Bit-wise Decomposition of M-ary Symbol Metric

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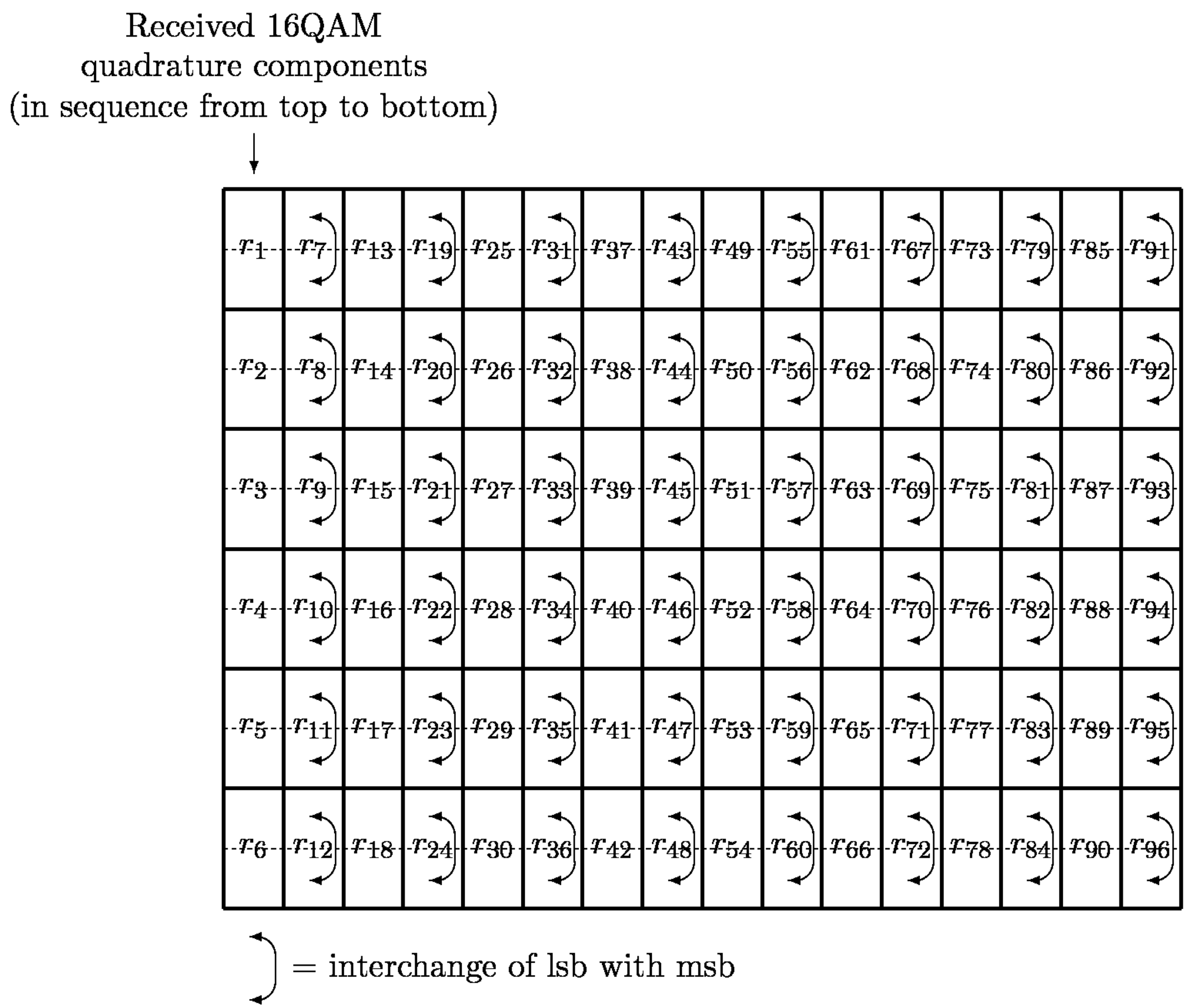
Outline

- Motivation
- Systematic Bit-wise Decomposition of M-ary Symbol Metric
- Performance Evaluation over the AWGN & Rayleigh Flat Fading Channel
- Performance Impact of Code Punctuation
- Realization of Systematic Bit-wise Decomposition Metric
- Conclusion

Motivation

- Motivation
  - The state-of-the-art wireless transmission technique of IEEE 802.11a/g incorporated high QAM into OFDM to achieve a high data rate.
  - The standard also specified a two-step bit-wise interleaver.
    - The first step maps adjacent code bits onto non-adjacent sub-carriers;
    - The second step permutes the code bits alternately onto less and more significant bits of the QAM constellation.
- Problem Formulation
  - Such an interleaver design, although straight and simple in concept, may restrict the potential structures of a receiver in practice.

- The first step of the de-interleaver

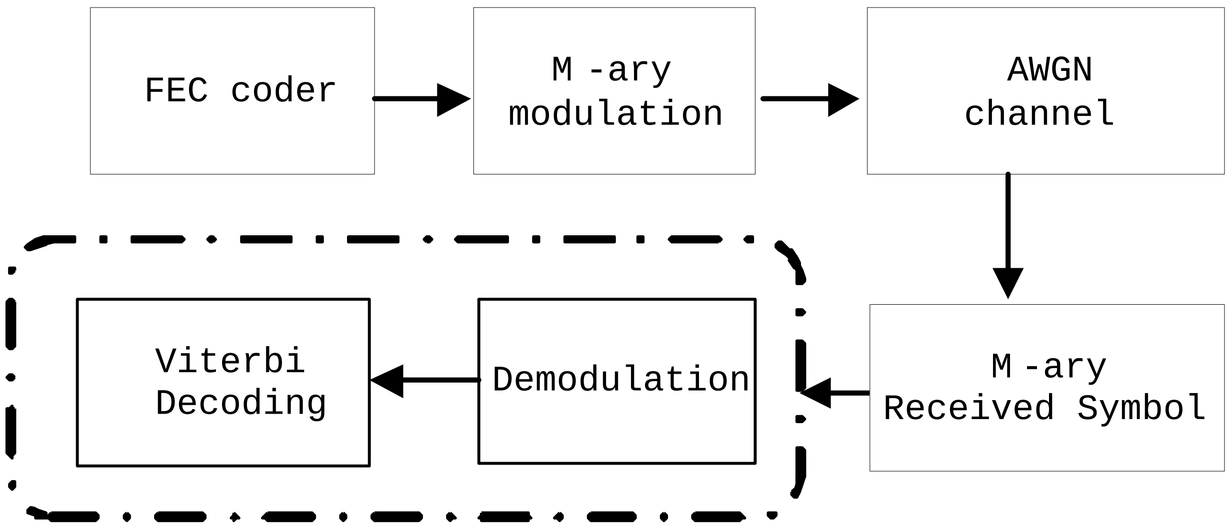


- The second step of the de-interleaver

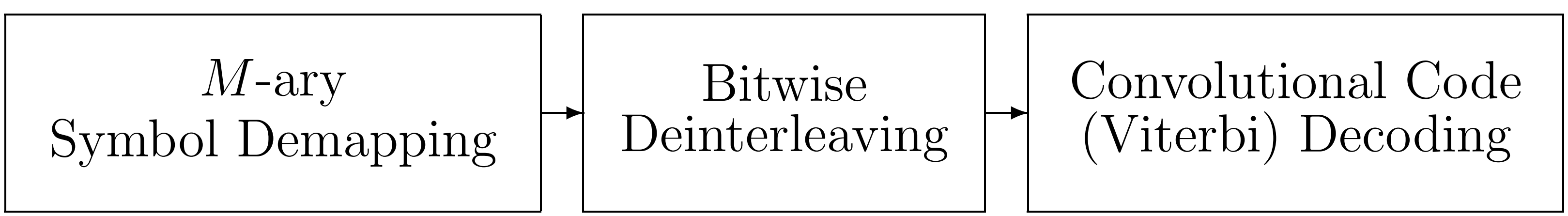
c_{1}	c_{2}	c_{3}	c_{4}	c_{5}	c_{6}	c_{7}	c_{8}	c_{9}	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}
c_{17}	c_{18}	c_{19}	c_{20}	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}	c_{31}	c_{32}
c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}	c_{41}	c_{42}	c_{43}	c_{44}	c_{45}	c_{46}	c_{47}	c_{48}
c_{49}	c_{50}	c_{51}	c_{52}	c_{53}	c_{54}	c_{55}	c_{56}	c_{57}	c_{58}	c_{59}	c_{60}	c_{61}	c_{62}	c_{63}	c_{64}
c_{65}	c_{66}	c_{67}	c_{68}	c_{69}	c_{70}	c_{71}	c_{72}	c_{73}	c_{74}	c_{75}	c_{76}	c_{77}	c_{78}	c_{79}	c_{80}
c_{81}	c_{82}	c_{83}	c_{84}	c_{85}	c_{86}	c_{87}	c_{88}	c_{89}	c_{90}	c_{91}	c_{92}	c_{93}	c_{94}	c_{95}	c_{96}
c_{97}	c_{98}	c_{99}	c_{100}	c_{101}	c_{102}	c_{103}	c_{104}	c_{105}	c_{106}	c_{107}	c_{108}	c_{109}	c_{110}	c_{111}	c_{112}
c_{113}	c_{114}	c_{115}	c_{116}	c_{117}	c_{118}	c_{119}	c_{120}	c_{121}	c_{122}	c_{123}	c_{124}	c_{125}	c_{126}	c_{127}	c_{128}
c_{129}	c_{130}	c_{131}	c_{132}	c_{133}	c_{134}	c_{135}	c_{136}	c_{137}	c_{138}	c_{139}	c_{140}	c_{141}	c_{142}	c_{143}	c_{144}
c_{145}	c_{146}	c_{147}	c_{148}	c_{149}	c_{150}	c_{151}	c_{152}	c_{153}	c_{154}	c_{155}	c_{156}	c_{157}	c_{158}	c_{159}	c_{160}
c_{161}	c_{162}	c_{163}	c_{164}	c_{165}	c_{166}	c_{167}	c_{168}	c_{169}	c_{170}	c_{171}	c_{172}	c_{173}	c_{174}	c_{175}	c_{176}
c_{177}	c_{178}	c_{179}	c_{180}	c_{181}	c_{182}	c_{183}	c_{184}	c_{185}	c_{186}	c_{187}	c_{188}	c_{189}	c_{190}	c_{191}	c_{192}

→ Convolutional code bits (from left to right)

- Traditional Symbol-wise Soft-decision Receiver



- New Bit-wise Soft-decision Receiver



Hard Decomposition of M-ary Symbol Metric

- Denote by $\boldsymbol{r} = (r_1, r_2, \ldots, r_K)$ the real-valued received vector when M-ary symbols $\boldsymbol{s} = (s_1,\ s_2,\ \ldots,\ s_K)$ that are mapped from an interleaved version of encoding output $\boldsymbol{c} = (c_1, c_2, \ldots, c_N) \in \{0, 1\}^N$ are transmitted.
- Assumed that the M-ray symbol transmission suffers AWGN $(n_1, n_2, \ldots, n_K)$. This forms a channel model of $r_i = s_i + n_i$ for $1 \le i \le K$.
- **Hard** decision first determines the convolution bit-streams from received scalars (through the nearest M-ary symbol). Then use Viterbi algorithm to decode the information bits after de-interleaving.

Soft Decomposition of M-ary Symbol Metric (1)

- A well-known result for AWGN channel model is that the maximum-likelihood decision upon the receipt of $\boldsymbol{r}$ is given by:

$$\begin{aligned} d_{ML}(\boldsymbol{r}) &= \arg\max_{\boldsymbol{s}\in\mathcal{S}} \Pr\{r_1,\ldots,r_K\,|s_1,\ldots,s_K\} \\ &= \arg\max_{\boldsymbol{s}\in\mathcal{S}} \frac{1}{(\pi N_0)^{K/2}} \exp\left\{-\sum_{i=0}^{K}\frac{(r_i-s_i)^2}{N_0}\right\} \\ &= \arg\min_{\boldsymbol{s}\in\mathcal{S}} \sum_{i=1}^{K}(r_i-s_i)^2 \end{aligned}$$

- Due to the non-linear (e.g., interleaving) relation between codewords and M-ary symbol words, the above formula cannot be equivalently transformed to the sum of code bit metrics.

Soft Decomposition of M-ary Symbol Metric (2)

- Target

$$\sum_{i=1}^{N} f_i(c_i, r) \;\doteqdot\; \sum_{i=1}^{K} (r_i - s_i)^2$$

- Squared error criterion for 16QAM

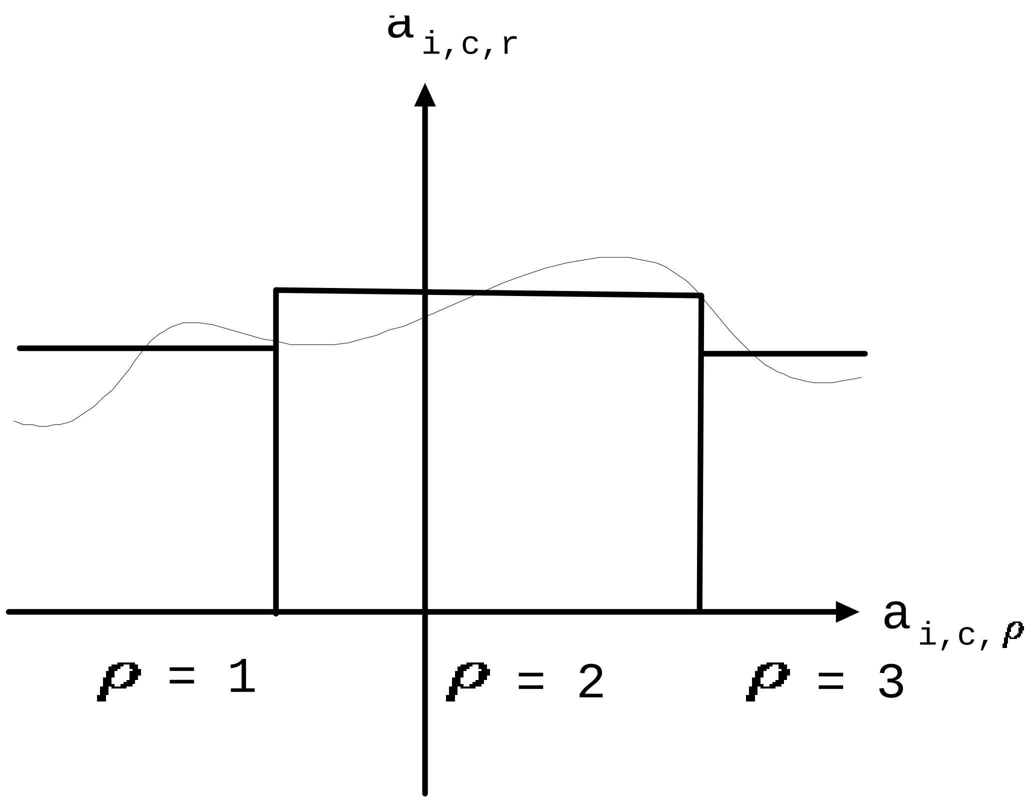
$$\min_{f_1, f_2} W(f_1, f_2) \;=\; \min_{f_1, f_2} E\left[\left(f_1(c, r) + f_2(\bar{c}, r) - [r - s(c, \bar{c})]^2\right)^2\right]$$

  – Without loss of generality, let

$$\begin{aligned} f_1(r, c) &= \frac{1}{2} r^2 + a_{1,c,r} r + b_{1,c,r} \\ f_2(r, c) &= \frac{1}{2} r^2 + a_{2,c,r} r + b_{2,c,r} \end{aligned}$$

Soft Decomposition of M-ary Symbol Metric (3)

- Relationship between $a_{2,c,r}$ and $a_{2,c,\rho}$



Soft Decomposition of M-ary Symbol Metric (4)

- For example, if $s(c, \bar{c})$ = Gray code mapping to 16QAM quadrature component,

$$W(f_1, f_2) = \frac{1}{4\sqrt{\pi N_0}} \sum_{\rho=1}^{q}$$

$$\left(\int_{\lambda_{\rho-1}}^{\lambda_\rho} \left[(a_{1,0,\rho} + a_{2,0,\rho} - 6) r + (b_{1,0,\rho} + b_{2,0,\rho} - 9) \right]^2 e^{-(r+3)^2/N_0} dr \right.$$

$$+ \int_{\lambda_{\rho-1}}^{\lambda_\rho} \left[(a_{1,0,\rho} + a_{2,1,\rho} - 2) r + (b_{1,0,\rho} + b_{2,1,\rho} - 1) \right]^2 e^{-(r+1)^2/N_0} dr$$

$$+ \int_{\lambda_{\rho-1}}^{\lambda_\rho} \left[(a_{1,1,\rho} + a_{2,1,\rho} + 2) r + (b_{1,1,\rho} + b_{2,1,\rho} - 1) \right]^2 e^{-(r-1)^2/N_0} dr$$

$$\left. + \int_{\lambda_{\rho-1}}^{\lambda_\rho} \left[(a_{1,1,\rho} + a_{2,0,\rho} + 6) r + (b_{1,1,\rho} + b_{2,0,\rho} - 9) \right]^2 e^{-(r-3)^2/N_0} dr \right)$$

Soft Decomposition of M-ary Symbol Metric (5)

- **⟨Algorithm Q⟩**

  Initialization: Input L ordered equations with indices 1, 2, ..., *L.*

  Step 1. For each $1 \le \rho \le L$, find the largest set A_ρ of equations
  - *that can be simultaneously satisfied by some solutions, and*
  - *that are contiguous in their equation numbers, and*
  - *that contains equation ρ (namely, the equation whose index equals ρ).*

  Step 2. Delete all duplicate sets among $A_1, A_2, \ldots, A_L$. (Let q be the number of sets remained.)

  Step 3. Output the remaining q (distinct) sets.

Soft Decomposition of M-ary Symbol Metric (6)

- Sub-optimum solution

$$
\begin{aligned}
s(0,0) = -3 &\Rightarrow a_{1,0,\rho} + a_{2,0,\rho} - 6 = 0, \quad b_{1,0,\rho} + b_{2,0,\rho} - 9 = 0 \quad (1)\\
s(0,1) = -1 &\Rightarrow a_{1,0,\rho} + a_{2,1,\rho} - 2 = 0, \quad b_{1,0,\rho} + b_{2,1,\rho} - 1 = 0 \quad (2)\\
s(1,1) = +1 &\Rightarrow a_{1,1,\rho} + a_{2,1,\rho} + 2 = 0, \quad b_{1,1,\rho} + b_{2,1,\rho} - 1 = 0 \quad (3)\\
s(1,0) = +3 &\Rightarrow a_{1,1,\rho} + a_{2,0,\rho} + 6 = 0, \quad b_{1,1,\rho} + b_{2,0,\rho} - 9 = 0 \quad (4)
\end{aligned}
$$

- Applying Algorithm Q to (1)–(4) results:

iteration	seed	set
1	(1), (2), (3)	A_1
2	(1), (2), (3)	—
3	(2), (3), (4)	A_2
4	(2), (3), (4)	—

Soft Decomposition of M-ary Symbol Metric (7)

- Sub-optimum solution

$$A_1(\rho=1) \quad : \quad \begin{cases} a_{1,0,1} = 2 - a_{2,1,1} \\ a_{1,1,1} = -2 - a_{2,1,1} \\ a_{2,0,1} = 4 + a_{2,1,1} \\ b_{1,0,1} = b_{1,1,1} = 1 - b_{2,1,1} \\ b_{2,0,1} = 8 + b_{2,1,1} \end{cases}$$

$$A_3(\rho=2) \quad : \quad \begin{cases} a_{1,0,2} = 2 - a_{2,1,2} \\ a_{1,1,2} = -2 - a_{2,1,2} \\ a_{2,0,2} = -4 + a_{2,1,2} \\ b_{1,0,2} = b_{1,1,2} = 1 - b_{2,1,2} \\ b_{2,0,2} = 8 + b_{2,1,2} \end{cases}$$

Soft Decomposition of M-ary Symbol Metric (8)

- Then we can find the rules:

$$\begin{cases} f_1^{16\text{QAM}}(c,r) &= c|r|\cdot\text{sgn}(-r) \\ f_2^{16\text{QAM}}(c,r) &= c(|r|-2) \end{cases}$$

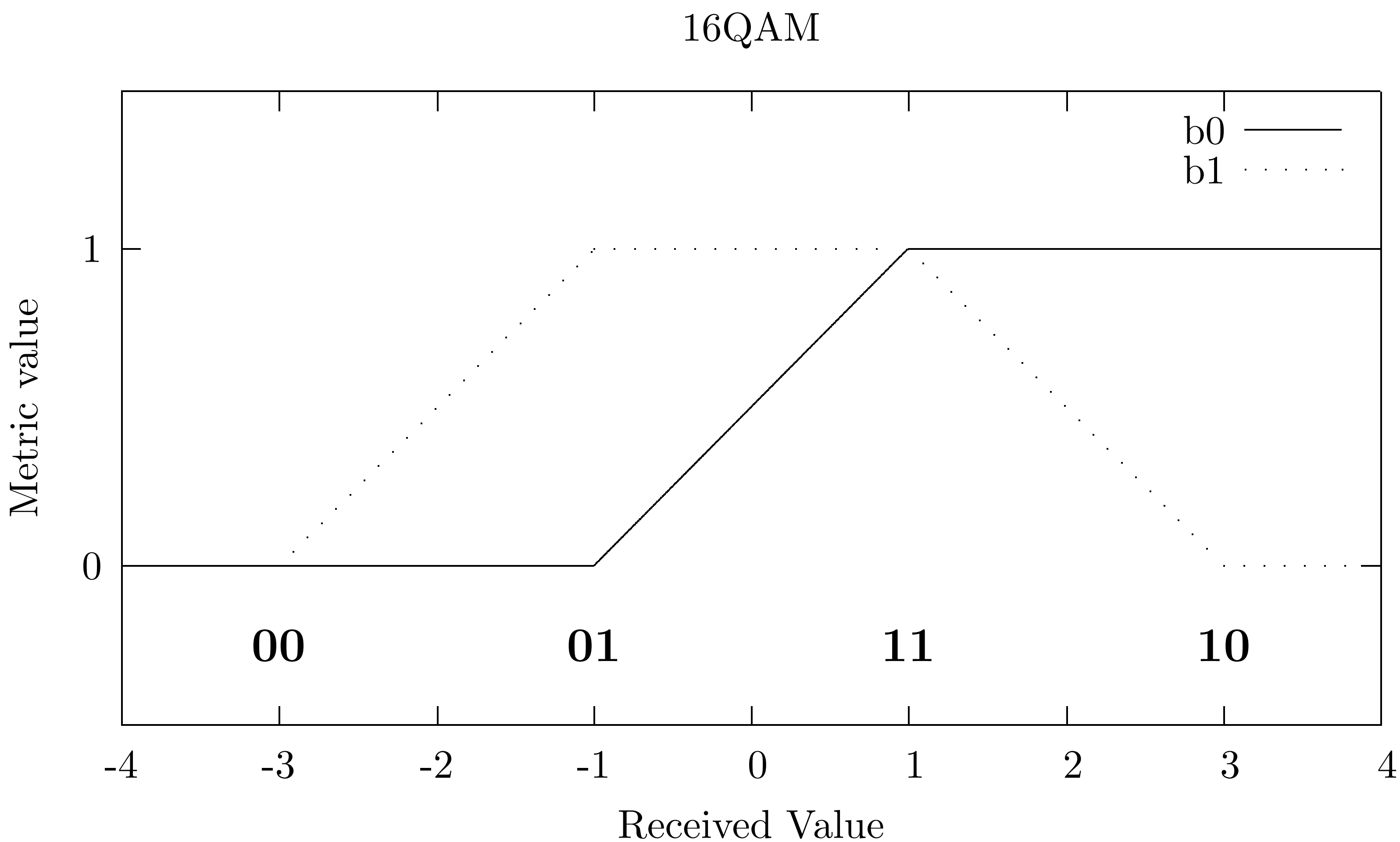
$$\begin{cases} f_1^{64\text{QAM}}(c,r) &= c(|r-4|+|r|+|r+4|-8)\cdot\text{sgn}(-r) \\ f_2^{64\text{QAM}}(c,r) &= f_1^{16\text{QAM}}(c,4-|r|) \\ f_3^{64\text{QAM}}(c,r) &= f_2^{16\text{QAM}}(c,|r|-4) \end{cases}$$

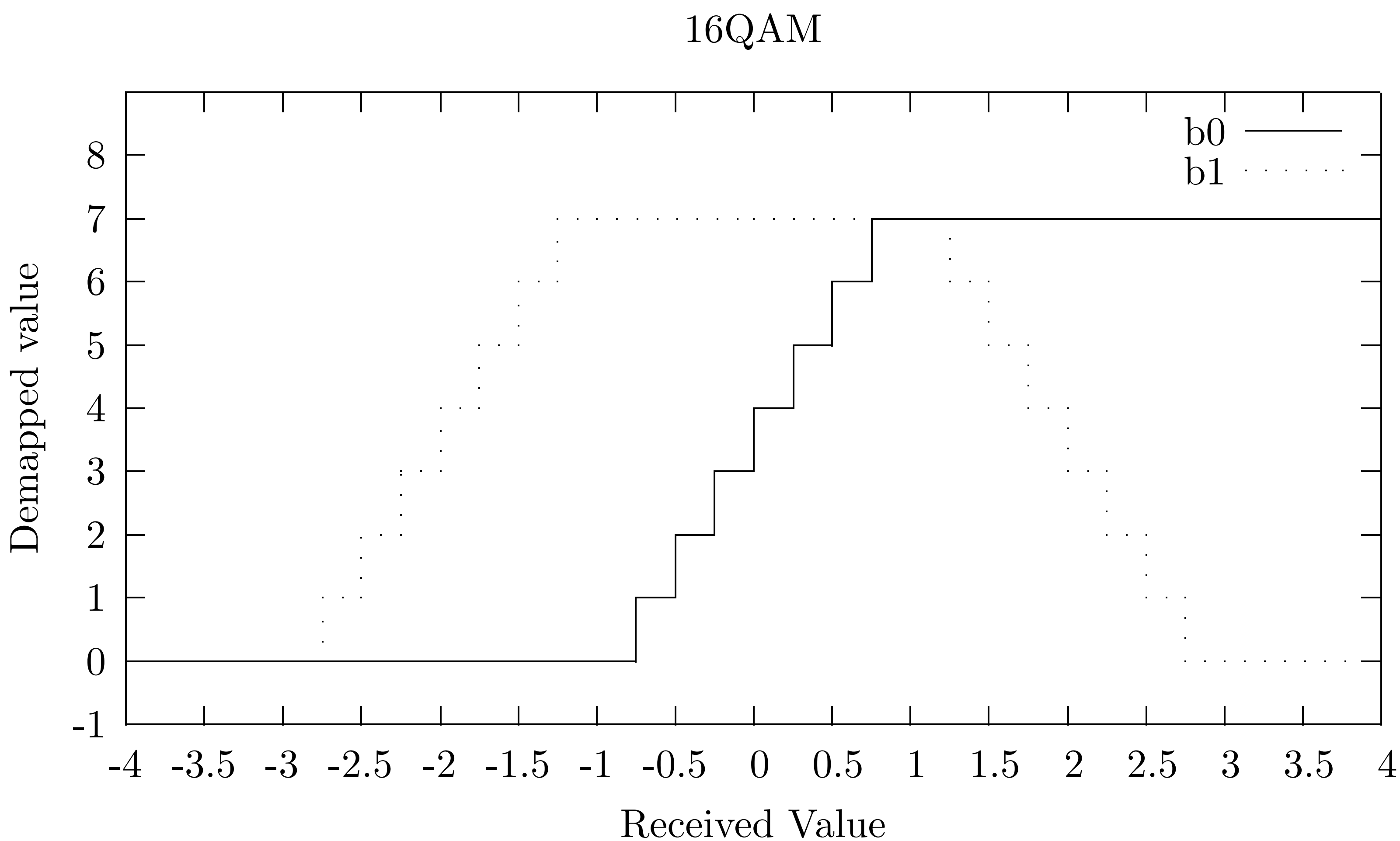
$m = \log_2(M)$ for M^2-QAM

$$\begin{cases} f_1^{(\text{m})}(c,r) &= c\cdot\text{sgn}(-r)\left(\sum_{i=-(m-2)}^{m-2}(|r+2ui|-|2ui|)\right) \\ f_j^{(\text{m})}(c,r) &= f_{j-1}^{(\text{m}-1)}\left(c,(-1)^j\left[2^{m-2}u-|r|\right]\right) \text{ for } 2\leq j\leq m \end{cases}$$

Other Soft Decomposition Metric Method (1)

- Soft-demapping for Bit-interleaved Coded Modulation
  - An intuitive approach to soft-demap bit-interleaved coded modulation signals is to directly quantize the received vector according to the *inverse* of modulation mapping function.
  - Gray code mapping is applied to 16QAM modulation, where $s(0,0) = -3$, $s(0,1) = -1$, $s(1,1) = 1$ and $s(1,0) = 3$, the inverse mapping for the first bit position is $b_0^{-1}(-3) = 0$, $b_0^{-1}(-1) = 0$, $b_0^{-1}(1) = 1$ and $b_0^{-1}(3) = 1$, and similarly the inverse mapping for the second bit position can be obtained as $b_1^{-1}(-3) = 0$, $b_1^{-1}(-1) = 1$, $b_1^{-1}(1) = 1$ and $b_1^{-1}(3) = 0$.

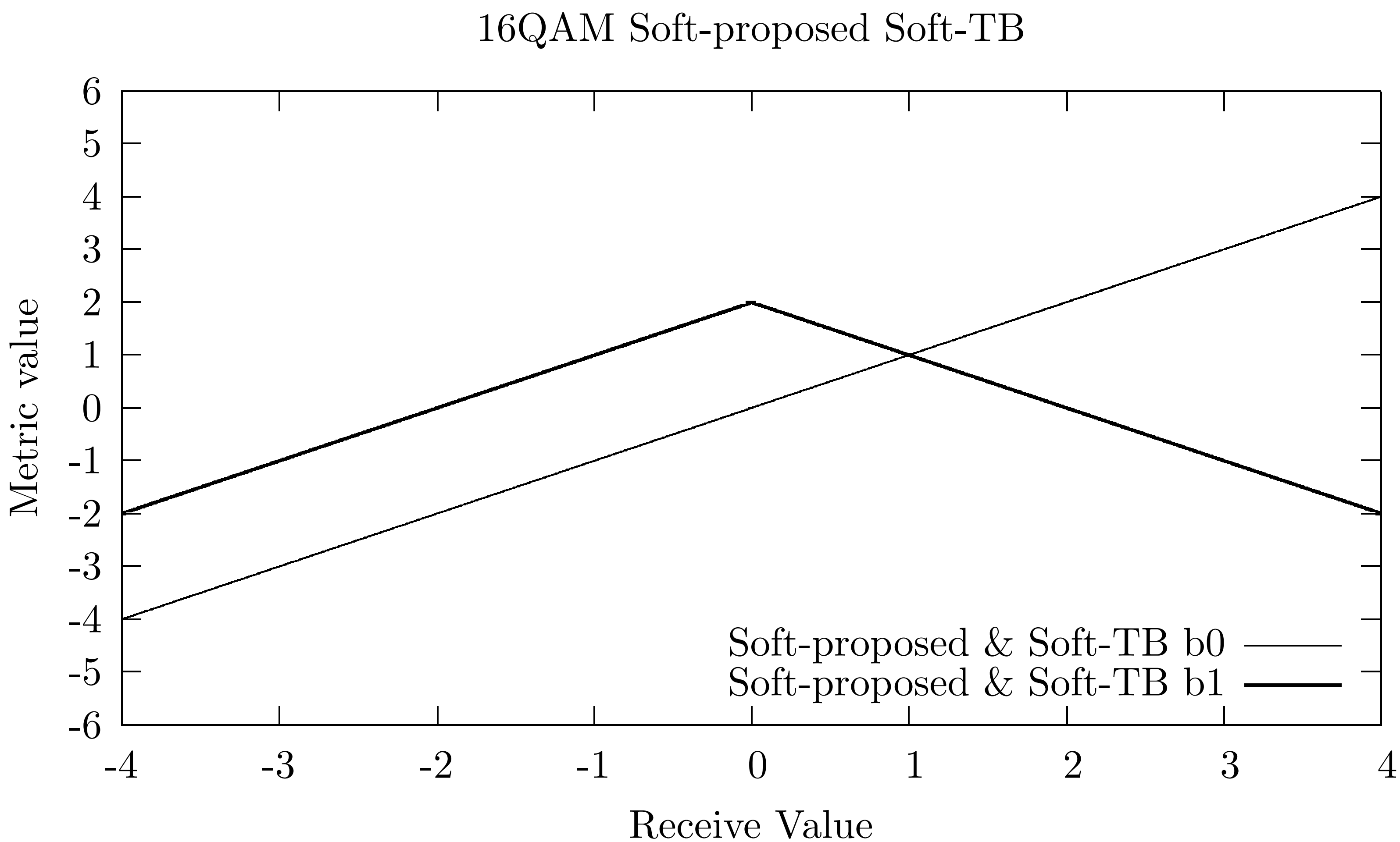

16QAM
b0
b1
1
0
Metric value
00
01
11
10
-4
-3
-2
-1
0
1
2
3
4
Received Value

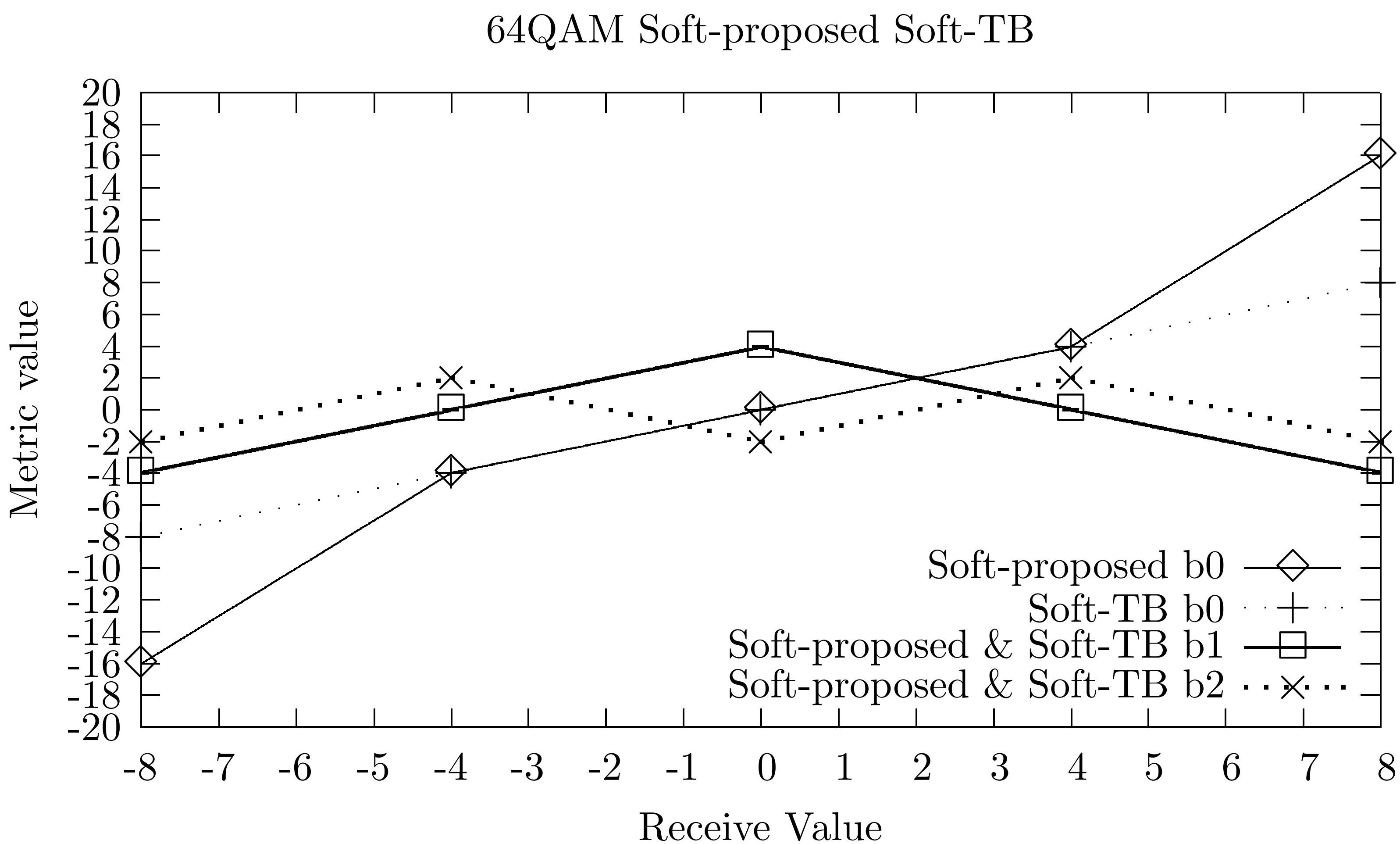

16QAM
b0
b1
Demapped value
Received Value
8
7
6
5
4
3
2
1
0
-1
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

Other Soft Decomposition Metric method (2)

- Bit Metrics Recursively Generated from Other First-bit Metric
  - In 2001, Tosato and Bisaglia proposed and examined a simplified soft-output demapper for binary interleaved COFDM with application to HIPERLAN/2.
  - We interestingly found that their proposed bit metrics $\{g_j^{(m)}(\cdot,\cdot)\}_{1\le j\le m, m\ge 1}$ for 2^{2m}-QAM can be equivalently expressed in terms of our recursive formula as:

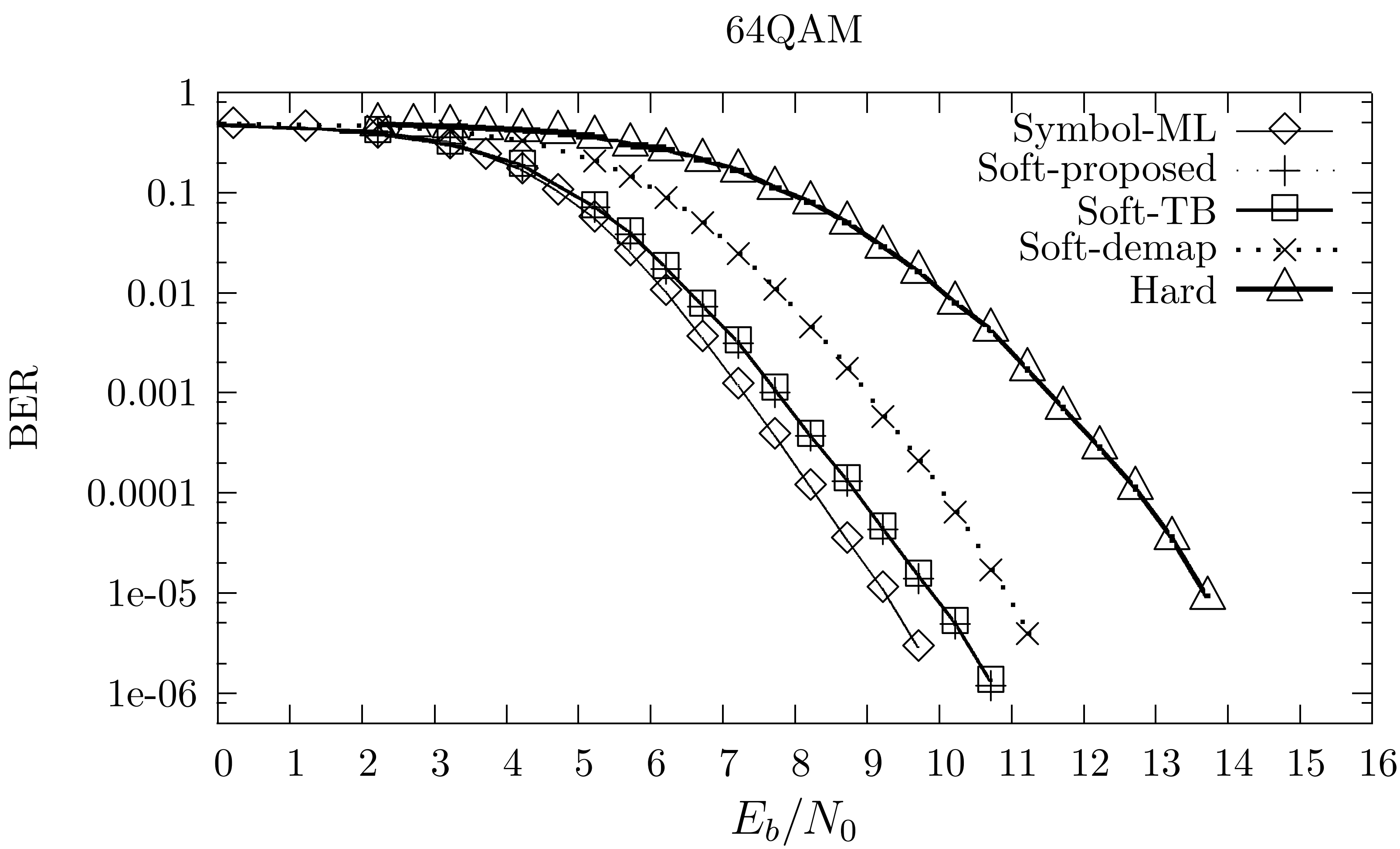
$$
\begin{aligned}
g_1^{(m)}(c,r) &= c(-r) \quad \text{for } m = 1,2,3,\ldots \\
g_j^{(\mathrm{m})}(c,r) &= g_{j-1}^{(\mathrm{m}-1)}\big(c,(-1)^j(2^{m-2}u-|r|)\big) \quad \text{for } 2 \le j \le m.
\end{aligned}
$$


16QAM Soft-proposed Soft-TB
Metric value
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-4
-3
-2
-1
0
1
2
3
4
Receive Value
Soft-proposed & Soft-TB b0
Soft-proposed & Soft-TB b1

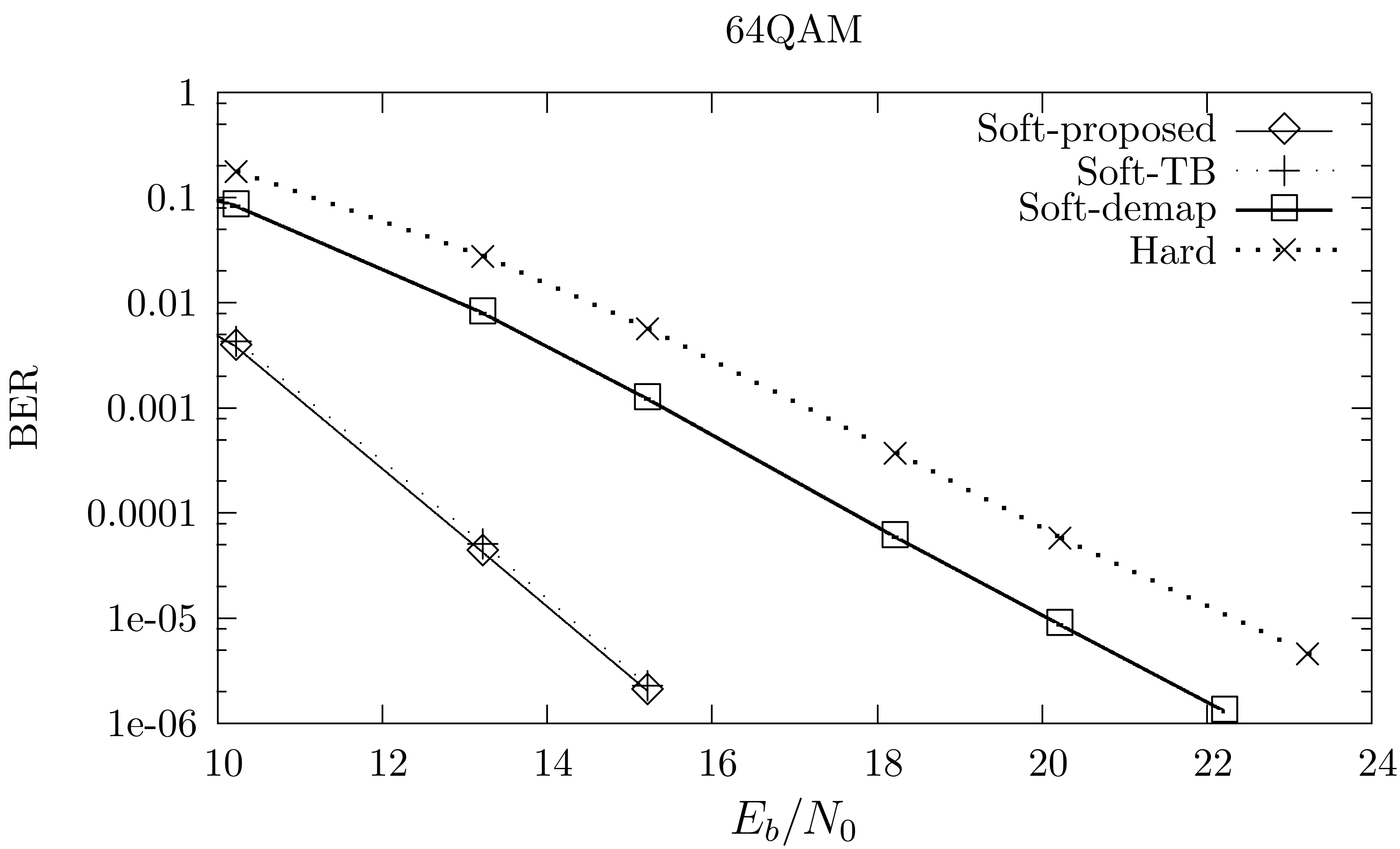


64QAM Soft-proposed Soft-TB
Metric value
Receive Value
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
Soft-proposed b0
Soft-TB b0
Soft-proposed & Soft-TB b1
Soft-proposed & Soft-TB b2

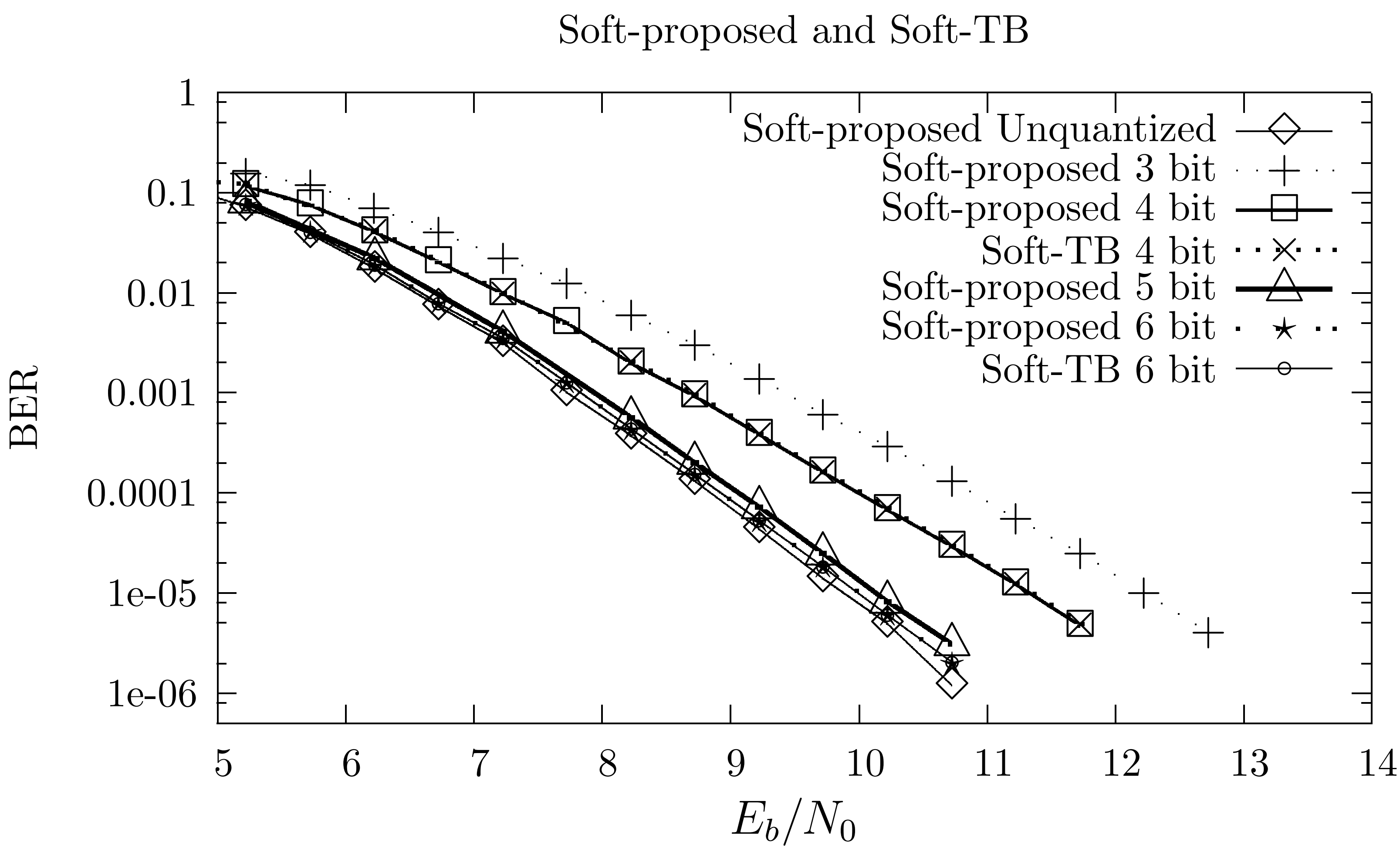
- At BER=10^{-5} under AWGN, Soft-proposed & Soft-TB have 3.9 dB over Hard and Soft-demap has only 2.9dB.



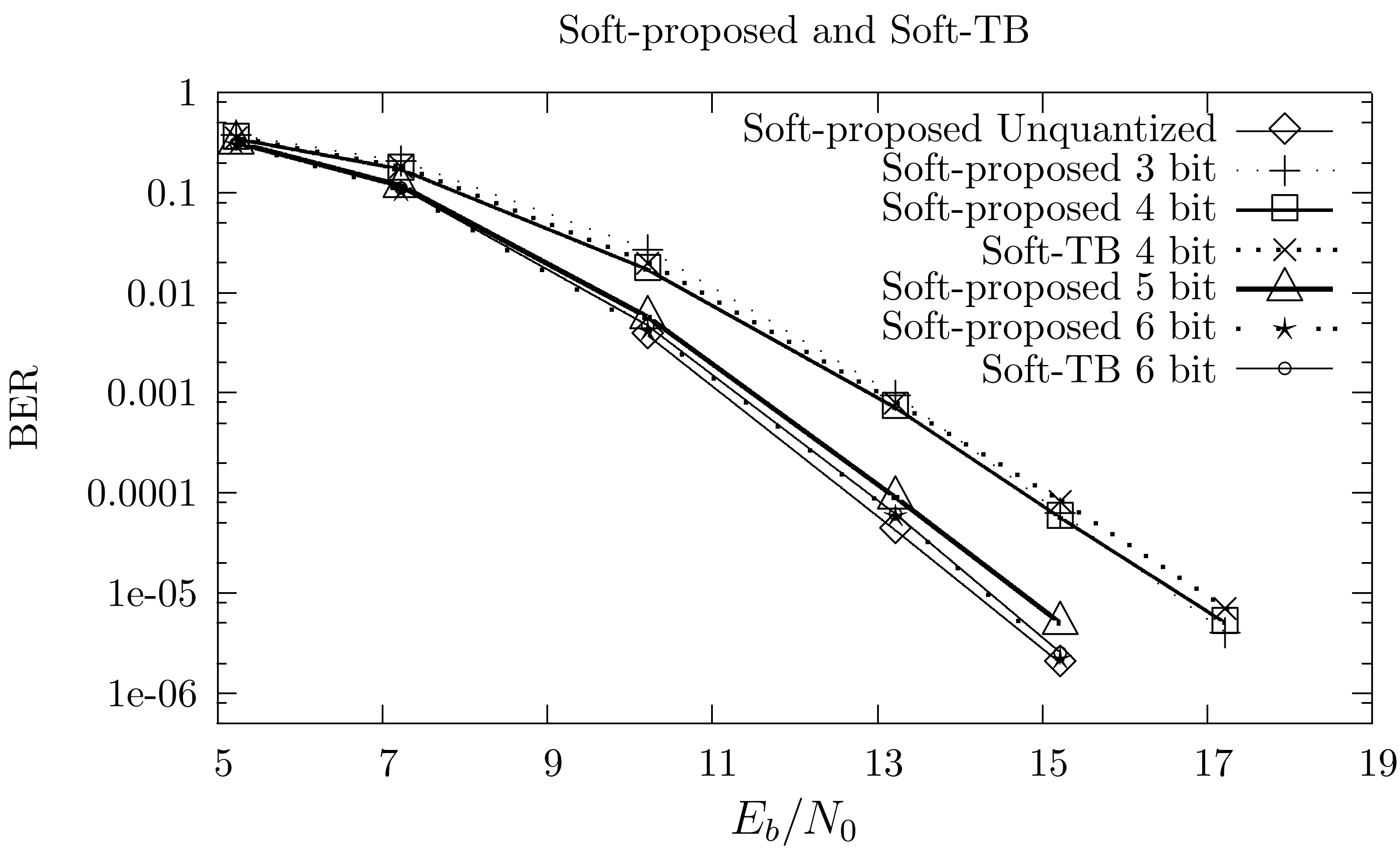
- At BER=10^{-5} under Rayleigh flat fading, Soft-proposed & Soft-TB have 8.1 dB over Hard and Soft-demap has only 2.0 dB.



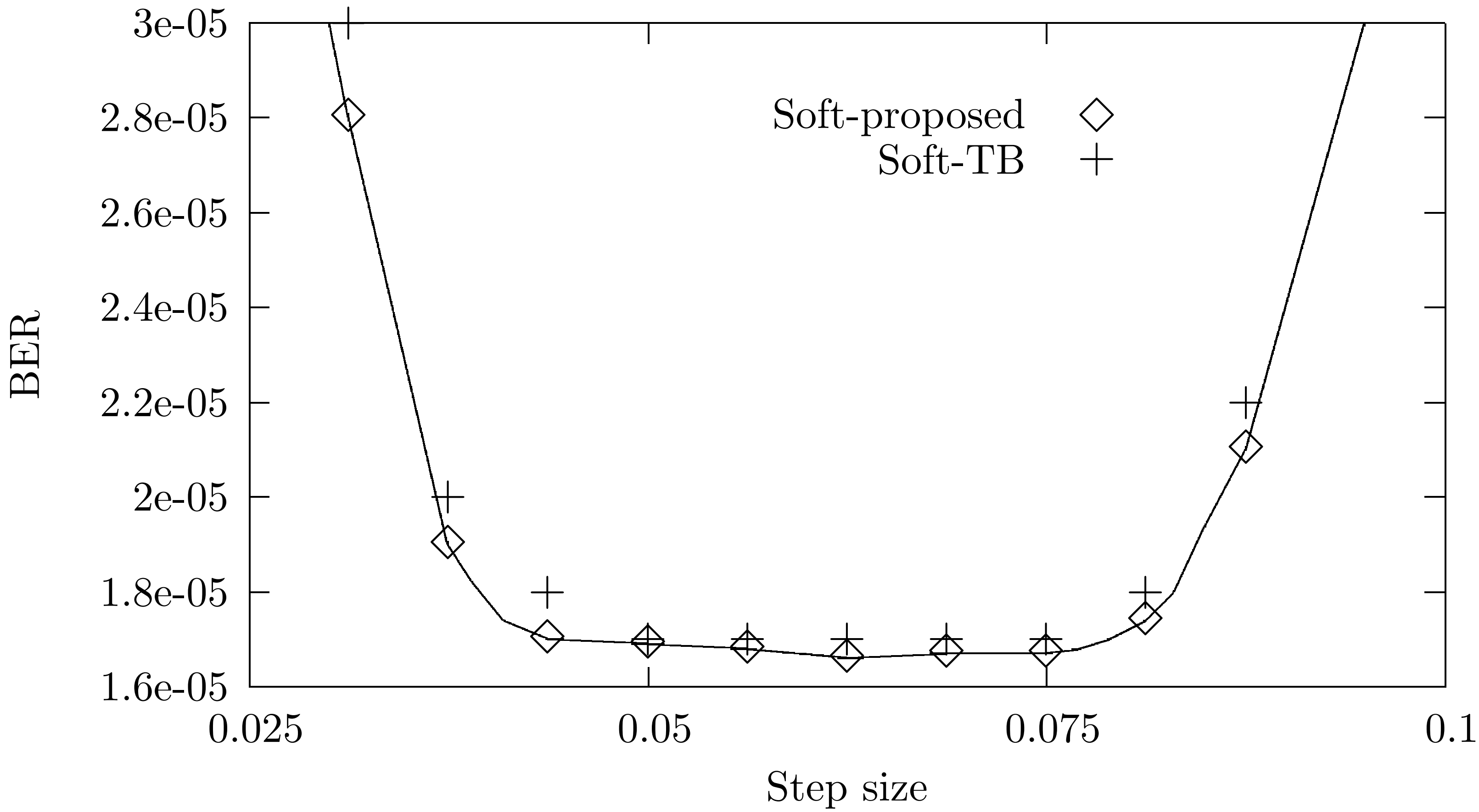
- At BER=10^{-5} under AWGN, adopting 32-level quantizer to Soft-proposed & Soft-TB only yields 0.11 dB performance loss.



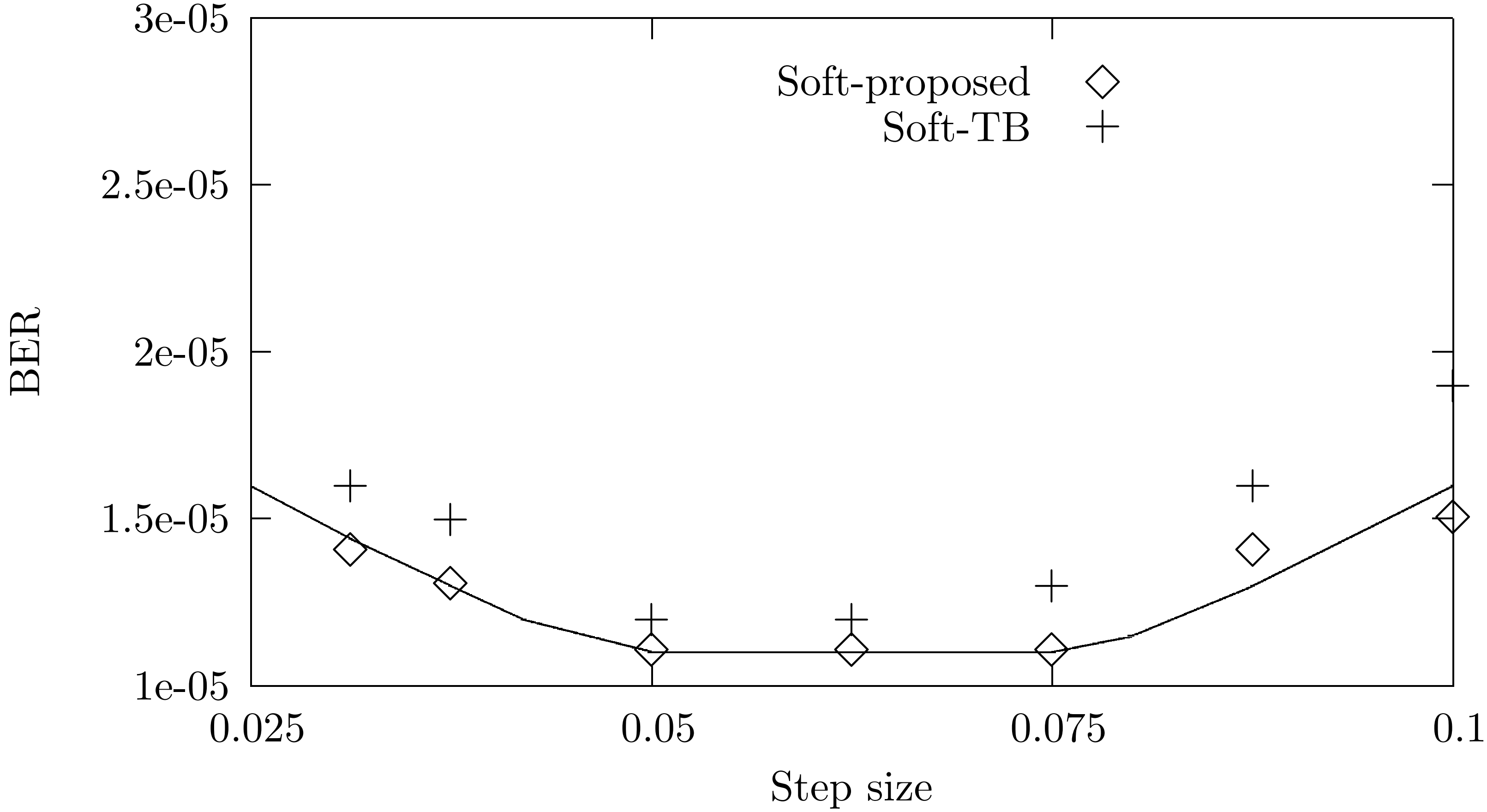
- At BER=10^{-5} under Rayleigh flat fading, 32-level quantizer yields performance loss extend to 0.3 dB.



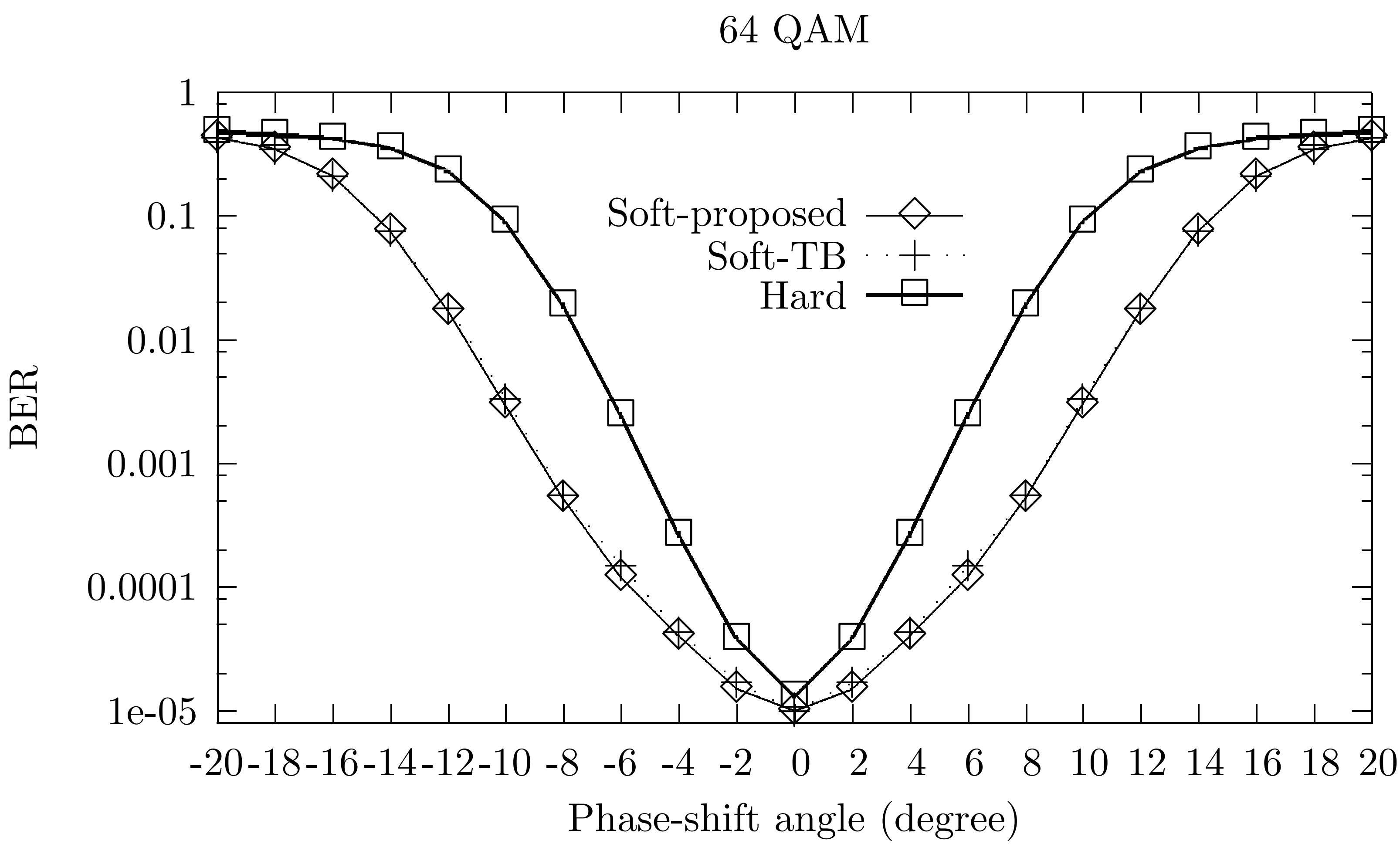
- Under AWGN, gain margin is $|0.04375 - 0.03125| \ / \ 0.03125 = |0.03125 - 0.01875| \ / \ 0.03125 = 40\%$ for BER :1.7 to 2.0×10^{-5}.



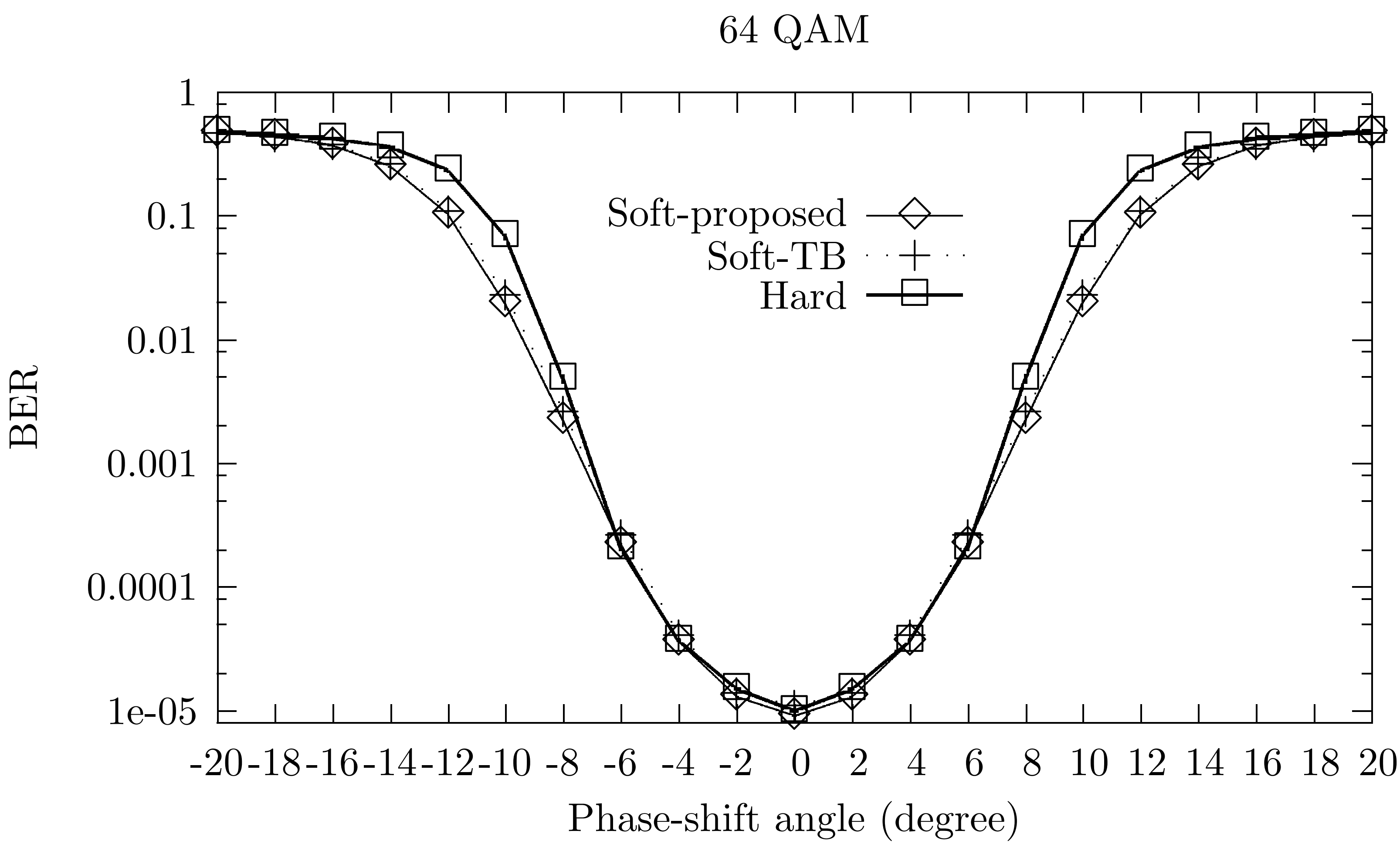
- The margin extends to $|0.03125 - 0.0625| \ / \ 0.0625 = |0.09375 - 0.0625| \ / \ 0.0625 = 50\%$ for BER : 1.2 to 1.5 $\times 10^{-5}$.



- For BER 1.0 to 10.0 $\times 10^{-5}$, Soft-proposed/Soft-TB, and Hard are $\pm 6^o$, and $\pm 3^o$ under AWGN.



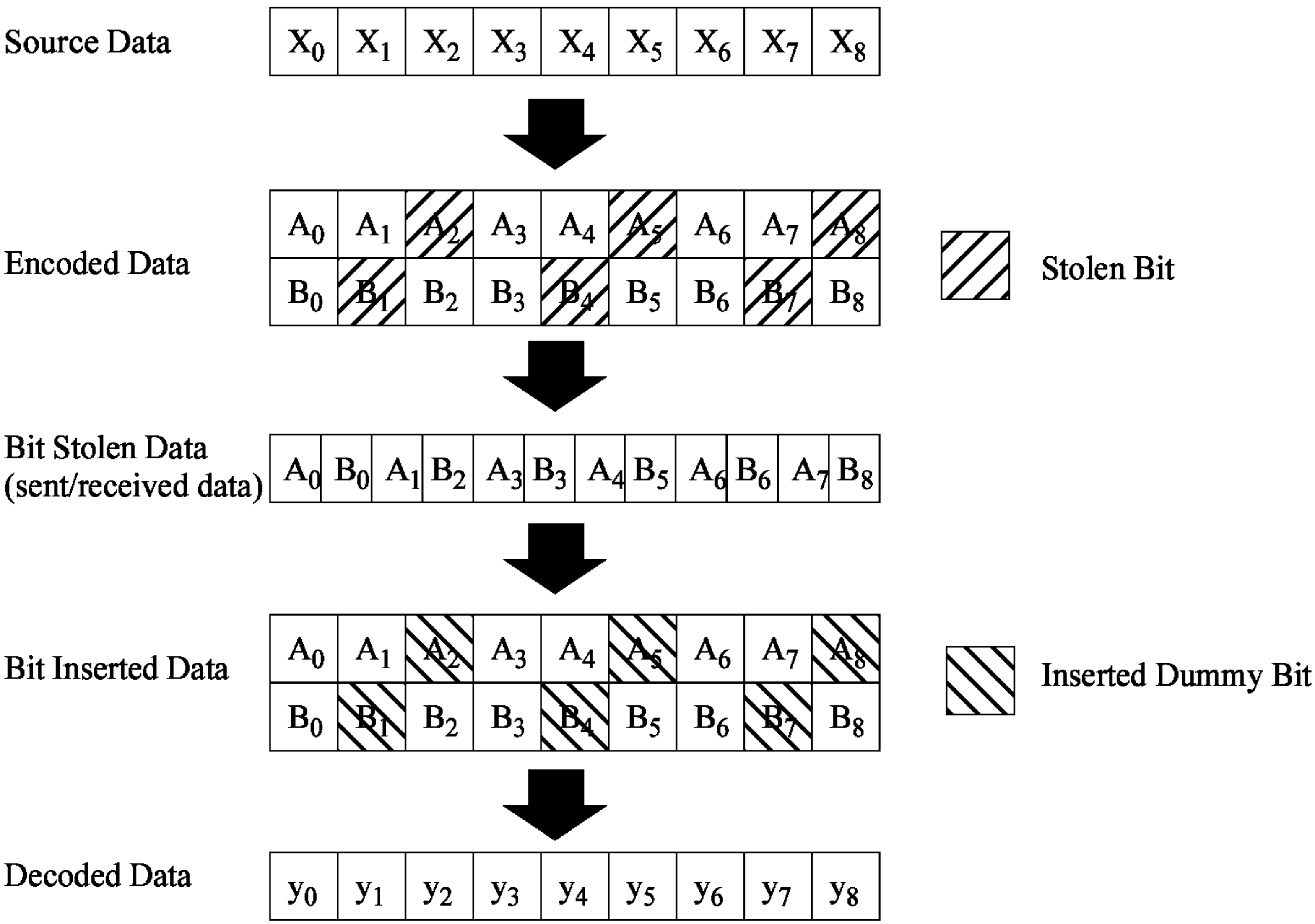
- For BER 1.0 to 10.0 $\times 10^{-5}$, Soft-proposed/Soft-TB, and Hard are $\pm 5^o$, and $\pm 5^o$ under Rayleigh flat fading.



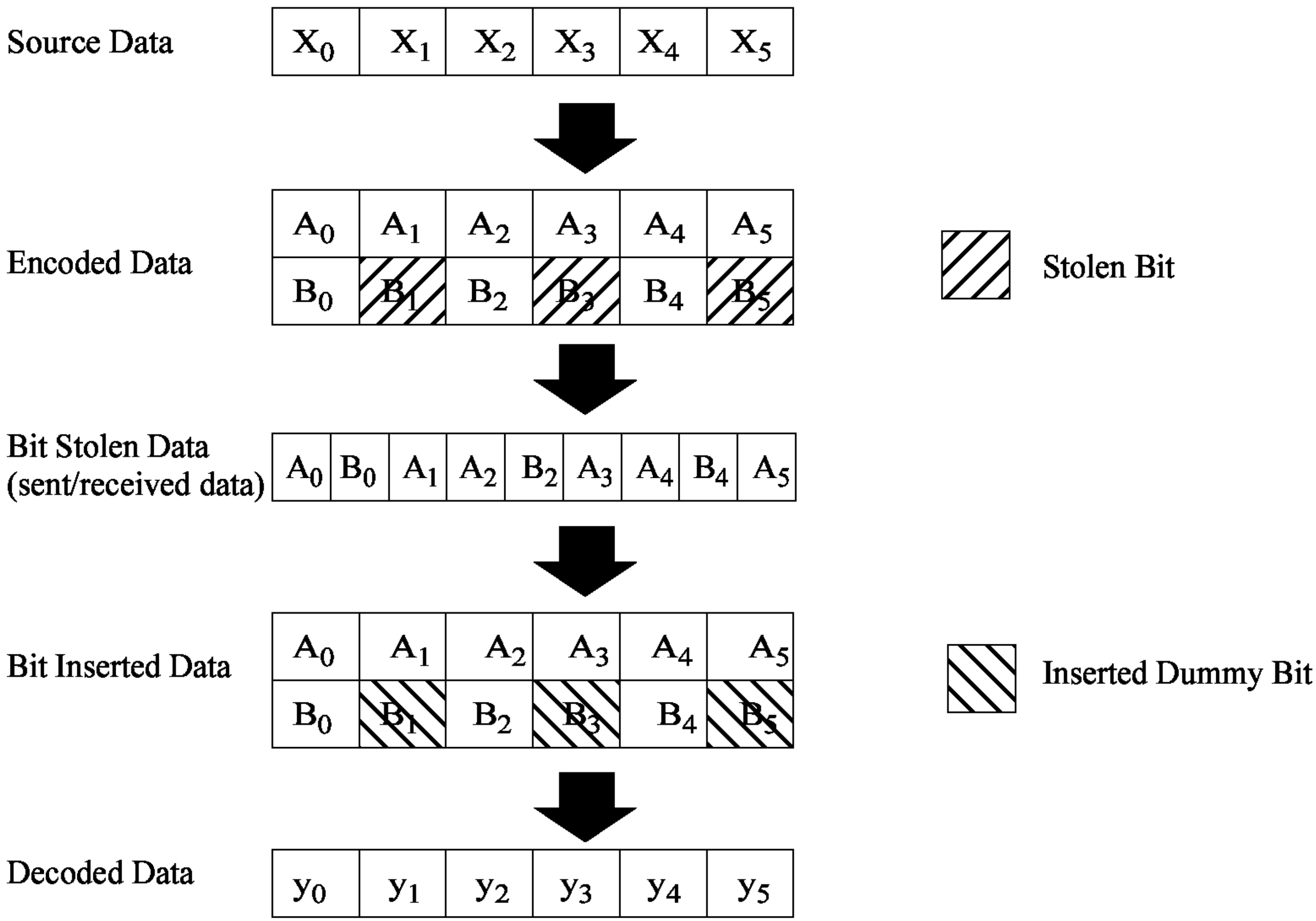
Performance Impact of Code Punctuation

- Description:
  - The DATA field of IEEE 802.11a/g, composed of SERVICE, PSDU, tail, and pad parts, shall be coded with a convolutional encoder of desired code rate R = 1/2, 2/3, or 3/4.
  - Code punctuation is a procedure to omit some of the encoded bits in the transmitter (thus reducing the number of transmitted bits and increasing the code rate) and inserting a dummy "zero" metric into the convolutional decoder at the positions of the received vector corresponding to that of the omitted bits.

Punctured Coding (r = 3/4)



Punctured Coding (r = 2/3)



802.11a/g rate dependent parameters

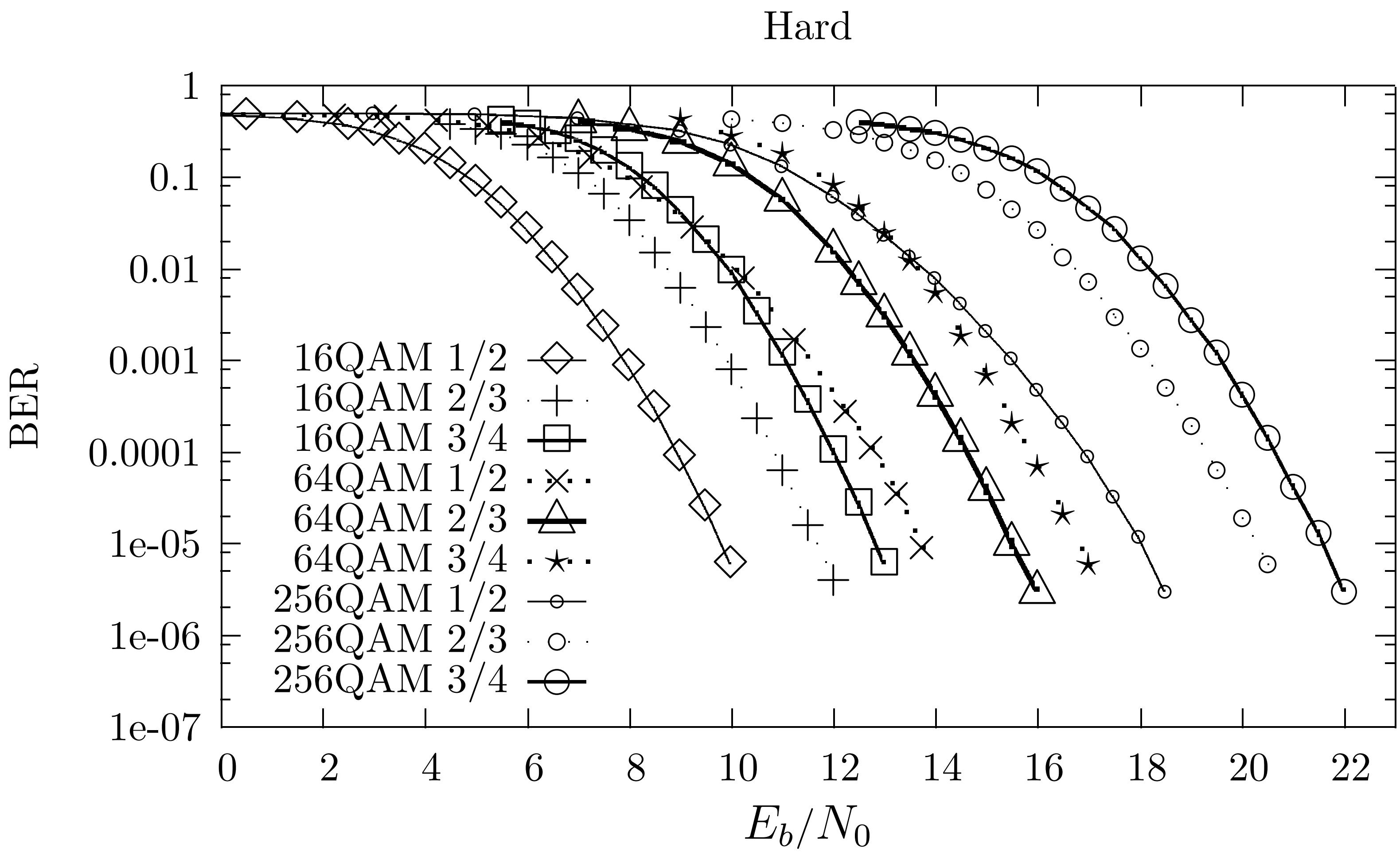
Table 78—Rate-dependent parameters

Data rate (Mbits/s)	Modulation	Coding rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPS})	Data bits per OFDM symbol (N_{DBPS})
6	BPSK	1/2	1	48	24
9	BPSK	3/4	1	48	36
12	QPSK	1/2	2	96	48
18	QPSK	3/4	2	96	72
24	16-QAM	1/2	4	192	96
36	16-QAM	3/4	4	192	144
48	64-QAM	2/3	6	288	192
54	64-QAM	3/4	6	288	216

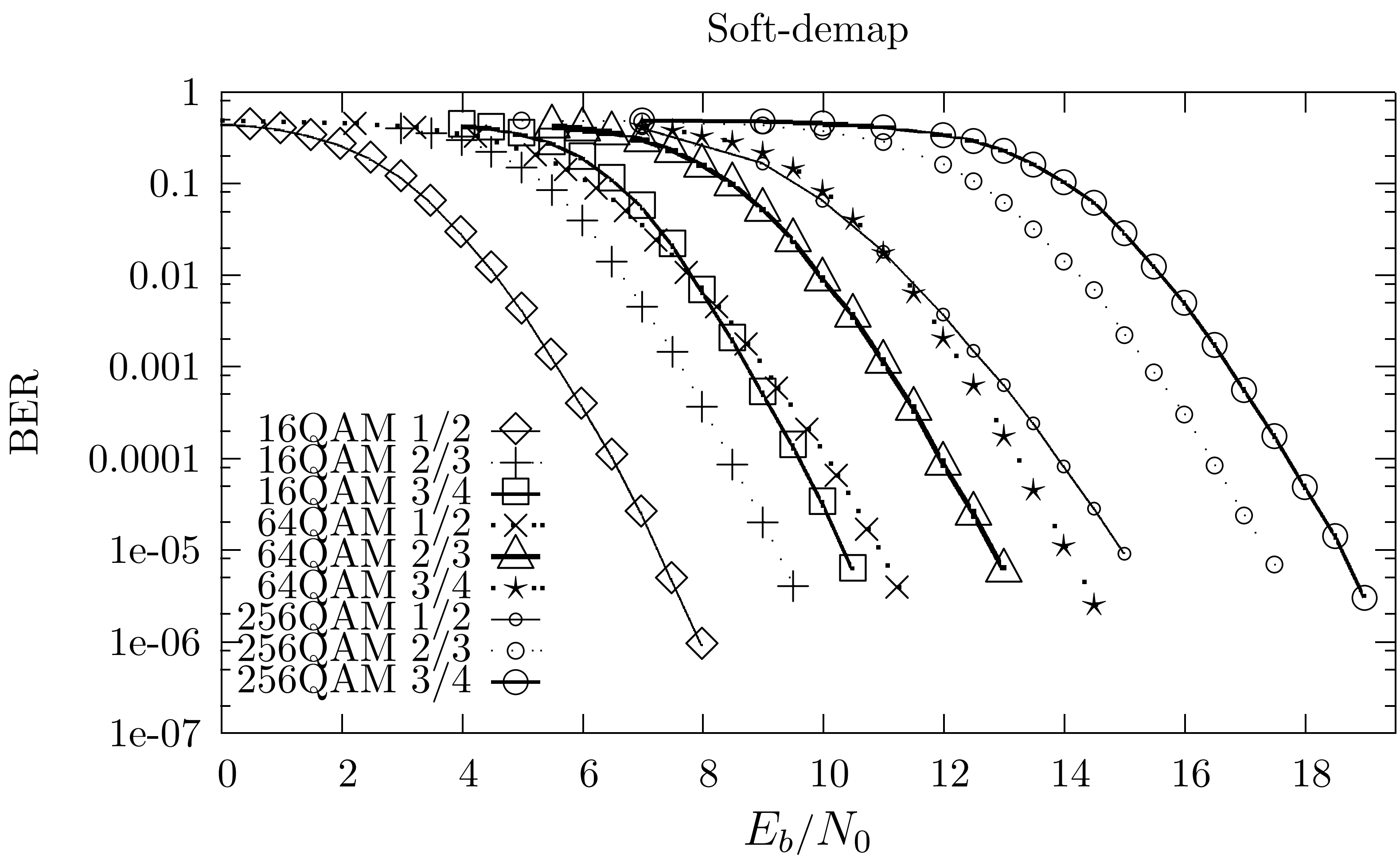
Punctured codes of Hard and Soft-demap

- Consistency to the standard selection:
  - At BER = 10^{-5}, the punctured code with code rate 3/4 under 16QAM modulation has 0.9 dB and 0.56 dB gains over the unpuctured code (with code rate 1/2) under 64QAM modulation for Hard and Soft-demap, respectively.
  - At the same bit error rate, the punctured code with code rate 2/3 under 64QAM modulation has 2.6 dB and 2.1 dB gains over the unpuctured code under 256QAM modulation for Hard and Soft-demap, respectively.

Hard under AWGN Channel



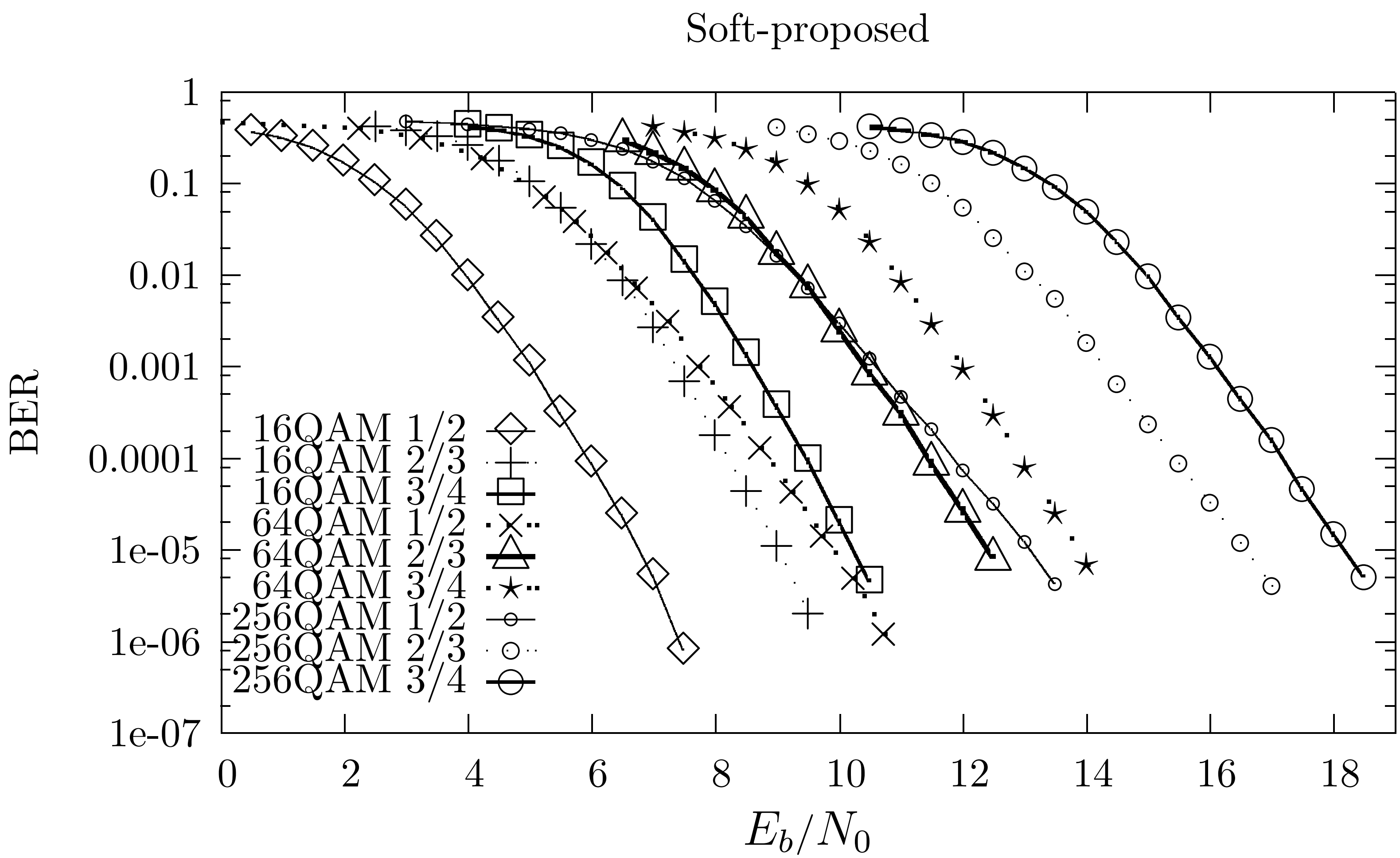
Soft-demap under AWGN Channel



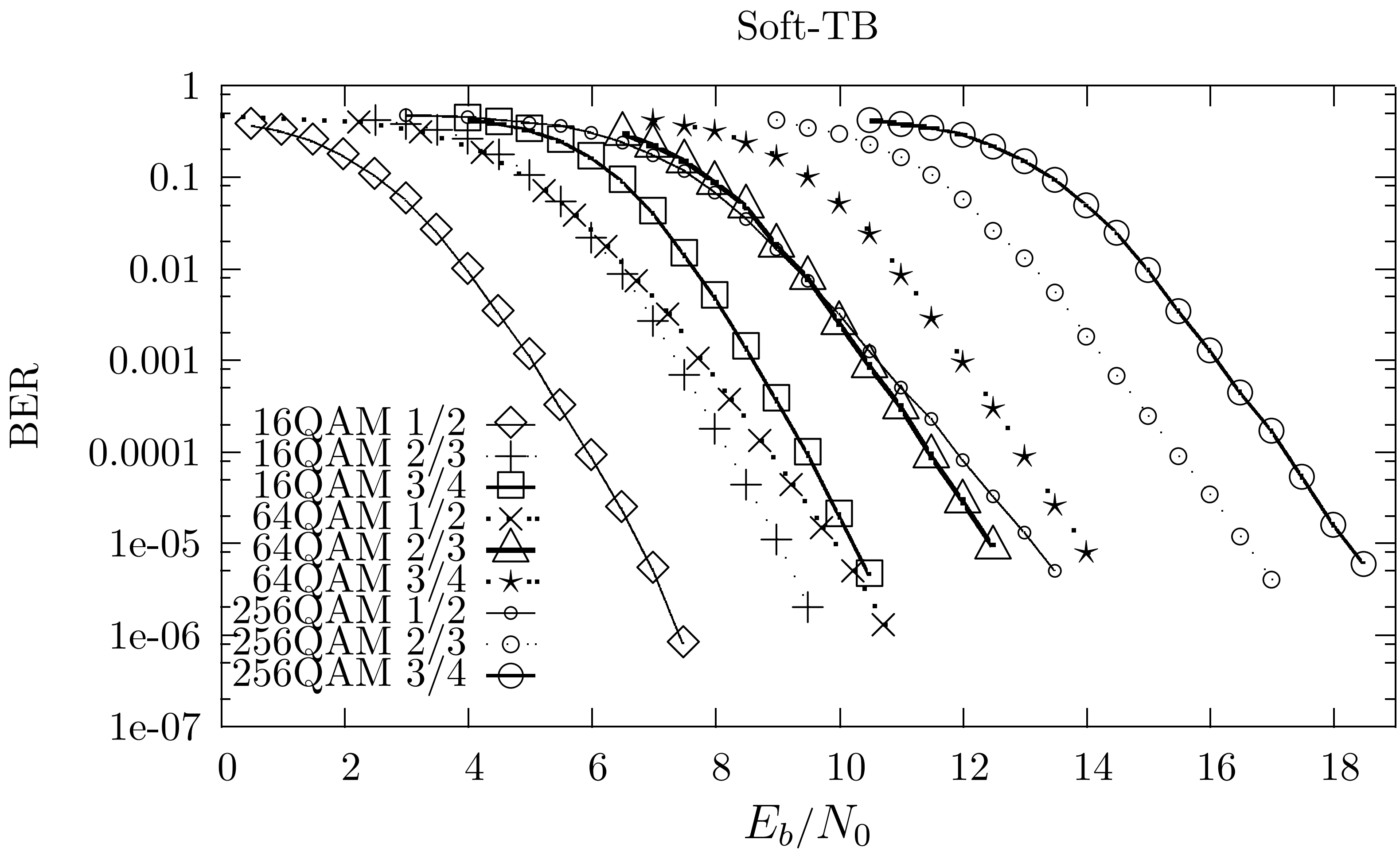
Performance of Soft-proposed and Soft-TB

- Consistency to the standard selection:
  - At BER $= 10^{-5}$, the code with code rate 2/3 under 64QAM modulation recommended in the standard still has 0.67 dB gain over that with code rate 1/2 under 256QAM modulation for Soft-proposed and Soft-TB.
- Inconsistency to the standard selection:
  - At the same bit error rate, the code with code rate 1/2 under 64QAM modulation has 0.33 dB gain over that with code rate 3/4 under 16QAM modulation for Soft-proposed and Soft-TB.

Soft-proposed under AWGN Channel



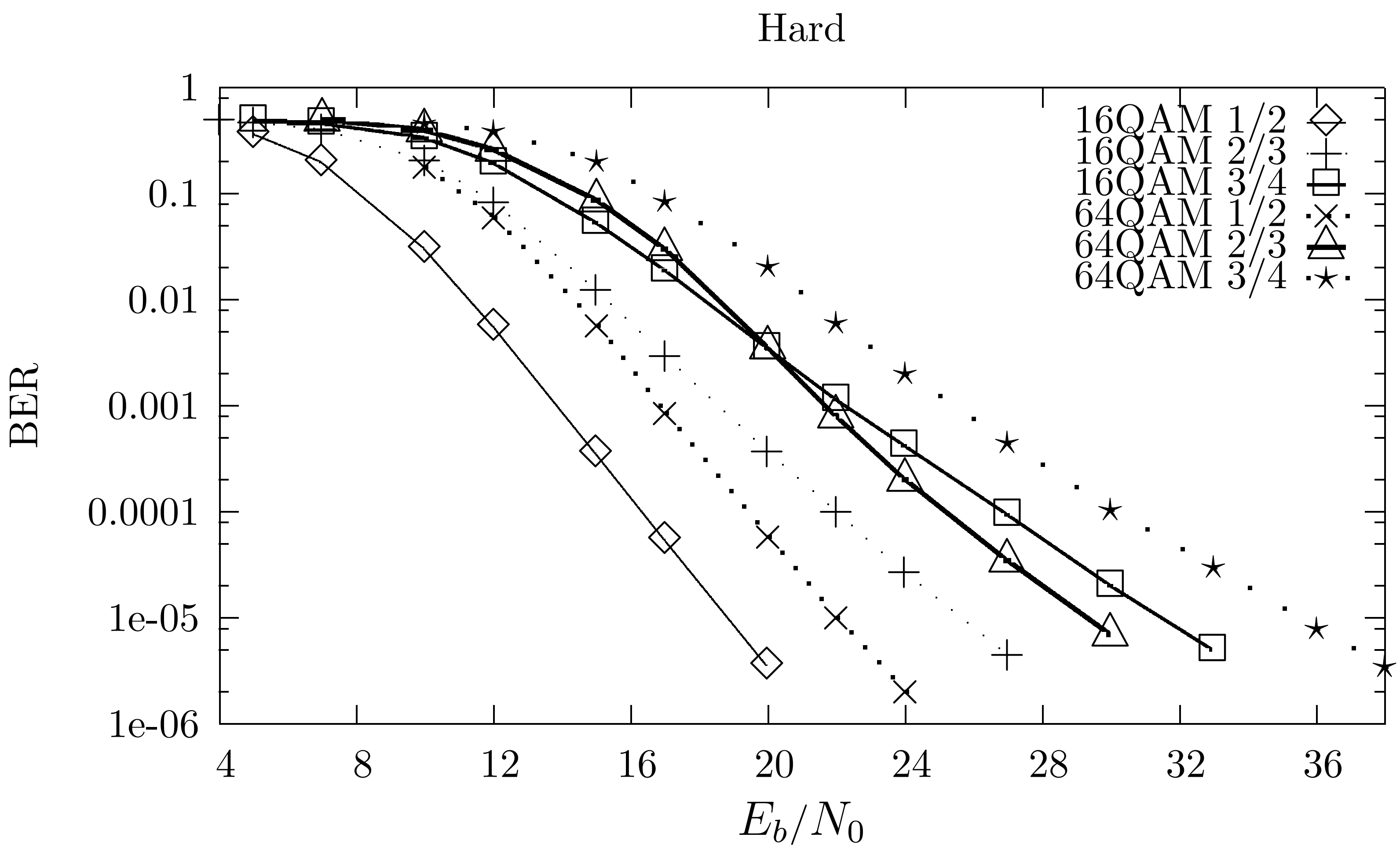
Soft-TB under AWGN Channel



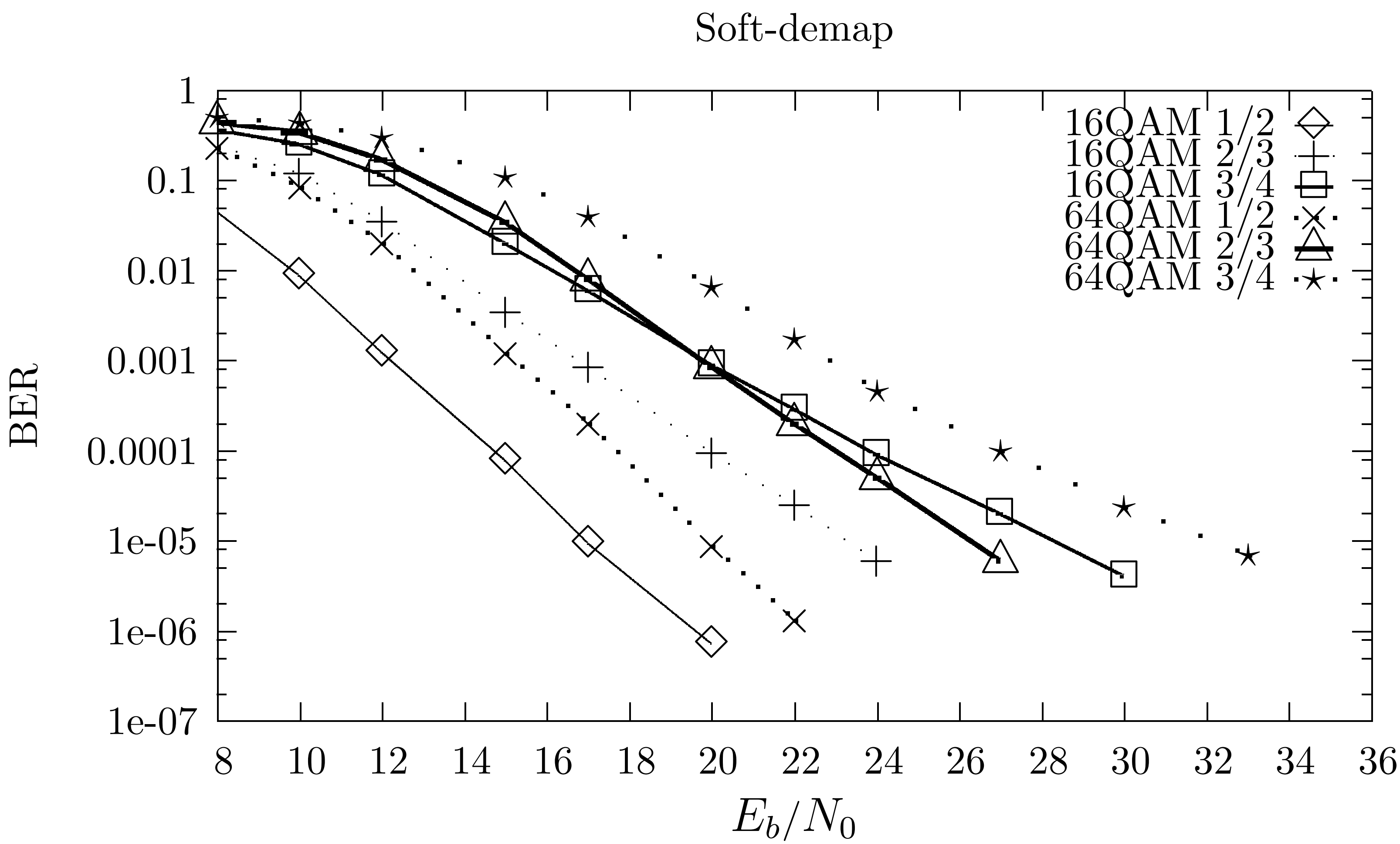
Performance under Rayleigh flat fading channel

- All inconsistency to the standard selection:
  - At BER = 10^{-5}, the punctured code with code rate 1/2 under 64QAM modulation has 9.3 dB, 8.3 dB, and 5.1 dB gain over the code with code rate 3/4 under 16QAM modulation for Hard, Soft-demap and Soft-proposed/Soft-TB, respectively.
  - This indicates that in a Rayleigh flat fading environment, the code with code rate 1/2 under 64QAM modulation performs better than the code with rate 3/4 under 16QAM modulation, no matter what receiver structure among Soft-proposed, Soft-TB, Soft-demap and Hard is employed.

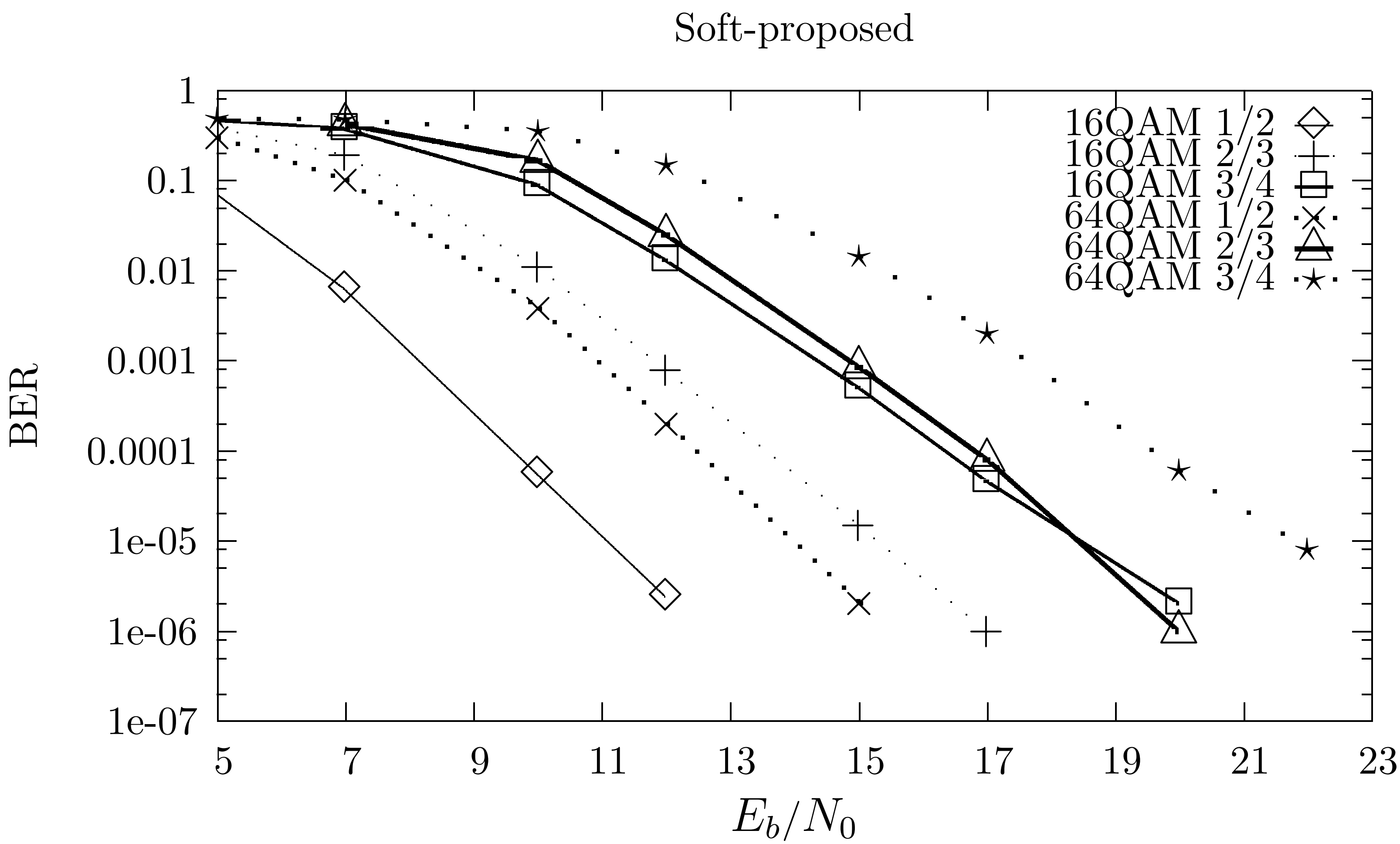
Hard under Rayleigh Flat Fading



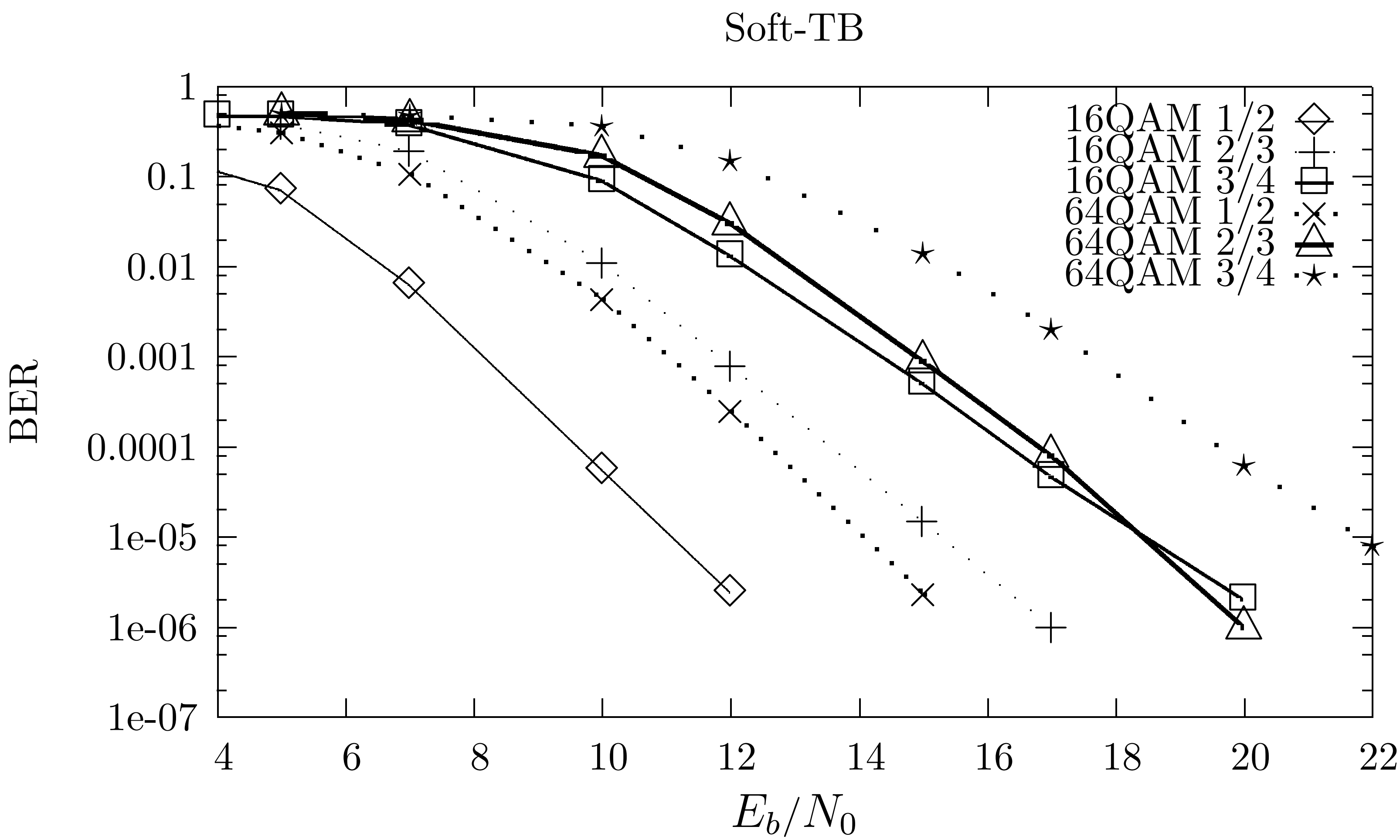
Soft-demap under Rayleigh Flat Fading



Soft-proposed under Rayleigh Flat Fading

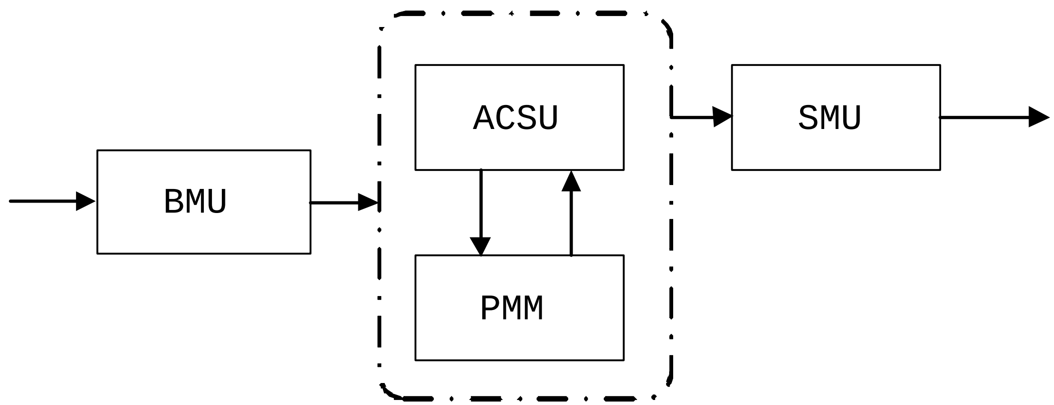


Soft-TB under Rayleigh Flat Fading



Realization of Soft-proposed & Soft-TB Metric

- Traditionally, realization of the Viterbi decoder can be divided into three units: BMU, ACSU/PMM and SMU.
- The input data is used in the branch metric unit (BMU) to calculate the branch metrics for each new time step.



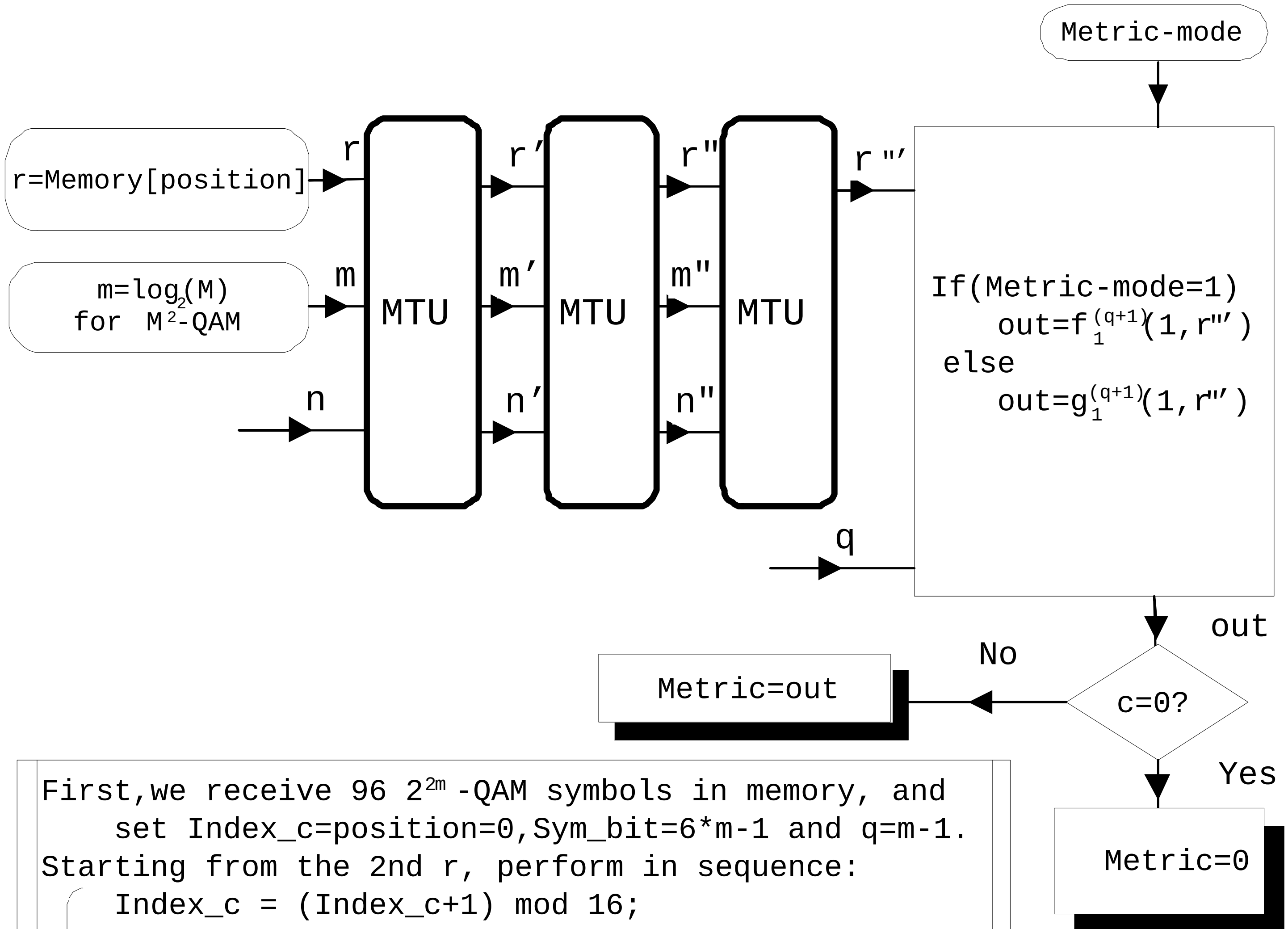
- These metrics are then fed to the add-compare-select unit (ACSU), which accumulates the branch metrics as the path metric (PM) stored in the path metric memory (PMM) according to the ACS-recursion.
- The survivor memory unit (SMU) processes the decisions which are being made in ACSU
- We now provide a systematic architecture for BMU, which avails the recursiveness nature of the proposed bit metric decomposition formulas.

Architecture Description (1)

- Our architecture only requires the first-bit function table, and can be applied for 2^{2m}-QAM modulations for every $2 \leq m \leq m_{\max}$, if $(m_{\max} - 2)$ metric transition units (MTUs) is serially connected.
- Our architecture requires 2 external input messages:
  - r: received value derived from the proper position of the 96 symbol block.
  - $m = \log_2(M)$: if M^2-QAM symbol is received.
- Our architecture includes 1 external control message:
  - n: keep the bit-order of symbol after interleaver.

Architecture Description (2)

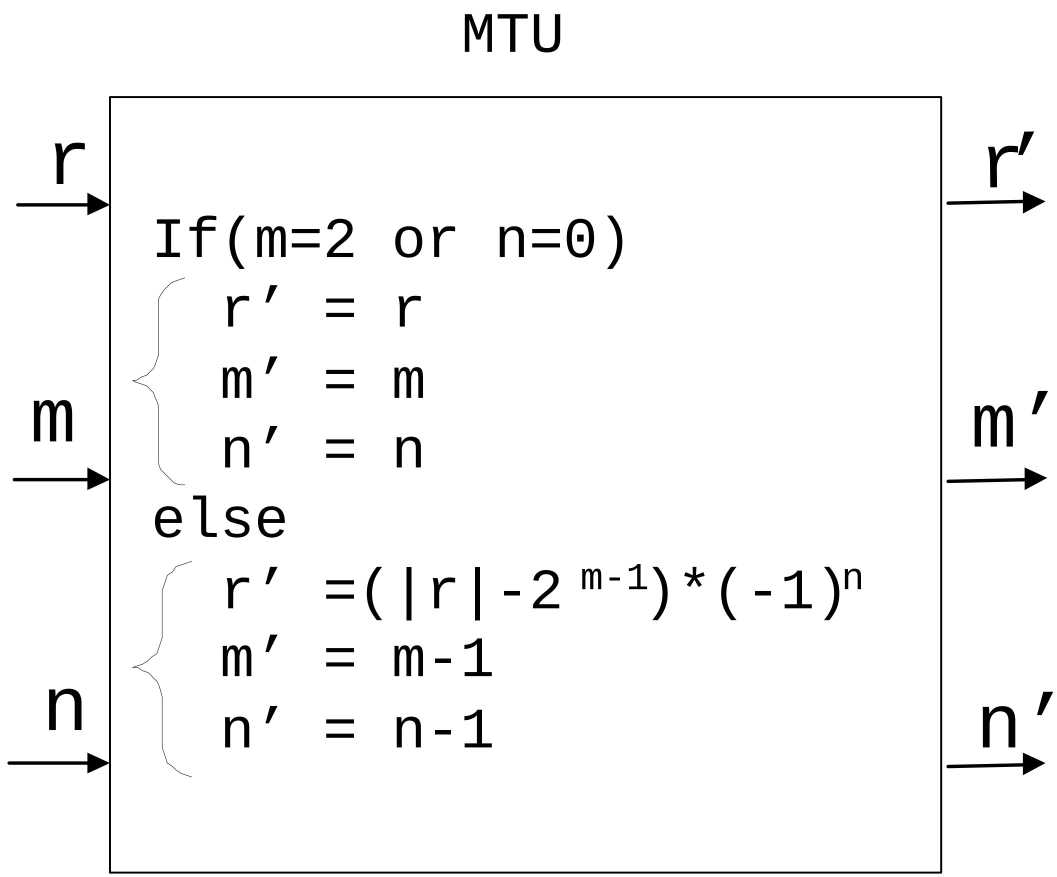
- Our architecture requires 4 internal control messages:
  - Index_c: used to keep the record of which column of QAM quadrature symbol the BMU is currently used.
  - Sym_bit: used to adjust the demapped bit number in each QAM symbol according to the interleaver rule.
  - position: record the position of the QAM quadrature component (in the 96 QAM symbol block) that is currently used.
  - q: used to determine the bit metric function number.



First, we receive 96 2^{2m}-QAM symbols in memory, and set Index_c=position=0, Sym_bit=6*m-1 and q=m-1.

Starting from the 2nd r, perform in sequence:

```
Index_c = (Index_c+1) mod 16;
if(Index_c=0) Sym_bit = Sym_bit-1;
position = 6*Index_c+5-⌊Sym_bit/m⌋;
q = (Index_c+Sym_bit) mod m;
n = m-1-q;
```


MTU
r
m
n
r′
m′
n′
If(m=2 or n=0)
r′ = r
m′ = m
n′ = n
else
r′ =(|r|-2 m-1)*(-1)n
m′ = m-1
n′ = n-1

Conclusion

- In this thesis, we obtained a recursive bit-decomposed metric formula through the approximation of symbol-based Euclidean metric.
- As anticipated, the proposed soft bit-decomposed metric and the simplified bit metrics (Soft-TB) perform better than Hard in all respects.
- A realization of the proposed soft bit-decomposed metric, which can perform all bit metric evaluation for 2^{2m}-QAM, where $2 \leq m \leq 5$, is also presented.