Design of Non-coherent Inner Convolutional Codes for Turbo-Coded OFDM System Over Frequency-Selective Fading Channels

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Outline

- Introduction
- Preliminaries
- Design of Inner Convolutional Code for Turbo-Coded OFDM System
- Simulation Results
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Introduction

Introduction

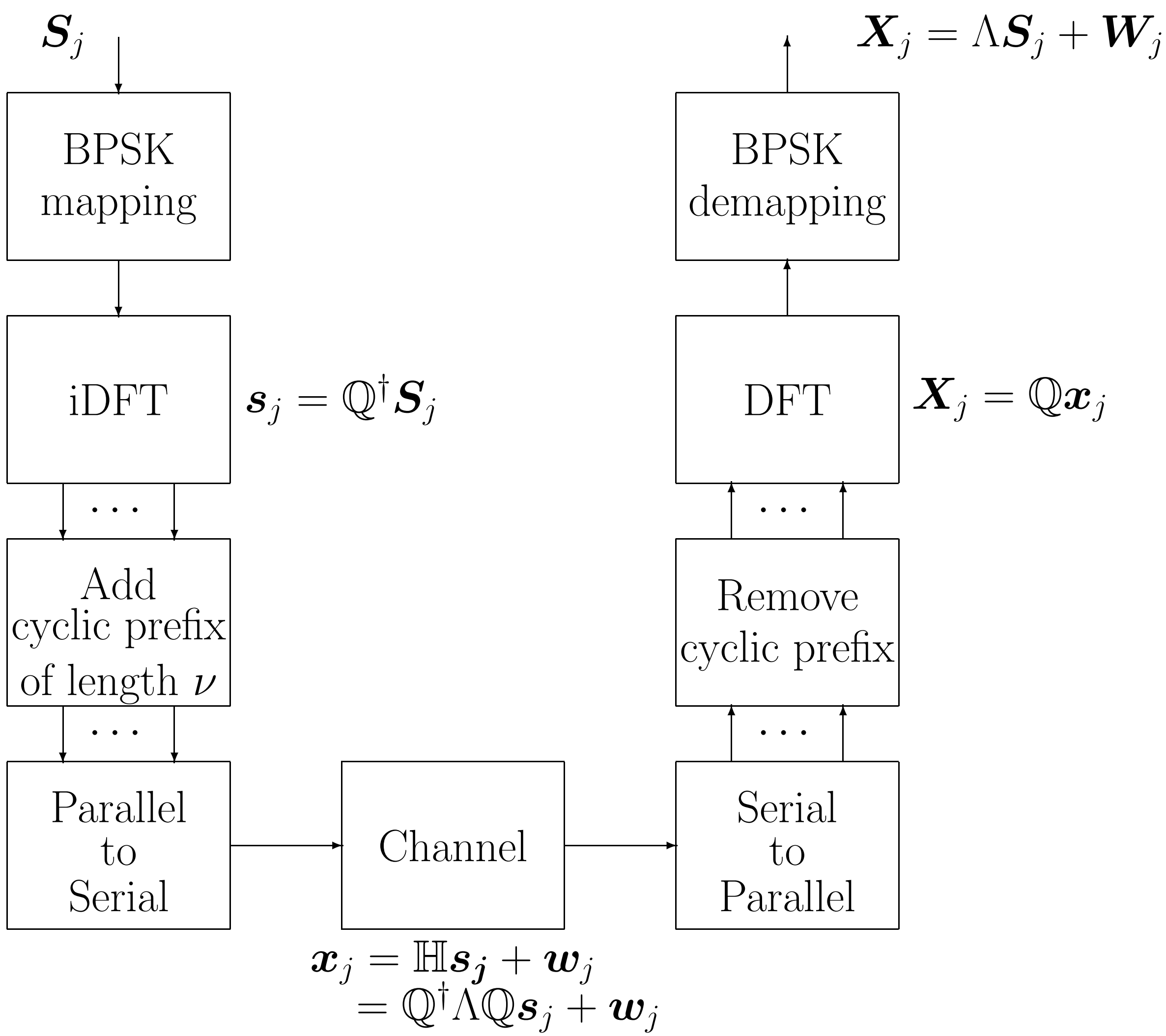
- We research on the transceiving of orthogonal frequency-division multiplexing (OFDM) modulated signals over the **unknown** frequency-selective fading channels in this thesis.
- Separate channel estimation and channel equalization may not be suited for a highly mobile environment, which motivates the use of **non-coherent detection** for communication systems over fading channels.
- Research results show that the **generalized likelihood ratio test** (GLRT) demodulator that jointly estimates channel and data can provide the good performance under a blind channel.

Contributions

- We propose an error correcting coding scheme that concatenates the turbo code (as an outer code) and convolutional code (as an inner code) with the OFDM system and the GLRT demodulator.
- The convolutional code is specifically designed for a blind environment, which we will conveniently refer to as the *non-coherent convolutional code*.
- We compare our design with the traditional system that equips the turbo code with a least square (LS) channel estimator and demonstrate that to replace the LS estimator with our non-coherent convolutional code can yield a better performance.

Preliminaries

A non-coherent OFDM system



A non-coherent OFDM system

- Since $\mathbb{H}$ is a circulant matrix, we can decompose it as:

$$\mathbb{H} = \mathbb{Q}^\dagger \Lambda \mathbb{Q} = \mathbb{Q}^\dagger \begin{bmatrix} \lambda_{N-1} & 0 & \cdots & 0 \\ 0 & \lambda_{N-2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_0 \end{bmatrix} \mathbb{Q}$$

  where Λ is a diagonal matrix with diagonals $\boldsymbol{\lambda} = \begin{bmatrix} \lambda_{N-1} & \lambda_{N-2} & \cdots & \lambda_0 \end{bmatrix}^T$ and $\mathbb{Q}$ is the DFT matrix.

- For derivation convenience, we rewrite the above system as:

$$\boldsymbol{X}_j = \mathbb{S}_j \boldsymbol{\lambda} + \boldsymbol{W}_j$$

  where $\mathbb{S}_j$ is the diagonal matrix corresponding to $\boldsymbol{S_j}$, which is given by:

$$\mathbb{S}_j = \begin{bmatrix} S_{1,j} & 0 & \cdots & 0 \\ 0 & S_{2,j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{N,j} \end{bmatrix}_{N \times N}$$

A non-coherent OFDM system

- It can be verified that at least two OFDM symbols are needed in order to secure a detection error $\geq$ that of a random guess in a fading channel that is blind to both transmitter and receiver.
- We thus assume that $T \geq 2T_s$, where T is the period during which fading coefficients $\boldsymbol{h}$ remain constant, and T_s is the symbol duration of an OFDM symbol.
- Let the two OFDM symbols be:

$$\boldsymbol{X}_1 = \mathbb{S}_1 \boldsymbol{\lambda} + \boldsymbol{W}_1 \tag{1}$$

and

$$\boldsymbol{X}_2 = \mathbb{S}_2 \boldsymbol{\lambda} + \boldsymbol{W}_2. \tag{2}$$

- We then combine (1) and (2) into:

$$\vec{\boldsymbol{X}} = \vec{\boldsymbol{S}}\boldsymbol{\lambda} + \vec{\boldsymbol{W}}$$

where

$$\vec{\boldsymbol{X}} = \begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{bmatrix}, \quad \vec{\boldsymbol{S}} = \begin{bmatrix} \mathbb{S}_1 \\ \mathbb{S}_2 \end{bmatrix} \quad \text{and} \quad \vec{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W}_1 \\ \boldsymbol{W}_2 \end{bmatrix}.$$

- Again, we can multiplex k OFDM symbols to make $\vec{\boldsymbol{S}} = \begin{bmatrix} \mathbb{S}_1 & \mathbb{S}_2 \cdots \mathbb{S}_k \end{bmatrix}^{\mathtt{T}}$ if $T \geqslant kT_s$. However, the system complexity will significantly grow as k getting large. In addition, the requirement of $T \geq kT_s$ will restrict the applicability of our design to a less mobile environment for a moderately large k. For these reasons, we will focus only on the case of $k = 2$ in this thesis.

Least Square Estimator for Fading Channels

- The least square estimator (LSE) intends to estimate $\boldsymbol{\lambda}$ such that $\|\vec{\boldsymbol{X}} - \vec{\boldsymbol{S}}\boldsymbol{\lambda}\|^2$ is minimized when $\vec{\boldsymbol{S}}$ is assumed known. We derive

$$J(\boldsymbol{\lambda}) = \|\vec{\boldsymbol{X}} - \vec{\boldsymbol{S}}\boldsymbol{\lambda}\|^2 = \vec{\boldsymbol{X}}^{\mathsf{T}}\vec{\boldsymbol{X}} - \vec{\boldsymbol{X}}^{\mathsf{T}}\vec{\boldsymbol{S}}\boldsymbol{\lambda} - \boldsymbol{\lambda}^{\mathsf{T}}\vec{\boldsymbol{S}}^{\mathsf{T}}\vec{\boldsymbol{X}} + \boldsymbol{\lambda}^{\mathsf{T}}\vec{\boldsymbol{S}}^{\mathsf{T}}\vec{\boldsymbol{S}}\boldsymbol{\lambda}.$$

- Note that $\vec{\boldsymbol{X}}^{\mathsf{T}}\vec{\boldsymbol{X}}$ is a constant scalar and hence we obtain the LSE of $\boldsymbol{\lambda}$ as:

$$\hat{\boldsymbol{\lambda}} = (\vec{\boldsymbol{S}}^{T}\vec{\boldsymbol{S}})^{-1}\vec{\boldsymbol{S}}^{T}\vec{\boldsymbol{X}}. \tag{3}$$

- Together with the turbo coding, the LSE provides a conventional system design to combat the channel fading.
- This conventional system will be used as a benchmark to be compared with for our proposed non-coherent scheme in this thesis.

GLRT Detection

- In a blind environment, the general LRT (GLRT) detection is given by:

$$\begin{aligned}\hat{\vec{\boldsymbol{S}}} &= \arg\min_{\vec{\boldsymbol{S}}\in\mathcal{S}}\left(\min_{\boldsymbol{\lambda}\in\mathcal{C}^N}\|\vec{\boldsymbol{X}}-\vec{\boldsymbol{S}}\boldsymbol{\lambda}\|^2\right)\\ &= \arg\min_{\vec{\boldsymbol{S}}\in\mathcal{S}}\|\vec{\boldsymbol{X}}-\vec{\boldsymbol{S}}\hat{\boldsymbol{\lambda}}\|^2\\ &= \arg\min_{\vec{\boldsymbol{S}}\in\mathcal{S}}\|\vec{\boldsymbol{X}}-\mathbb{P}_{\vec{\boldsymbol{S}}}\vec{\boldsymbol{X}}\|^2 \end{aligned} \tag{4}$$

where $\mathcal{C}$ is the set of complex numbers, $\mathcal{S}$ is the set of all possible transmission signal $\vec{\boldsymbol{S}}$, $\hat{\boldsymbol{\lambda}}$ is given by (3), and

$$\mathbb{P}_{\vec{\boldsymbol{S}}} = \vec{\boldsymbol{S}}(\vec{\boldsymbol{S}}^\dagger\vec{\boldsymbol{S}})^{-1}\vec{\boldsymbol{S}}^\dagger \tag{5}$$

GLRT Detection

- Taking (5) into (4) yields:

$$\hat{\vec{\boldsymbol{S}}} = \arg\min_{\vec{S}\in\mathcal{S}} \|\vec{\boldsymbol{X}} - \mathbb{P}_{\vec{\boldsymbol{S}}}\vec{\boldsymbol{X}}\|^2 = \arg\max_{\vec{S}\in\mathcal{S}} \sum_{i=1}^{N} |S_{i,1}X_{i,1}^* + S_{i,2}X_{i,2}^*|^2$$

- The result implies that the receiver cannot distinguish between $(S_{i,1}, S_{i,2}) = (1, -1)$ and $(S_{i,1}, S_{i,2}) = (-1, 1)$, and also $(S_{i,1}, S_{i,2}) = (1, 1)$ and $(S_{i,1}, S_{i,2}) = (-1, -1)$. So we shall fix one of $S_{i,1}$ and $S_{i,2}$ to ensure the detectability of information at the receiver as exemplified below:

$$\vec{\boldsymbol{S}} = \begin{bmatrix} \mathbb{S}_1 \\ \mathbb{S}_2 \end{bmatrix} = \begin{bmatrix} S_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & S_{N,1} \\ -1 & 0 & 0 & \cdots & 0 \\ 0 & S_{2,2} & 0 & \cdots & 0 \\ 0 & 0 & S_{3,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

Design of Inner Convolutional Code for Turbo-Coded OFDM System

Design of Noncoherent Convolutional Code

- In order to construct a good convolutional coding structure without resorting to simulations, we adopt the "non-coherent" Euclidean distances among codewords as our design criterion, i.e., $\|\vec{\boldsymbol{X}} - \mathbb{P}_{\vec{\boldsymbol{S}}}\vec{\boldsymbol{X}}\|^2$.
- Consider the system that transmits a convolutional codeword $\boldsymbol{b} = \begin{bmatrix} b_1 & b_2 & \cdots & b_N \end{bmatrix}$ of length N over a frequency-selective fading channel as follows:

$$\boldsymbol{y} = \mathbb{B}\boldsymbol{h} + \boldsymbol{v}$$

where the codeword matrix is given by:

$$\mathbb{B} = \begin{bmatrix} b_1 & 0 & \cdots & 0 \\ b_2 & b_1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_N & b_{N-1} & \ddots & 0 \\ 0 & b_N & \ddots & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_N \end{bmatrix}_{L\times P}$$

Design of Noncoherent Convolutional Code

- The "non-coherent" Euclidean distance between codewords i and j can be equivalently expressed as:

$$d_{i,j} = \|\text{vec}(\mathbb{P}_{B_i}) - \text{vec}(\mathbb{P}_{B_j})\|^2 \tag{6}$$

where

$$\mathbb{P}_B = \mathbb{B}(\mathbb{B}^T\mathbb{B})^{-1}\mathbb{B}^T.$$

- In order to analyze the **cumulative distance function** (CDF) of the "non-cohrent" Euclidean distances of a convolutional codeword, we quantify $d_{i,j}$ into 1000 levels and compute the CDF functions for two convolutional codes under comparison.

Design of Noncoherent Convolutional Code

- Denote by C_M and C_N the CDF functions of two convolutional codes with structure M and N, respectively. We then do the following computation in which we only take half of the level function values for this comparison:

$$a = \sum_{\texttt{level}=1}^{500} \texttt{index}(\texttt{level})$$

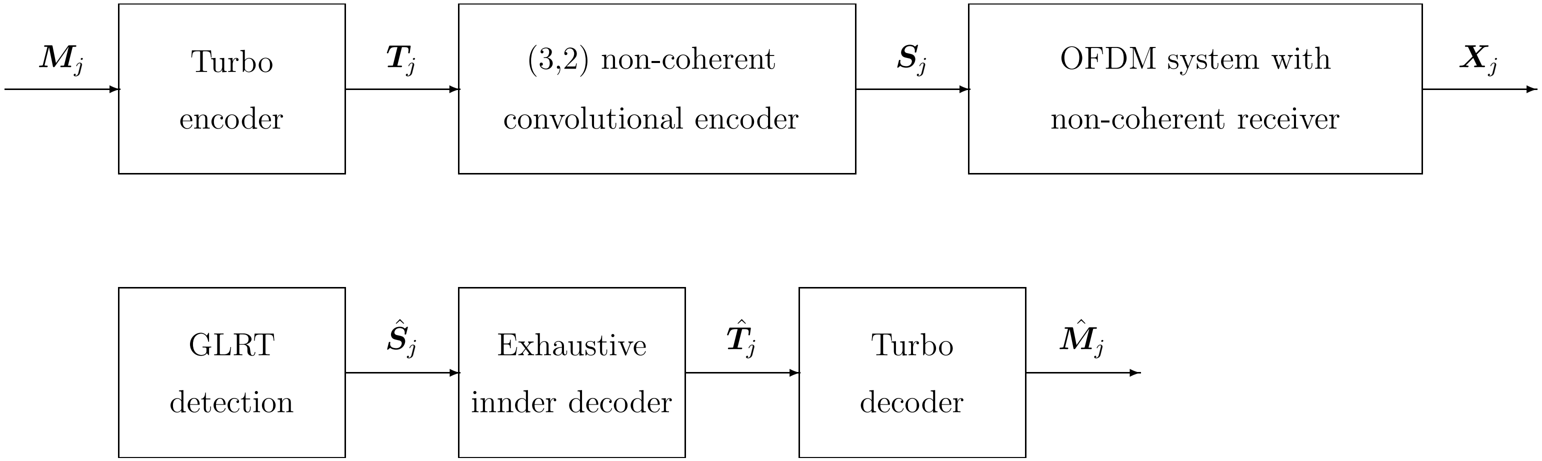
where

$$\texttt{index}(\texttt{level}) = \begin{cases} 1, & \text{if} \quad C_M(\texttt{level}) < C_N(\texttt{level}) \\ -1, & \text{if} \quad C_M(\texttt{level}) > C_N(\texttt{level}) \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

- From our observations, we note that if $a > 0$, then structure M mostly will yield a better performance than structure N. Hence, we propose to compare all the convolutional code design through this way, and determine the best one that can possibly yield a good (near optimal) error performance.

Concatenated Coding System Under Fading Channels

- The concatenated coding system we consider is depicted as below:



- The idea behind our concatenated system is that the traditional turbo code design is based on the assumption that the system can perfectly recover the fading effect and suffers only the additive white Gaussian noise. However, this may not be entirely true in practical application.

- So instead of targeting a perfect channel estimator and equalizer, we use a non-coherent convolutional code to exempt from the need of designing channel estimator and equalizer.

- In this thesis, three turbo codes with different RSC component codes and interleavers will be tested. They are respectively
  1. the (37,21) turbo code proposed by Berrou and Glavieux
  2. one of the 3GPP/LTE specified turbo codes
  3. the turbo code with generator matrix $\left[1 \;\; (1+D+D^2+D^4)/(1+D^3+D^4)\right]$ plus a uniform interleaver as introduced in Shu Lin's book.
- To test how effective our internal non-coherent convolutional code is, we choose practically short codeword lengths for the turbo code which are respectively 12288, 12288, and 3000, with information lengths respectively to be 4096, 4096 and 1000.

- The non-coherent convolutional codes that we design and test in this thesis have four different codeword lengths: six, eight, ten and twelve. For convenience, we will refer to them as type 3, type 2, type 1 and type 0 in the sequel.

Simulation Results

Table 1: Convolutional codes identified by the proposed method and the optimal convolutional code designs obtained from simulations. The codeword lengths vary from $N = 6$ to $N = 12$. The code rate is 1/2.

type	codeword length	code selected	optimal code
3	$N = 6$	7-5	5-6 or 7-5
2	$N = 8$	5-6	4-7 or 5-6 or 7-5
1	$N = 10$	7-4	7-4
0	$N = 12$	7-4	5-7 or 7-4 or 7-5

Table 2: Convolutional codes identified by the proposed method. The codeword lengths vary from $N = 6$ to $N = 12$. The code rate is 2/3. Since there are many choices for rate 2/3 codes, to identify the optimal code structures via simulations turns out to be infeasible. Hence, we did not show the optimal code design in this table.

type	codeword length	code selected
3	$N = 6$	7-6; 7-5
2	$N = 8$	5-6; 7-5
1	$N = 10$	5-6; 7-5
0	$N = 12$	7-5; 7-6

Here, "5-1" means that the convolutional code structure is defined via generator matrix $\left[1 + D^2 \quad D^2\right]$. Similarly, the codes marked "5-2", "5-3", "5-4", "5-5", "5-6" and "5-7" are defined via the generator matrices $\left[1 + D^2 \quad D\right]$, $\left[1 + D^2 \quad D + D^2\right]$, $\left[1 + D^2 \quad 1\right]$, $\left[1 + D^2 \quad 1 + D^2\right]$, $\left[1 + D^2 \quad 1 + D\right]$ and $\left[1 + D^2 \quad 1 + D + D^2\right]$, respectively.

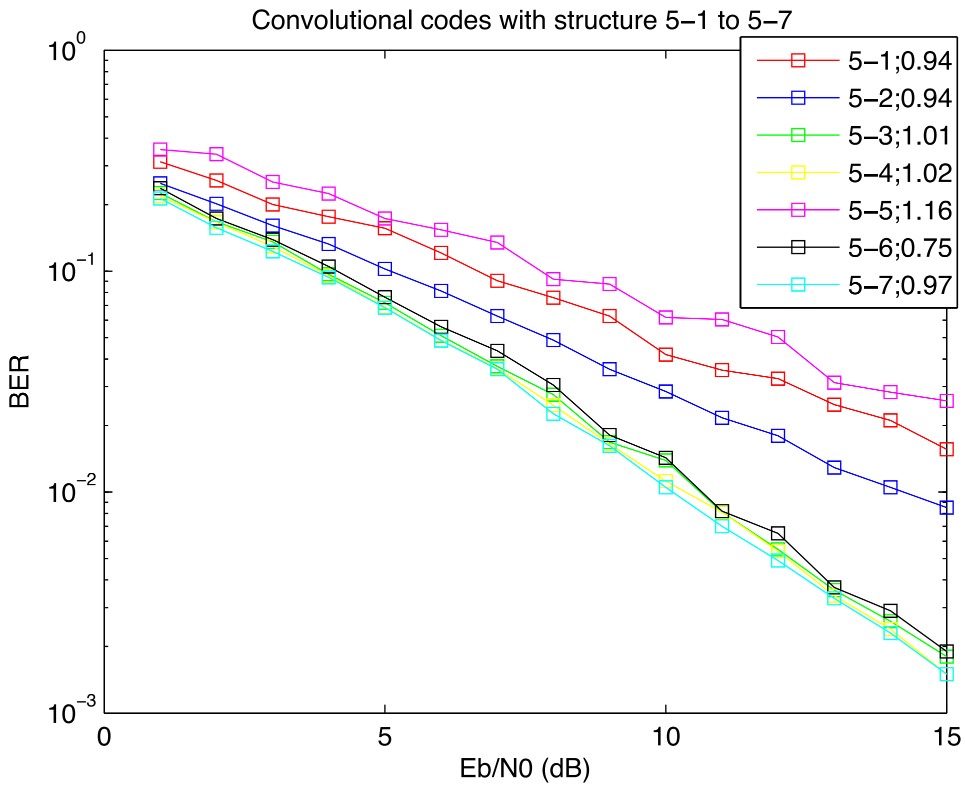


Figure 1: Performances of non-coherent convolutional codes under blind fading channels. Here, the codeword length $N = 10$ and the channel memory order is $\nu = 2$. The code rate is $1/2$.

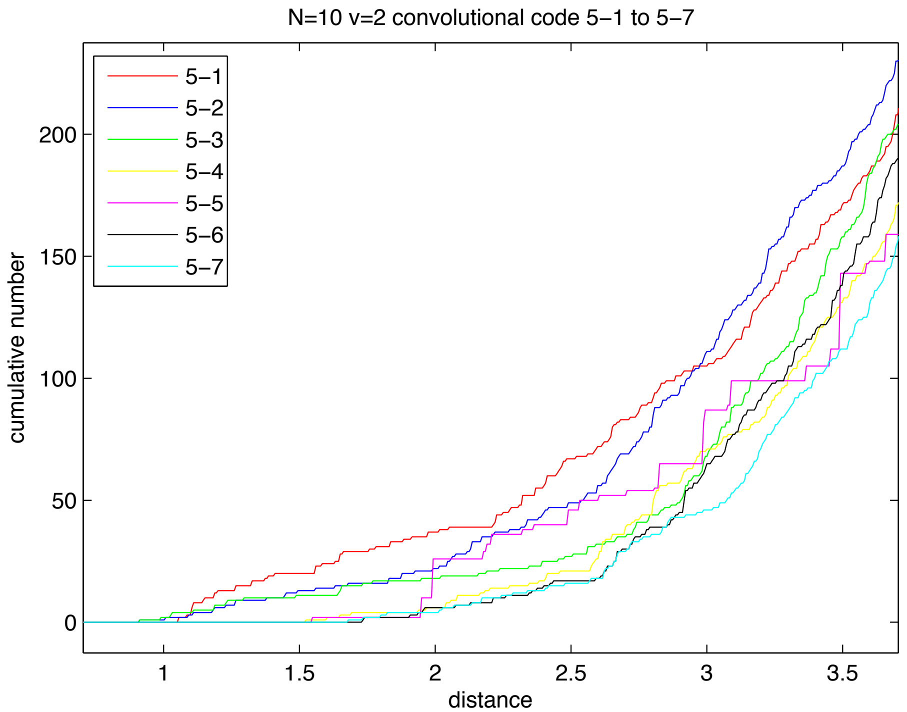


Figure 2: Cumulative distance function (CDF) of the convolutional codes under test. The code rate is 1/2.

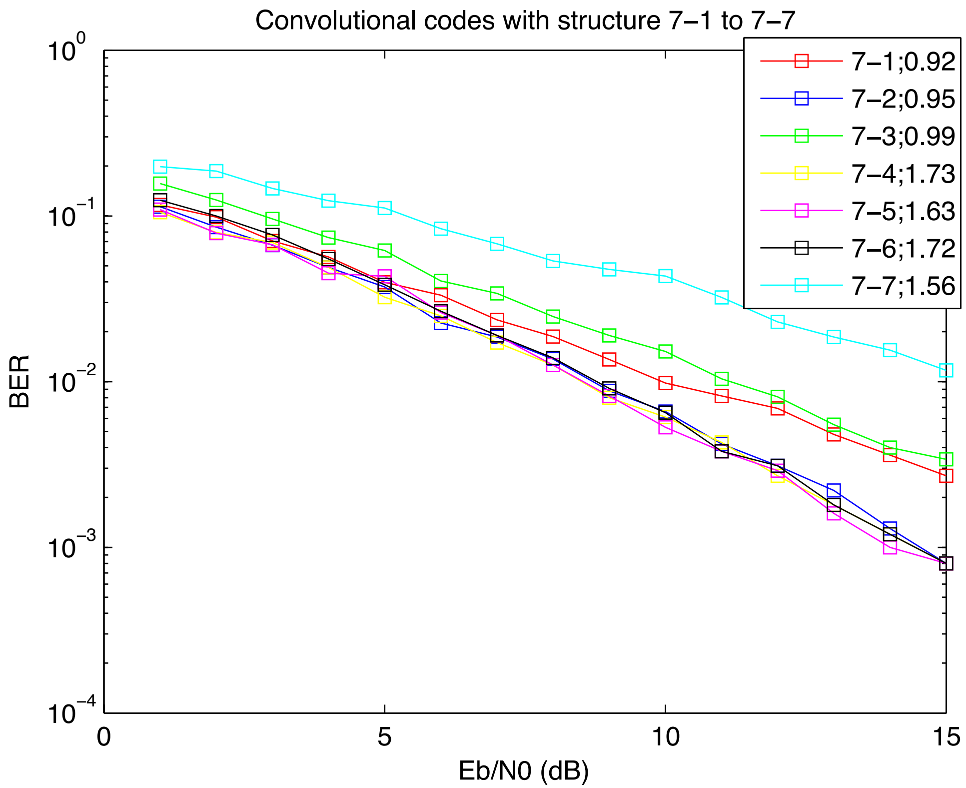


Figure 3: Performances of non-coherent convolutional codes under blind fading channels. Here, the codeword length $N = 10$ and the channel memory order is $\nu = 2$. The code rate is $1/2$.

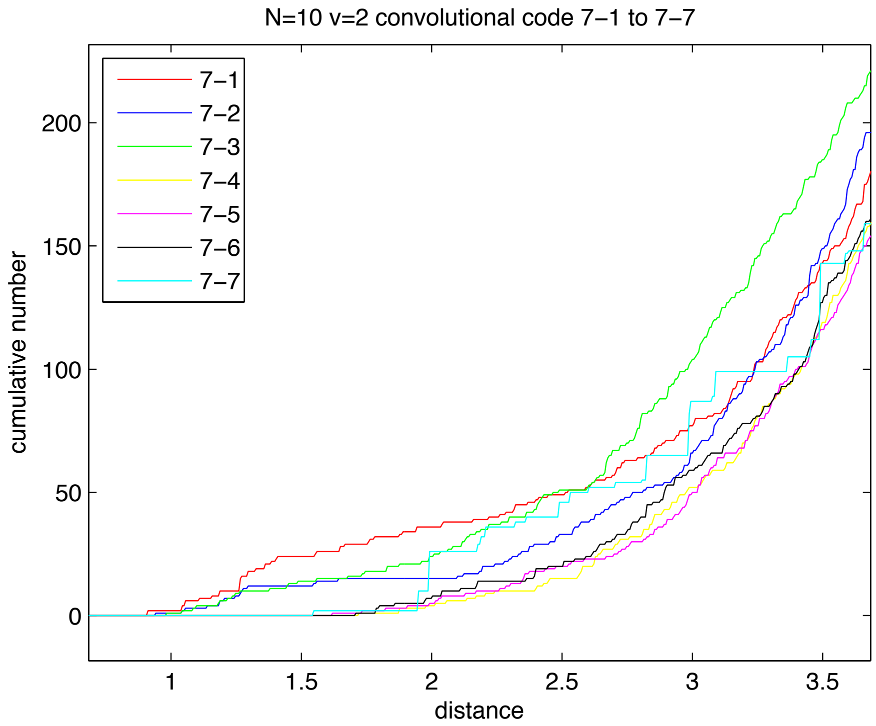


Figure 4: Cumulative distance function (CDF) of the convolutional codes under test. The code rate is 1/2.

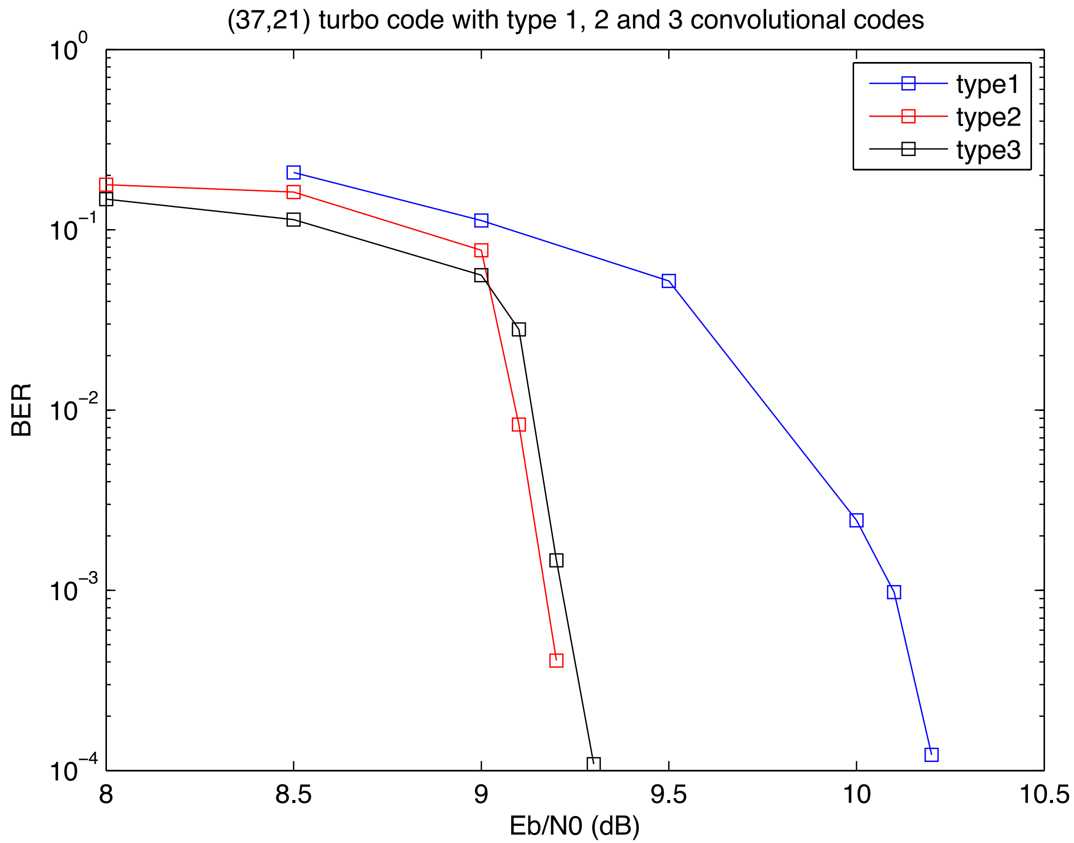


Figure 5: Performances of the concatenated coding system. The turbo code used is the (37, 21) code proposed by Berrou and Glavieux. The codeword lengths of the type 1, 2 and 3 convolutional codes are ten, eight and six, respectively. The code rate is 2/3.

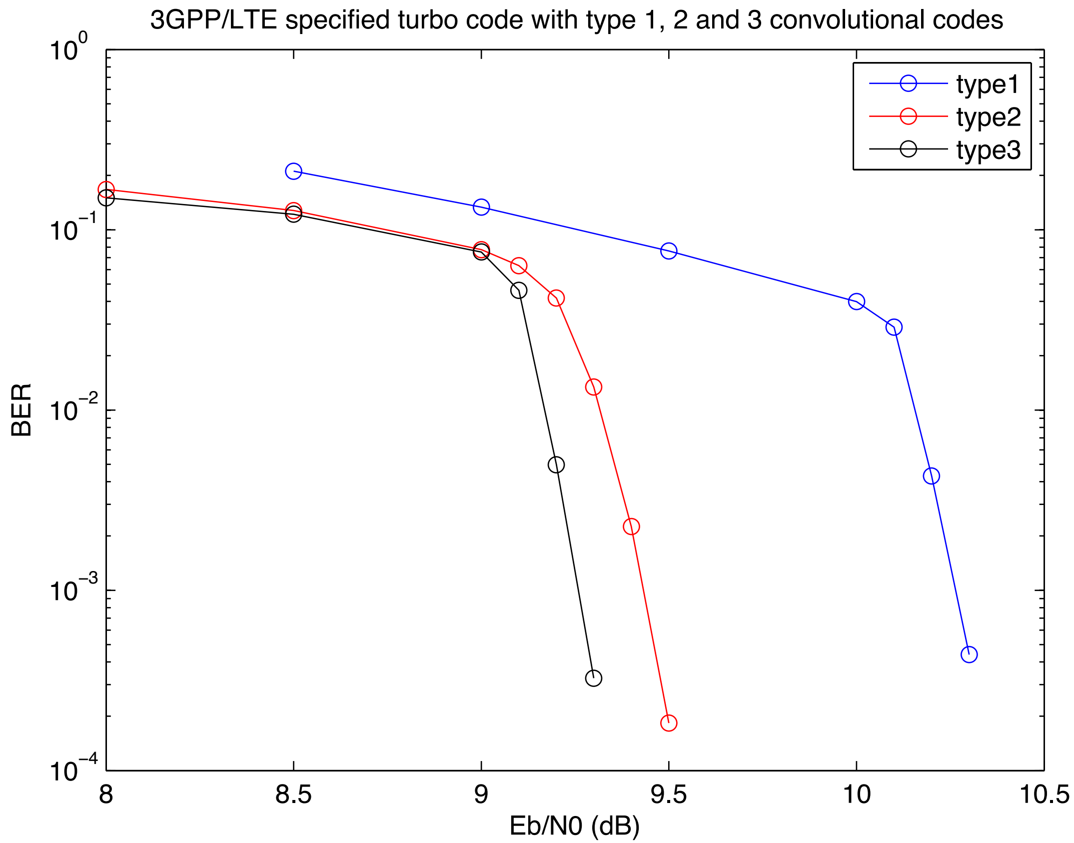


Figure 6: Performances of the concatenated coding system. The turbo code used is the one specified in 3GPP/LTE. The codeword lengths of the type 1, 2 and 3 convolutional codes are ten, eight and six, respectively. The code rate is 2/3.

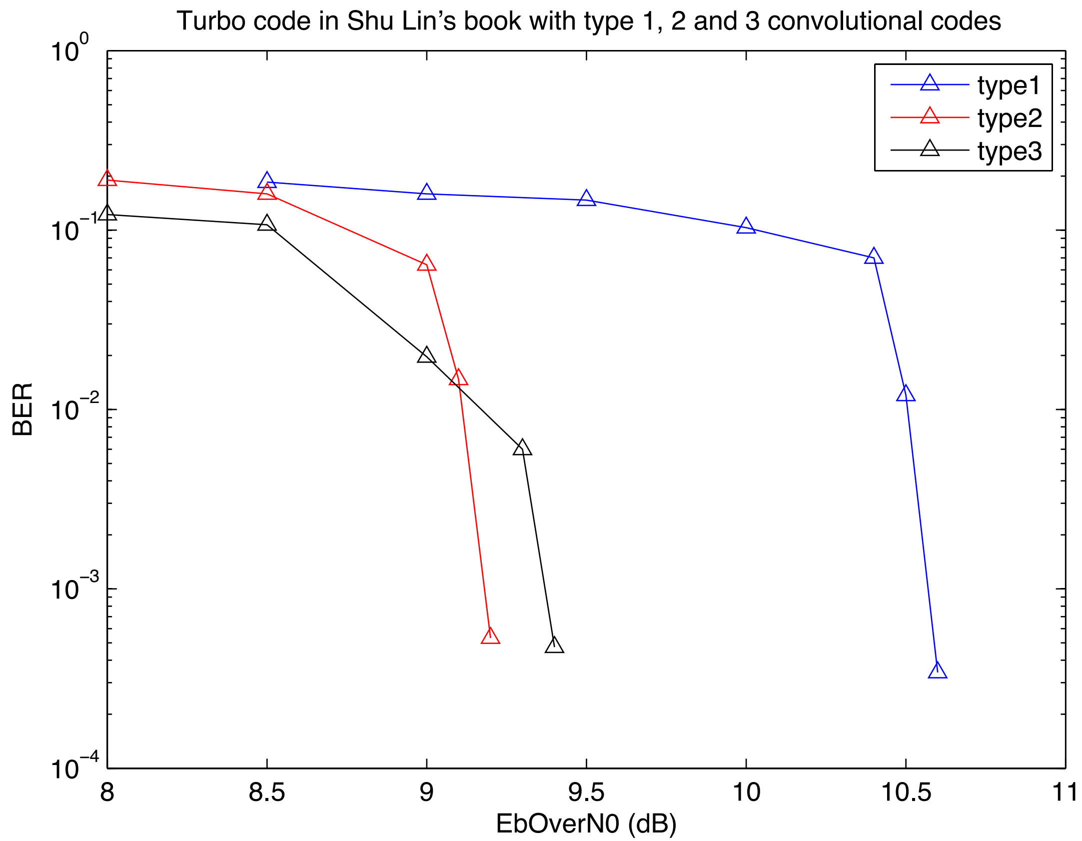


Figure 7: Performances of the concatenated coding system. The turbo code used is the one specified in S. Lin's book. The codeword lengths of the type 1, 2 and 3 convolutional codes are ten, eight and six, respectively. The code rate is 2/3.

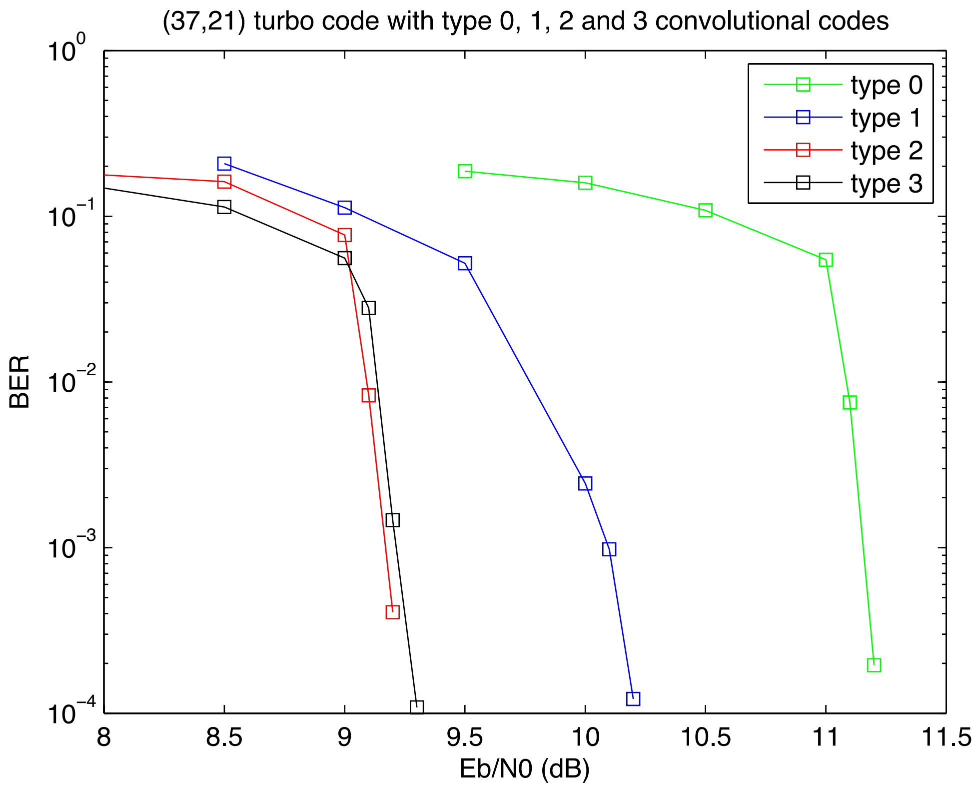


Figure 8: Performances of the concatenated system that employs the $(37, 21)$ turbo code concatenated with the non-coherent convolutional codes of type 0, 1, 2, 3. Here, the codeword lengths at type 0, 1, 2, 3 codes are respectively 12, 10, 8 and 6. The code rate is 2/3.

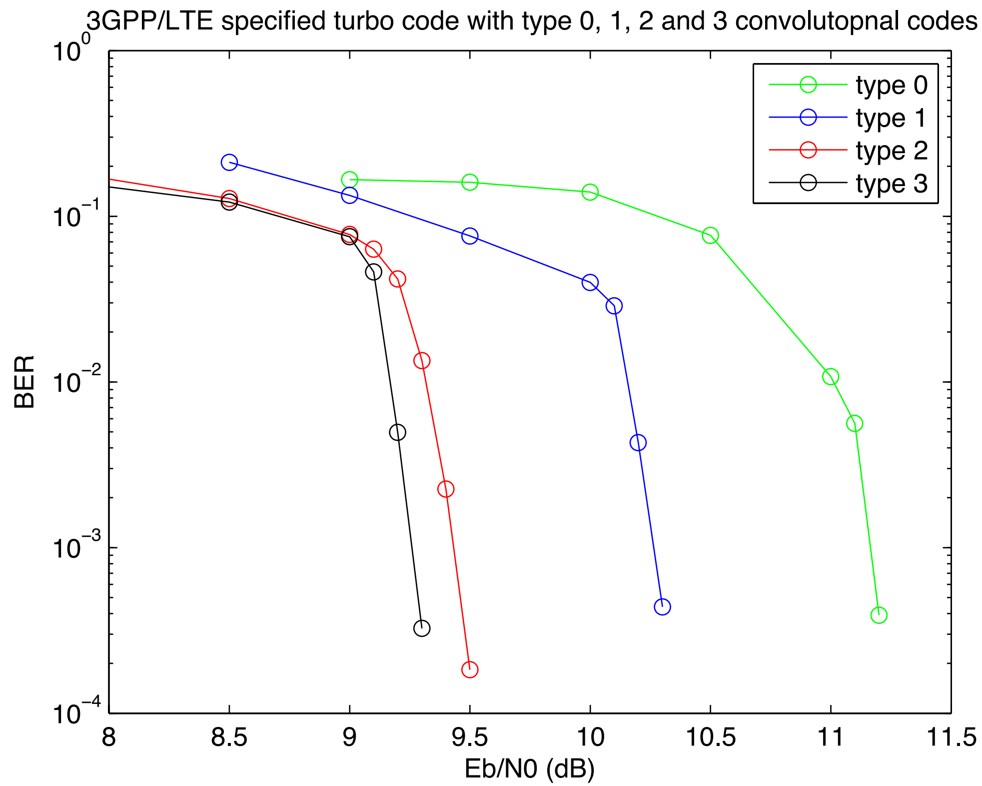


Figure 9: Performances of the concatenated system that employs the 3GPP/LTE specified turbo code concatenated with the non-coherent convolutional codes of type 0, 1, 2, 3. Here, the codeword lengths at type 0, 1, 2, 3 codes are respectively 12, 10, 8 and 6. The code rate is 2/3.

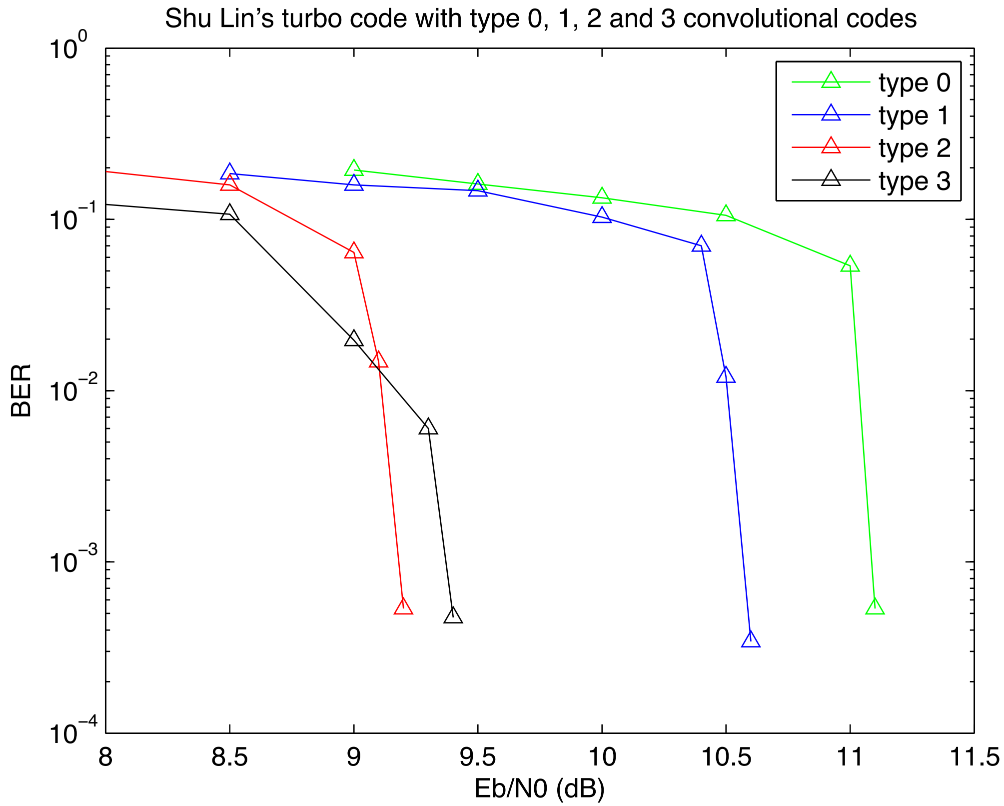


Figure 10: Performances of the concatenated system that employs the turbo code from S. Lin's book concatenated with the non-coherent convolutional codes of type 0, 1, 2, 3. Here, the codeword lengths at type 0, 1, 2, 3 codes are respectively 12, 10, 8 and 6. The code rate is 2/3.

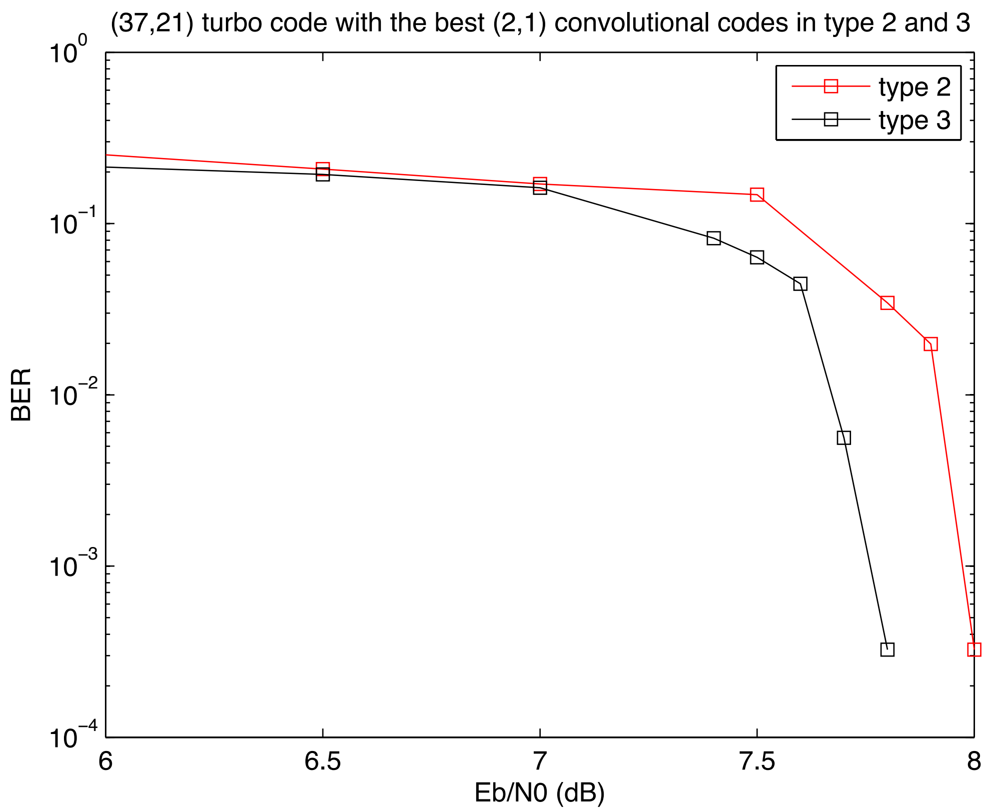


Figure 11: Performances of the concatenated system that employs the $(37, 21)$ turbo code concatenated with the optimal non-coherent convolutional codes of type 2 and 3. The code rate is $1/2$.

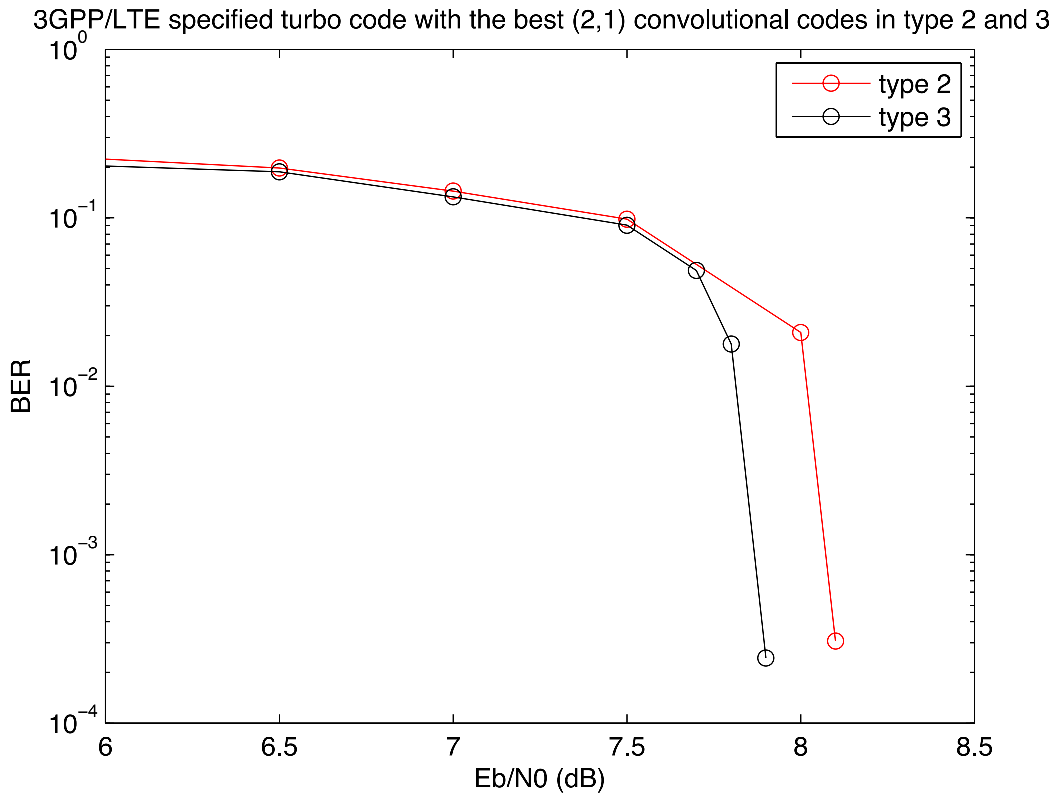


Figure 12: Performances of the concatenated system that employs the 3GPP/LTE specified turbo code concatenated with the optimal non-coherent convolutional codes of type 2 and 3. The code rate is 1/2.

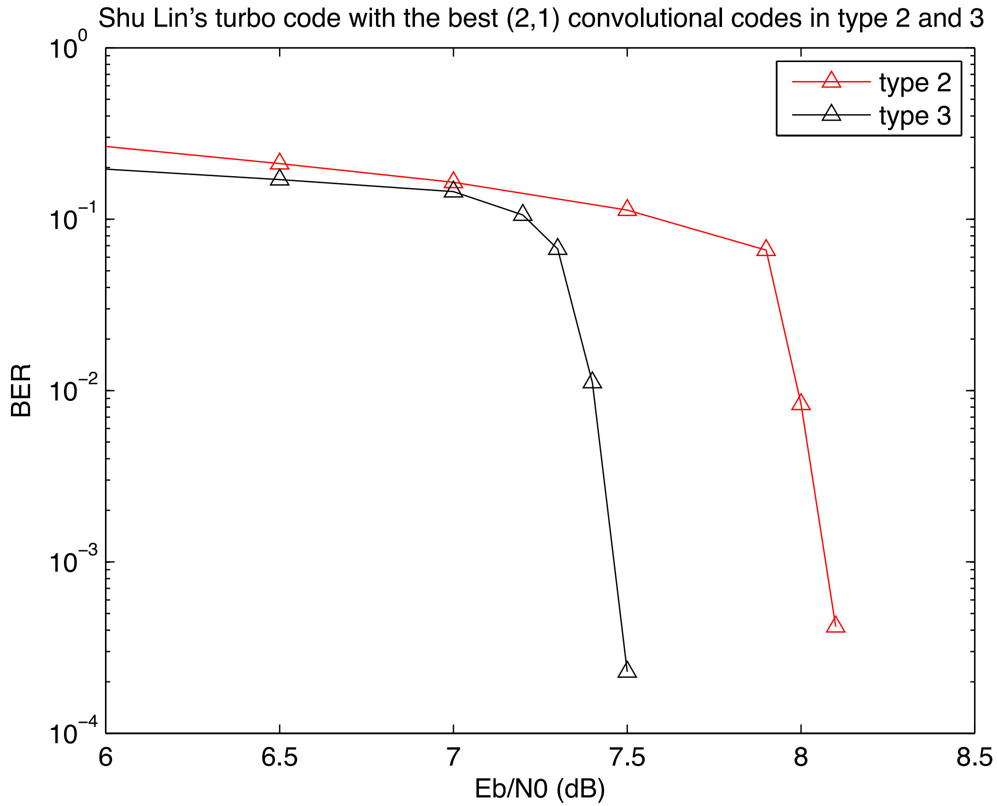


Figure 13: Performances of the concatenated system that employs the turbo code from S. Lin's book concatenated with the optimal non-coherent convolutional codes of type 2 and 3. The code rate is 1/2.

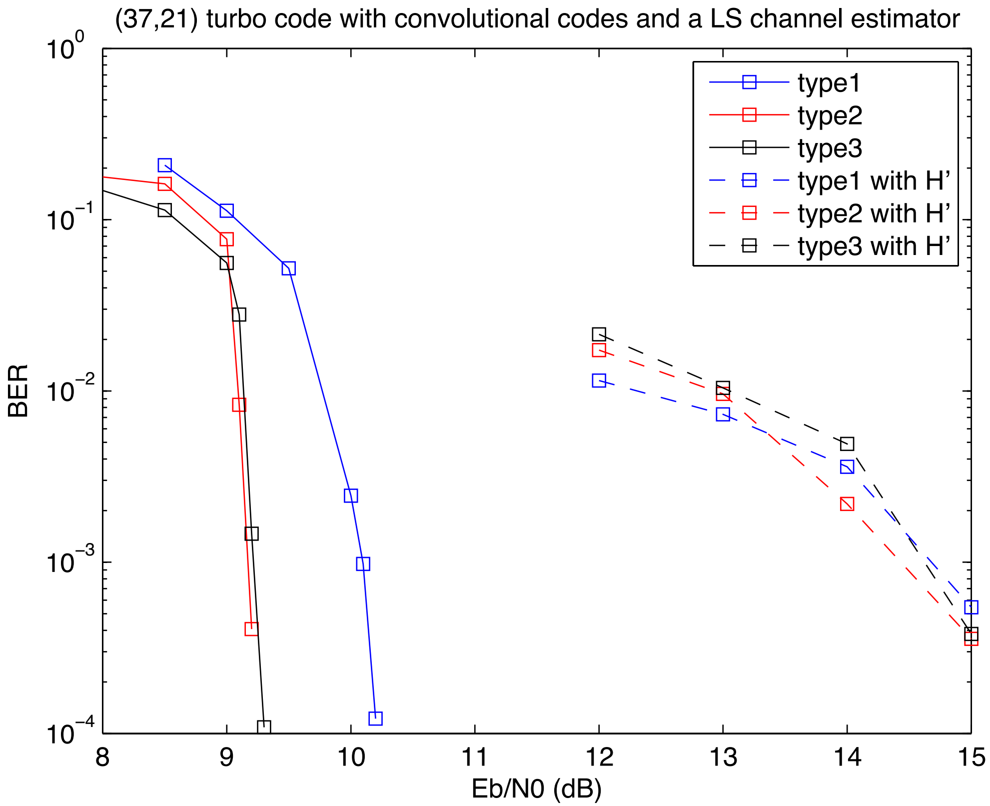


Figure 14: Performances of a conventional system that employs the $(37, 21)$ turbo code with a least square (LS) channel estimator instead of our non-coherent convolutional code. Here, type 1, type 2 and type 3 similarly denote the sizes of the LS estimator, i.e., 10, 8 and 6. The curves in Figure 5 are also illustrated for comparison. The code rate is $2/3$.

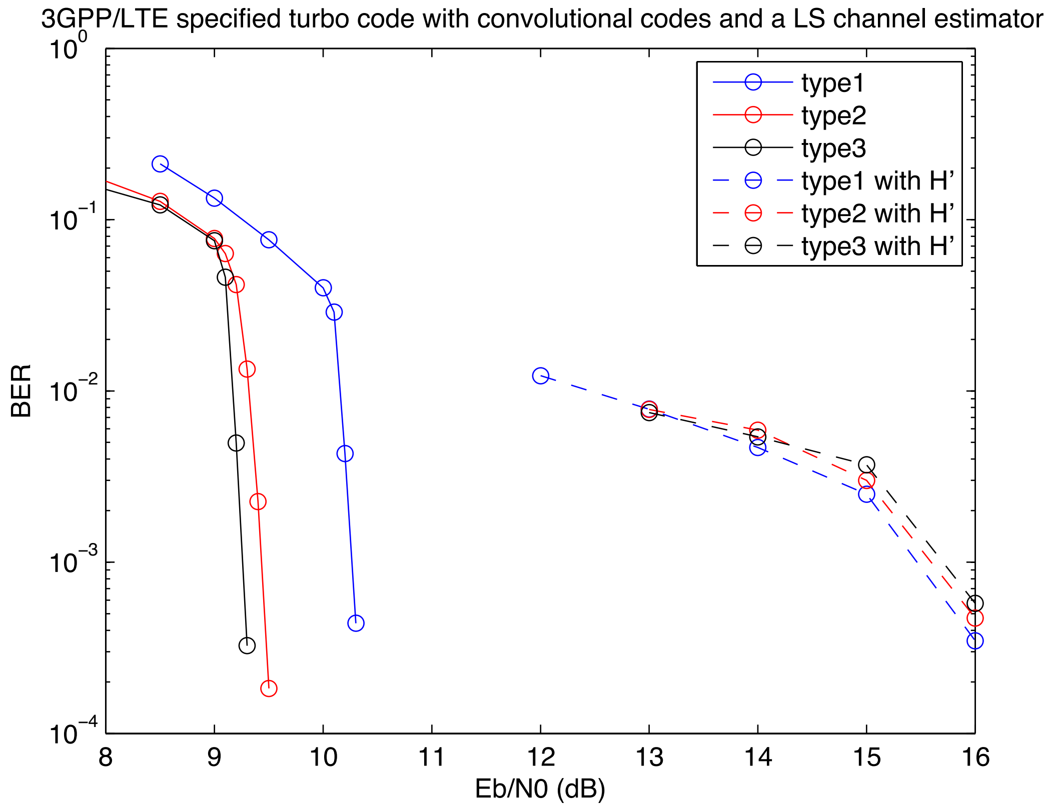


Figure 15: Performances of a conventional system that employs the 3GPP/LTE specified turbo code [?] with a least square (LS) channel estimator instead of our non-coherent convolutional code. Here, type 1, type 2 and type 3 similarly denote the sizes of the LS estimator, i.e., 10, 8 and 6. The curves in Figure 6 are also illustrated for comparison. The code rate is 2/3.

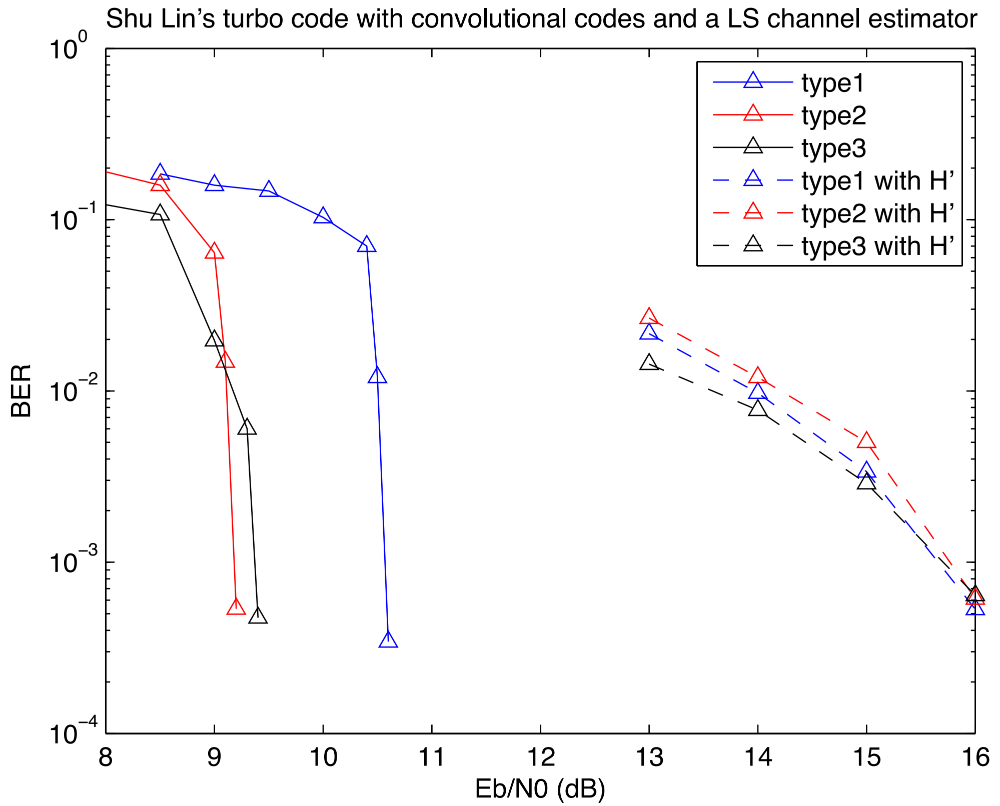


Figure 16: Performances of a conventional system that employs the turbo code from S. Lin's book with a least square (LS) channel estimator instead of our non-coherent convolutional code. Here, type 1, type 2 and type 3 similarly denote the sizes of the LS estimator, i.e., 10, 8 and 6. The curves in Figure 7 are also illustrated for comparison. The code rate is 2/3.

Conclusion and Future Work

Conclusion

- We propose a simple non-coherent-distance-based method to select a good convolutional code to co-work with the outer turbo code in an unknown frequency-selective fading environment.
- By using the selected inner non-coherent convolutional code and the corresponding GLRT decoding criterion, a better performance than the conventional LS estimator can be obtained when the codeword length/estimation window size is small.
- When the code rate is half, the codes that our method locates are the ones with the simulated optimal performance. When the code rate increases to 2/3, however, our method may locate the second-best convolutional code structure in the sense of simulated performance.
- A side observation from our simulations that the proposed concatenated system prefers using a shorter inner convolutional code. This could be due to that in a dynamic fading environment, it may be better to perform frequent channel estimation based on a smaller window and resort the error protection task to the long outer turbo code.

Future Work

- When code rates increase, our method may not select the best code in performance. So some modification may be needed when the code rates further increase.
- We restrict the modulation schemes to BPSK in our design and simulations. It shall be more practical to extend our design to a higher order modulations like QPSK and 16-QAM.

Thank You.