D/T_s}, e^{-j2\pi\tau_2 D/T_s}, \ldots, e^{-j2\pi\tau_L D/T_s}\} \subseteq \sigma(\boldsymbol{U}),$$

the set consisting of the eigenvalues of $\boldsymbol{U}$.

$[Proof]$:

$$\begin{aligned}\tilde{\boldsymbol{R}}_{(\hat{L}+1,\hat{L}+1)} &= \tilde{\boldsymbol{F}}_{P,(\backslash\hat{L}+1)}\Sigma\tilde{\boldsymbol{F}}^H_{P,(\backslash\hat{L}+1)} \\ &= \lim_{\epsilon\to 0}\begin{bmatrix}\tilde{\boldsymbol{F}}_{P,(\backslash\hat{L}+1)} & q_{L+1} & \cdots & q_{\hat{L}}\end{bmatrix}\begin{bmatrix}\Sigma & 0 \\ 0 & \Sigma_\epsilon\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{F}}^H_{P,(\backslash\hat{L}+1)} \\ q^H_{L+1} \\ \vdots \\ q^H_{\hat{L}}\end{bmatrix} \\ &= \lim_{\epsilon\to 0}\boldsymbol{Q}\tilde{\Sigma}\boldsymbol{Q}^H \qquad (3.18)\end{aligned}$$

where $q_{L+1}, q_{L+2}, ..., q_{\hat{L}}$ are $\hat{L}\times 1$ column vectors, which form basis of Null space of $\tilde{\boldsymbol{F}}_{P,(\backslash\hat{L}+1)}$ with dimension $\hat{L}-L$, $\Sigma_\epsilon = diag\{\epsilon,\epsilon,...,\epsilon\}$ is $\hat{L}-L\times\hat{L}-L$ matrix, and

$$\tilde{\boldsymbol{F}}_{P,(\backslash\hat{L}+1)} = \begin{bmatrix} W_1^0 & W_2^0 & \cdots & W_L^0 \\ W_1^D & W_2^D & \cdots & W_L^D \\ \vdots & \vdots & \vdots & \vdots \\ W_1^{(\hat{L}-1)D} & W_2^{(\hat{L}-1)D} & \cdots & W_L^{(\hat{L}-1)D}\end{bmatrix} \qquad (3.19)$$

As well, $\tilde{\boldsymbol{R}}_{(\hat{L}+1,1)}$ can be expressed as

$$\begin{aligned}\tilde{\boldsymbol{R}}_{(\hat{L}+1,1)} &= \tilde{\boldsymbol{F}}_{P,(\backslash\hat{L}+1)}\Sigma\Phi\tilde{\boldsymbol{F}}^H_{P,(\backslash\hat{L}+1)} \\ &= \lim_{\epsilon\to 0}\boldsymbol{Q}\tilde{\Sigma}\begin{bmatrix}\Phi & 0 \\ 0 & \tilde{\Phi}\end{bmatrix}\boldsymbol{Q}^H \qquad (3.20)\end{aligned}$$

where $\tilde{\Phi} = diag\{e^{j\theta_{L+1}}, e^{j\theta_{L+2}}, ..., e^{j\theta_{\hat{L}}}\}$ and θ_i can be any arbitrary angle between $0\sim 2\pi$, $L+1\leq i\leq\hat{L}$.

We again follow the same step as (3.11) and (3.12) to combine (3.18) and (3.20) to form $\hat{\boldsymbol{U}}$, it can be given as

$$\begin{aligned} \hat{\boldsymbol{U}} &= (\tilde{\boldsymbol{R}}^{-1}_{(\hat{L}+1,\hat{L}+1)} \tilde{\boldsymbol{R}}_{(\hat{L}+1,1)})^H \\ &= \boldsymbol{Q} \begin{bmatrix} \Phi^H & 0 \\ 0 & \tilde{\Phi}^H \end{bmatrix} \boldsymbol{Q}^{-1} \end{aligned} \tag{3.21}$$

From (3.21), it can be seen that (3.13) is just the first $(L \times L)$ principal submatrix of $\hat{\boldsymbol{U}}$. Therefore, the whole channel parameters are completely preserved in (3.21). As the above statement, the channel of multipath delays are corresponding to eigenvalue of $\hat{\boldsymbol{U}}$ and we can acquired those τ_l's by extracting the diagonal element of Φ^H, if the channel path L can be correctly estimated.

Let $\lambda_1, \lambda_2, ..., \lambda_{\hat{L}}$ be the $\hat{L}$ eigenvalue of $\boldsymbol{U}$, and

$$\hat{\tau}_i = \frac{arg\{\lambda_i^*\}T_s}{2\pi D} \quad for \ 1 \leq i \leq \hat{L} \tag{3.22}$$

Proposition I asserts that $\{\tau_1, \tau_2, ..., \tau_L\} \subseteq \{\hat{\tau}_1, \hat{\tau}_2, ..., \hat{\tau}_{\hat{L}}\}$. Hence, if L is over-estimated as $\hat{L}$, L among the $\hat{L}$ computed $\hat{\tau}_l$'s in (3.22) are the true path delays. As will be seen in the simulation section, such a nice feature can enhance the robustness of the proposed method against the mismatch between L and the estimated $\hat{L}$.

3.5 Discussions

Some discussions regarding the practice of the proposed approach when noise is introduced are given below. The complexity of the proposed approach as well as its comparison with the existing works is also investigated.

1. When noise is present, the autocorrelation matrix of $\hat{\boldsymbol{H}}_{P,k}$ becomes

$$\begin{aligned} \boldsymbol{R} &= E\left\{\boldsymbol{H}_{P,k}\boldsymbol{H}_{P,k}^{\mathrm{H}}\right\} + E\left\{\boldsymbol{v}_{P,k}\boldsymbol{v}_{P,k}^{\mathrm{H}}\right\} \\ &= \boldsymbol{F}_P \Sigma \boldsymbol{F}_P^H + \sigma^2 \boldsymbol{I}_P, \end{aligned} \tag{3.23}$$

   where $\boldsymbol{I}_P$ is the $P \times P$ identity matrix. An additional term $\sigma^2 \boldsymbol{I}_P$ is accordingly added to $\boldsymbol{F}_P \Sigma \boldsymbol{F}_P^{\mathrm{H}}$.

2. The exact autocorrelation matrix $\boldsymbol{R}$ cannot be obtained in practical situations. Rather, given a sequence of the channel frequency response estimates $\hat{\boldsymbol{H}}_{P,k},\ 1 \le k \le K$, $\boldsymbol{R}$ can be estimated via

$$\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{k=0}^{K-1} \hat{\boldsymbol{H}}_{P,k}\hat{\boldsymbol{H}}_{P,k}^{H}. \tag{3.24}$$

   Based on $\hat{\boldsymbol{R}}$ in (3.24), one can further resort to existing model-order selection schemes, such as minimum description length (MDL) [10] and Akaike's information criterion (AIC) [11], to obtain an estimate of L.

   In case that the noise power σ^2 is known, one can come up with an $\hat{\boldsymbol{R}}_0$ free from noise corruption as

$$\hat{\boldsymbol{R}}_0 = \hat{\boldsymbol{R}} - \sigma^2 \boldsymbol{I}_P. \tag{3.25}$$

Such a technique has been employed in blind channel estimation [12], [13] for improving the robustness against noise.

3. For clarity, the proposed approach is summarized as an algorithm in Table I. The floating-point operation (flop) count evaluation of the proposed algorithm is given in the Appendix. Table II lists the amounts of the required flops of the proposed approach as well as three existing methods, namely, Yang's scheme [4], Raghavendra's scheme [5], Liu's scheme [3]; for $L = 3$ and $N = 512$, the results are plotted in Figure 1 with respect to different numbers of pilot subcarriers. It can be seen that Liu's algorithm results in the lowest complexity; however, this method is developed exclusively to the scenario that each delay is an integer multiple of the sampling period. Compared with ESPRIT-based solution by Yang [4] and Raghavendra [5], the proposed approach benefits from reduced-dimensional processing and does reduce the total flop cost.

4. It is noted that in the ESPRIT-based algorithm [4] and [5], the computations required for delay estimation include a matrix inversion operation and two EVDs (refer to, e.g. [6], for more detail). By contrast, delay estimation via the proposed algorithm merely calls for one matrix inversion (step 4) followed by one EVD (step 5). Hence, our approach involves less EVD operations. In addition to the reduction in flop counts, such a

Table I Algorithm outline

Proposed Channel Estimation Algorithm	
Step 1:	Estimate the frequency responses on the pilot subcarriers via (3.2).
Step 2:	Calculate the sample autocorrelation matrix $\hat{\boldsymbol{R}}$ via (3.24).
Step 3:	Perform the EVD of $\hat{\boldsymbol{R}}$ for estimating L, i.e., the number of paths, via (3.24).
Step 4:	Compute the matrix $\boldsymbol{U}$ via (3.13).
Step 5:	Perform the EVD of $\boldsymbol{U}$ for delay estimation via (3.14).
Step 6:	Estimate the channel frequency responses via (3.15) and (3.16).

nice feature can also reduce the arithmetic error accumulations caused by finite-sample covariance matrix estimation errors. This will then enhance the delay estimation accuracy as well as the overall channel estimation performance (as will be seen in the simulation section).

5. When the number of paths L is over-estimated (thus $\hat{L} > L$), the truncated autocorrelation matrix $\tilde{\boldsymbol{R}}$ in (3.7) is of dimension $(\hat{L}+1) \times (\hat{L}+1)$. The two associated submatrices $\tilde{\boldsymbol{R}}_{/(\hat{L}+1,1)}$ and $\tilde{\boldsymbol{R}}_{/(\hat{L}+1,\hat{L}+1)}$ respectively in (3.11) and (3.12) are thus of size $\hat{L} \times \hat{L}$. Particularly in the noiseless case, since $\tilde{\boldsymbol{F}}_{P/(\hat{L}+1)}$ is of dimension $\hat{L} \times L$ and $\text{rank}(\Sigma) = L$, the matrix $\tilde{\boldsymbol{R}}_{/(\hat{L}+1,\hat{L}+1)}$ will be singular and hence non-invertible. However, when channel noise is present, the diagonal perturbation term $\sigma^2 \boldsymbol{I}_P$ in the autocorrelation

Table II Flop count evaluations (N: number of subcarriers, P: number of pilot subcarries, L: number of channel paths, K: number of OFDM symbols)

	Flop Cost
Yang's scheme [4]:	$(48-2K)P^3+(6KP+3)P^2+(24L^2-2L+12)P+10PK$ $+104L^3-24L^2+2L+8(P+N)LK-2(L+N)K.$
Raghavendra's scheme [5]:	$(48-2K)P^3+(6KP+3)P^2+(24L^2-2L+12)P+10PK$ $+104L^3-24L^2+2L+8(P+N)LK-2(L+N)K.$
Liu's scheme [3]:	$16P^3+8KP^2+7P^2+8(P+N)LK-2(L+N)K+6PK$ $+10P.$
Proposed scheme:	$48P^3+4KP^2+8(P+N)LK-2(L+N)K+4PK+2P^2$ $+2P+112L^3+2L^2.$

matrix $\boldsymbol{R}$ in (3.23) will guarantee the invertibility of $\tilde{\boldsymbol{R}}_{/(\hat{L}+1,\hat{L}+1)}$, thereby the matrix $\boldsymbol{U}$ in (3.13) still being well-defined. As a result, even though the idea behind the proposed method is developed under the assumption $\hat{L}=L$, it can still work in the practical noisy environment when $\hat{L}>L$. Through simulations, the proposed algorithm still performs well under an over-estimate of L.

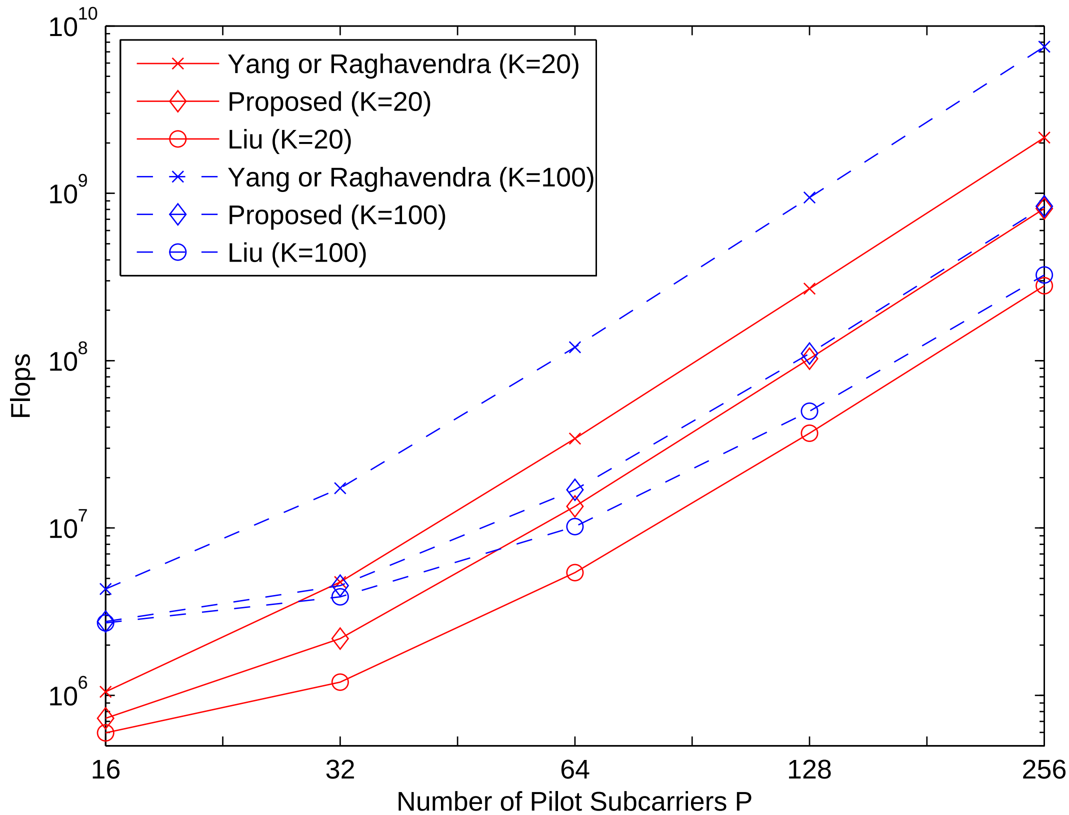

Figure 3.1: Flop counts of three methods (the amount of symbols used for covariance matrix estimation is $K = 20$ and $K = 100$; the total number of subcariers is N=512).

Chapter 4

Flop-Count Analysis of the Proposed Algorithm

Based on the proposed algorithm outlined in Table I, the number of arithmetic operations, counting both (complex) addition and multiplication, required in each step is given below

Step 1. From (3.2), PK multiplications are needed.

Step 2. From (3.24) and by exploiting the Hermitian structure of $\hat{\boldsymbol{R}}$, $\frac{(K+1)P(P+1)}{2}$ multiplications and $\frac{(K-1)P(P+1)}{2}$ additions are needed.

Step 3. An EVD of $\hat{\boldsymbol{R}}$ is needed for the channel path L estimation. $6P^3$ multiplications and $6P^3$ additions are needed.

Step 4. *To form (3.13), $7L^3 + L^2$ multiplications and $7L^3 - L^2$ additions are needed for taking the inverse of $\boldsymbol{R}_{(m,L+1)}^{-1}$ through SVD decomposition. Then L^3 multiplications and $L^3 - L^2$ additions are needed for multiplying $\boldsymbol{R}_{(m,1)}$ by $\boldsymbol{R}_{(m,L+1)}^{-1}$. In this step, $6P^3+8L^3+L^2$ multiplications and $6P^3 + 8L^3 - 2L^2$ additions are needed.*

Step 5. *An EVD of $\boldsymbol{U}$ is performed for the channel delay estimation. $6L^3$ multiplications and $6L^3$ additions are needed.*

Step 6. *Computation of the channel impulse responses via using (**??**) and (3.16) requires $(P+N)LK$ multiplications and $(P+N)LK-(L+N)K$ additions.*

Since a complex addition (multiplication, respectively) needs two (six, respectively) flops [3], the total number of flops is computed as

$$48P^3 + 4KP^2 + 8(P + N)LK - 2(L + N)K + 4PK + 2P^2 + 2P + 112L^3 + 2L^2.$$

Chapter 5

Simulation Results

This section examines and summarizes the performance of the proposed scheme by simulations.

We adopt the the same system setup as in [5]. In summary, the number of subcarriers is $N = 128$, and $P = 8$ equally-spaced QPSK pilots are inserted in each OFDM symbol, and the length of CP is 8. The system occupies a bandwidth of 10MHz with a center carrier frequency at 5GHz. The wireless channel has $L = 3$ taps. The channel path delays are uniformly distributed over [0, 4.5 μs], and the power delay profile of the channel gains obeys the exponential distribution with pdf $f(\tau) = e^{-\beta\tau}$ with $\beta = 1/4$. The communication environment is quasi-static fading such that the channel gains remain invariant per coherent interval spanning $K = 40$ OFDM symbol periods. As in [5], the performance metric for channel estimation is the normalized mean square error (NMSE), defined as $E[\|\boldsymbol{H}_k - \hat{\boldsymbol{H}}_k\|^2])/E[\|\boldsymbol{H}_k\|^2]$.

Figure 2 compares the proposed scheme with Liu's method [3] and the ESPRIT-based algorithm by Raghavendra *et. al* [5] in terms of the NMSE. It can be seen that Liu's method results in a high error floor because our environment violates its strict assumption that each delay must be an integer multiple of the sampling duration. The figure also shows that in comparison with Raghavendra's ESPRIT-based algorithm, the proposed solution yields smaller NMSE.

Figure 3 further depicts the resultant bit error rate (BER) performances of the three methods. The BER curve obtained with perfect channel knowledge is also plotted as the benchmark. As can be seem from the figure, the proposed approach outperforms the other two schemes to be compared, despite its lower algorithmic complexity. As has been mentioned in the previous section, such a performance advantage is benefited from the reduction of the EVD processing in the delay estimation stage. To further justify this, Figure 4 shows the NMSEs of the proposed and the ESPRIT-based methods when the channel noise vector is artificially set to be of the form $\boldsymbol{v}_k = \varepsilon[1, 1, ..., 1]^{\mathrm{T}}$, where ε is a positive real number; under such a fixed noise pattern, the mismatch between the true and estimated channels is incurred entirely by the accumulated arithmetic errors due to the noise perturbation. From the figure, it can be seen that our method does reduce the NMSE.

Finally, the performance of our method is also testified when the number of paths L is over-estimated. With the estimated $\hat{L}$ fixed to be 4, 5, and 6, Figure 5 compares the NMSEs of the proposed approach and Raghavendra's algorithm. The result shows that the proposed scheme is quite robust against the over-estimation of L: the incurred increase in NMSE is very slight as $\hat{L}$ is enlarged from 4 to 6.

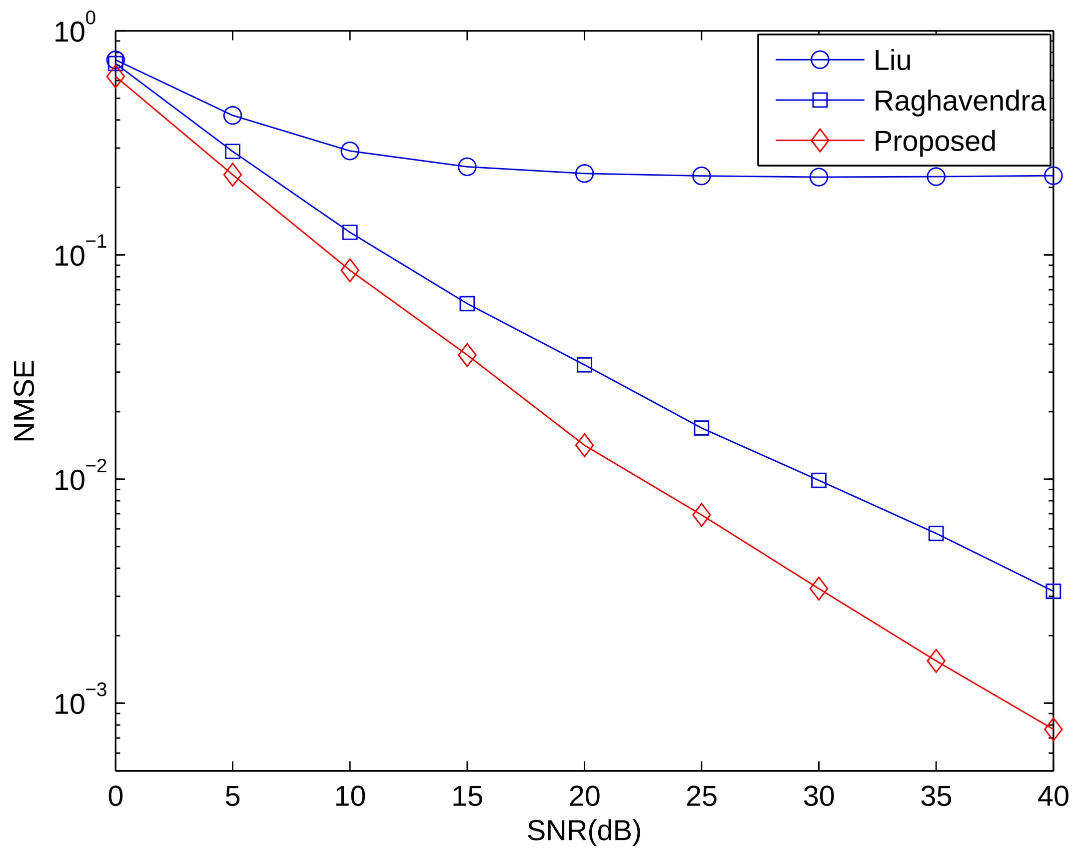

Figure 5.1: NMSEs of the channel estimates obtained by three different methods.

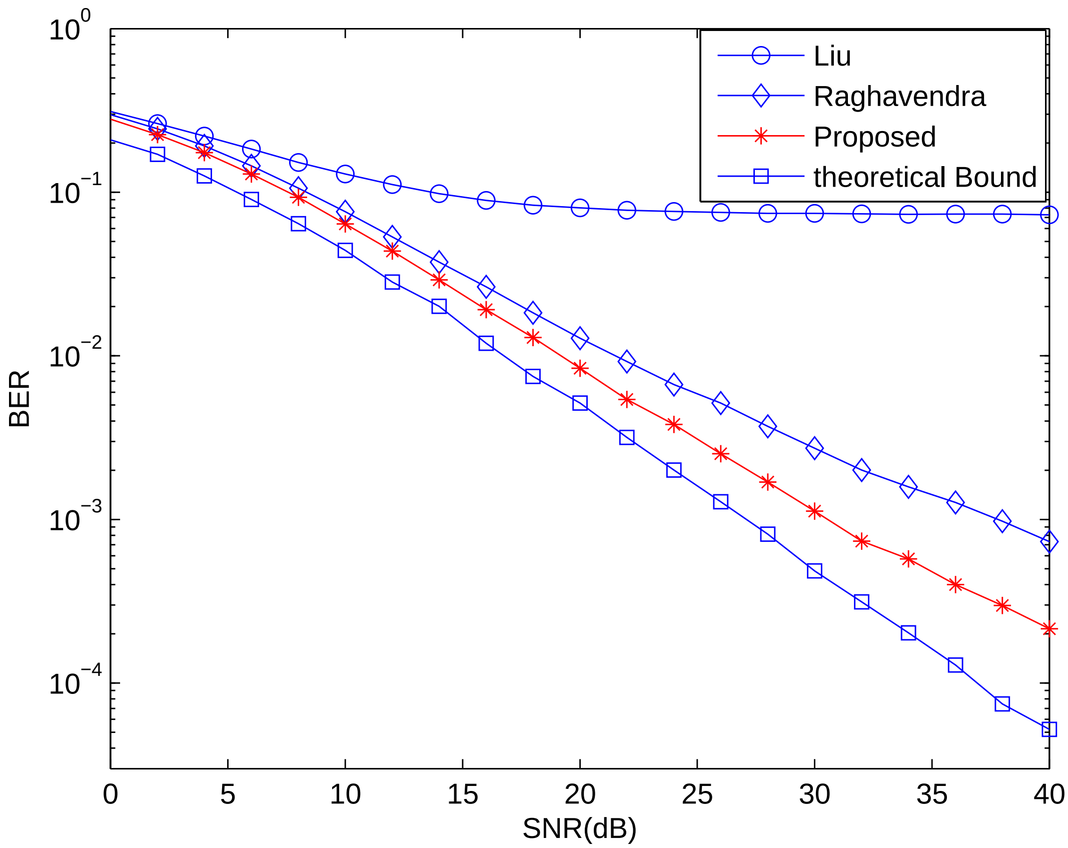

Figure 5.2: BERs of three considered channel estimation schemes.

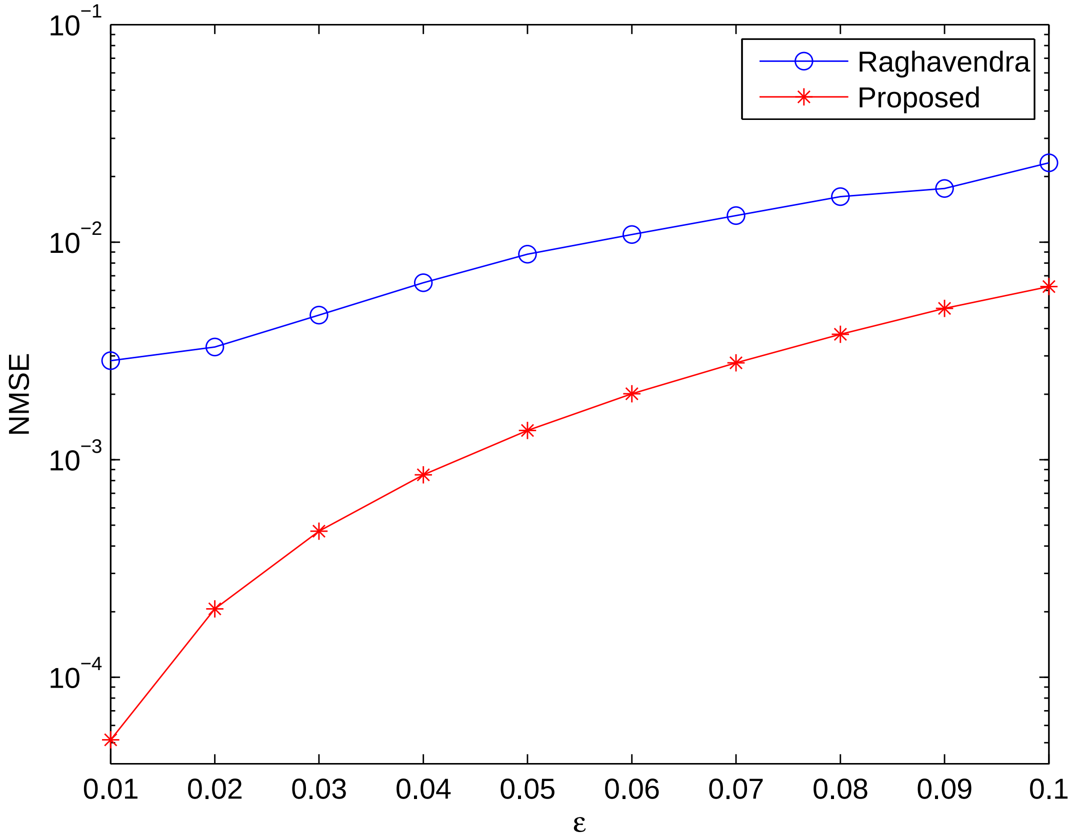

Figure 5.3: NMSEs of the proposed and Raghavendra's ESPRIT-based algorithms when the noise vector is $\boldsymbol{v}_k = \varepsilon[1, 1, ..., 1]^{\mathrm{T}}$.

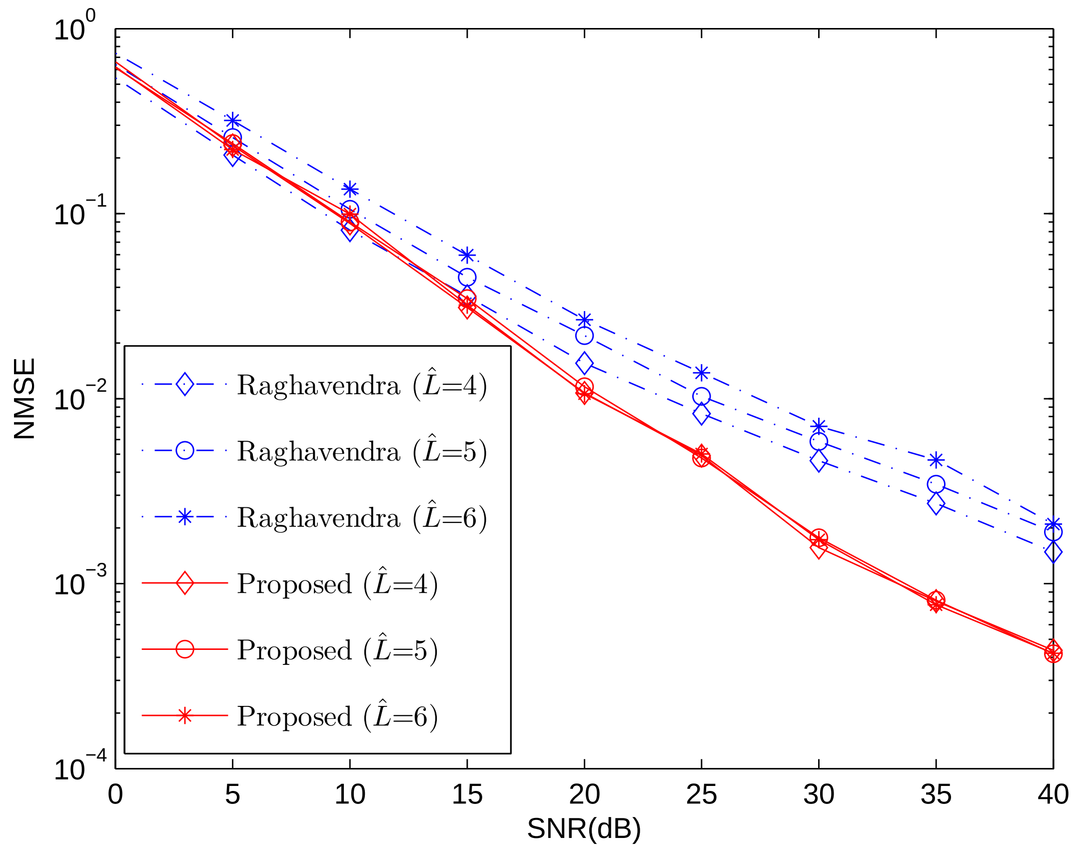

Figure 5.4: NMSEs when the number of channel paths is over-estimated.

Chapter 6

Conclusion

This thesis has presented a new model-based channel estimation scheme for OFDM systems. The proposed method was designed based on the general assumption that the multi-path delays are not necessarily integer multiples of the sampling period. The main computations involved in our method are: (i) the inversion of a sub-matrix of the autocorrelation matrix of the estimated pilot frequency responses, and (ii) an EVD of a matrix product. When being compared with the existing ESPRIT-based algorithm, the proposed approach benefits from the reduced-dimensional processing and thus results in a lower algorithmic complexity. Through simulations, we also show that the proposed solution can yield an improved channel estimation performance as well as a lower symbol decision error rate.

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