Maximum-likelihood Soft-decision Sequential Decoding Algorithms (MLSDA) for Convolutional Codes

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Outlines

- ❑ Introduction
- ❑ Background of Maximum-likelihood Soft-decision Sequential Decoding Algorithm (MLSDA)
- ❑ Computational Complexity of MLSDA on AWGN channels
- ❑ MLSDA with finite stack-size constraint on AWGN channels
- ❑ MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels
- ❑ Conclusions

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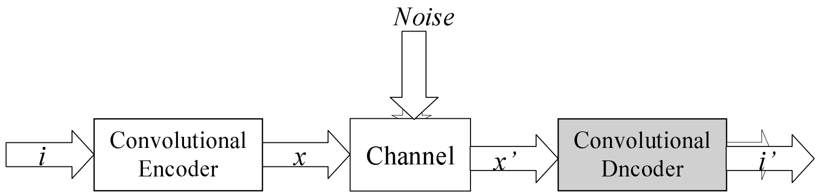
Introduction

- A convolutional coding system with additive noise is considered.
  - *In this thesis, we focus on the decoding algorithm.*



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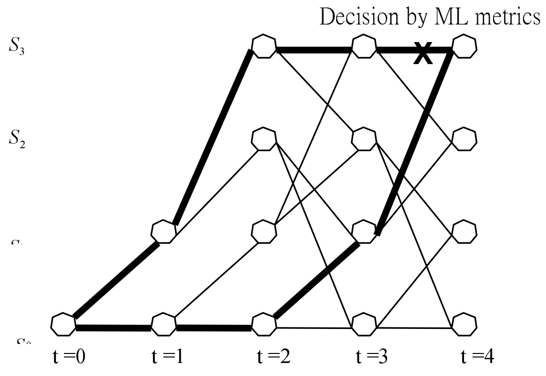
Introduction

- Viterbi Decoding Algorithm
  - *The Viterbi algorithm suffers from a decoding complexity to error probability tradeoff, which prevents it from being applied to codes with long constraint lengths.*



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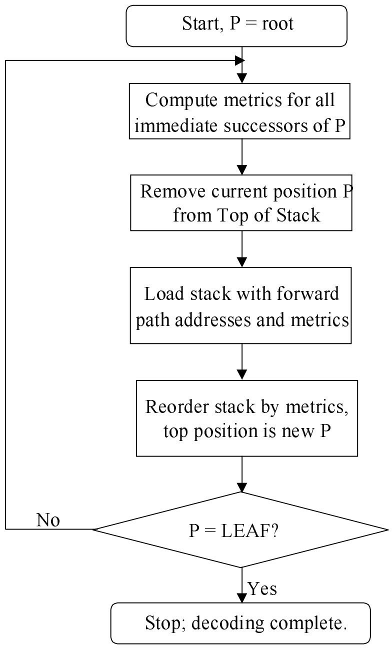
Introductions

- ❑ ZJ algorithm with Fano metrics, operating over a code tree
  - – *Fano metric for code rate R*

$$M\left(r_i^{(j)} \middle| x_i^{(j)}\right) = \log_2\left[\frac{P\left(r_i^{(j)} \middle| x_i^{(j)}\right)}{P\left(r_i^{(j)}\right)}\right] - R$$

where $x_i^{(j)}$ *and* $r_i^{(j)}$ *are respectively the* i^{th} *bit of the* j^{th} *transmitted block, and* i^{th} *received bit at* j^{th} *received block.*

- ❑ ZJ algorithm does not always yield ML decision.



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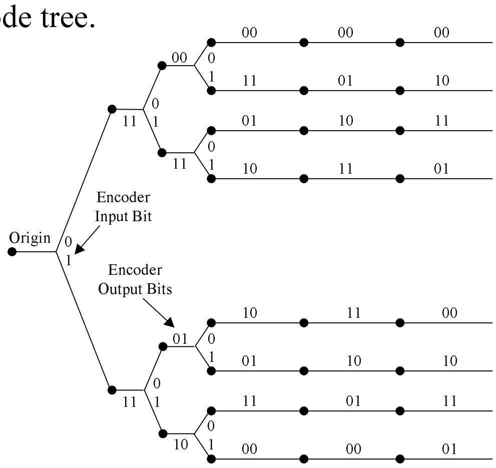
Introduction

- ❑ The stack decoding algorithm with Fano metric, operating over a code tree.



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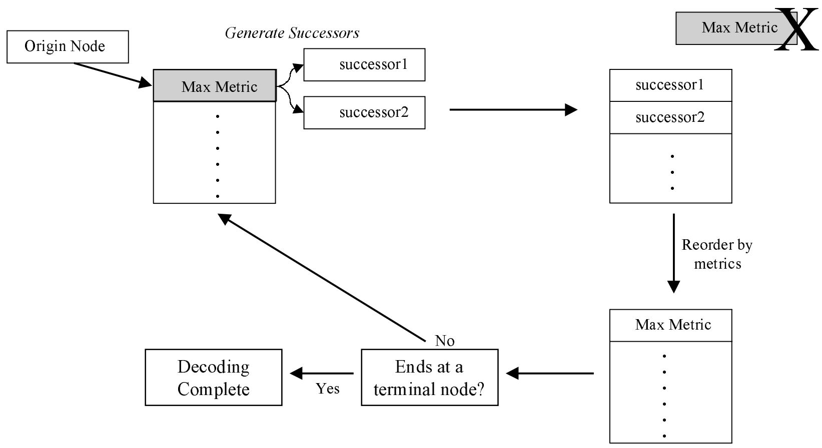
Introductions

- The ZJ algorithm with Fano metric, operating over a code tree.



Background of MLSDA

- (Han and Chen) Tree-based *Maximum-likelihood* Soft-decision Sequential Decoding Algorithm
  - *The conventional sequential decoding algorithm with a new metric*

$$M\left(r_i^{(j)} \middle| x_i^{(j)}\right) = (y_i^{(j)} \oplus x_i^{(j)}) \left| \log_2 \left[\frac{P\left(r_i^{(j)} \middle| x_i^{(j)} = 0\right)}{P\left(r_i^{(j)} \middle| x_i^{(j)} = 1\right)} \right] \right|$$

where $x_i^{(j)}$ *and* $r_i^{(j)}$ *are respectively the* i^{th} *bit of the* j^{th} *transmitted block and* i^{th} *received bit at* j^{th} *received block, and*

$$y_i^{(j)} = \begin{cases} 1, & \text{if } \log_2 \left[\frac{P\left(r_i^{(j)} \middle| x_i^{(j)} = 0\right)}{P\left(r_i^{(j)} \middle| x_i^{(j)} = 1\right)} \right] < 0; \\ 0, & \text{otherwise.} \end{cases}$$

Background of MLSDA

- (Han and Chen) Tree-based *Maximum-likelihood* Soft-decision Sequential Decoding Algorithm on **AWGN** channels
  - *The conventional sequential decoding algorithm with a new metric*

$$M\left(r_i^{(j)} \middle| x_i^{(j)}\right) = (y_i^{(j)} \oplus x_i^{(j)}) \left| r_i^{(j)} \right|$$

where $x_i^{(j)}$ *and* $r_i^{(j)}$ *are respectively the* i^{th} *bit of the* j^{th} *transmitted block and* i^{th} *received bit at* j^{th} *received block, and*

$$y_i^{(j)} = \begin{cases} 1, & \text{if } r_i^{(j)} < 0; \\ 0, & \text{otherwise.} \end{cases}$$

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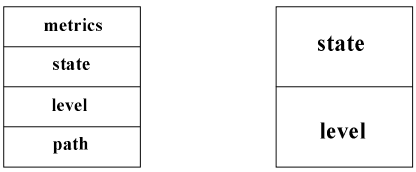
Background of MLSDA

- Trellis-based MLSDA
  - *By employing two stacks--an* Open Stack *and a* Closed Stack-- *instead of one, MLSDA can operate over a code trellis*
  - *The Open Stack contains all paths ending at the frontier part of the trellis, which have been explored by the algorithm.*
  - *The Closed Stack stores the information of the ending states and ending levels of the paths, which had been the top paths of the Open Stack..*



OPEN STACK CLOSED STACK

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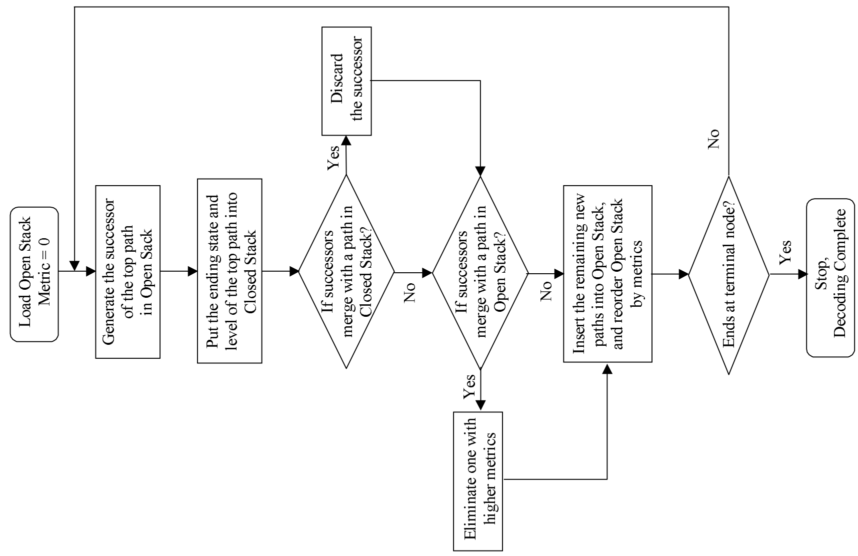
The Trellis-based MLSDA



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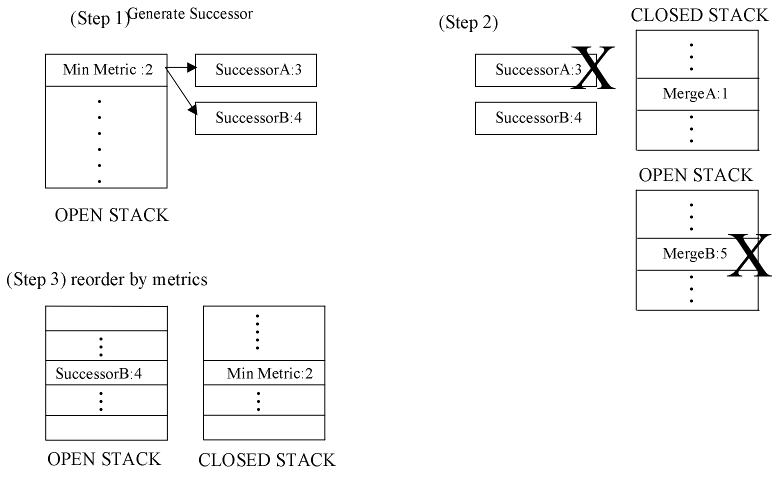
Background of MLSDA

- Trellis-based MLSDA



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Background of MLSDA

❑ Successor generation

metrics
state
level
path

OPEN STACK

0.36
011111
6
11111

OPEN STACK

0.96
111110
7
1111110

SuccessorA

$r_6 = \left(r_6^{(0)}, r_6^{(1)}\right) = (0.1, 0.5)$ *implies* $y_6 = \left(y_6^{(0)}, y_6^{(1)}\right) = (0,0)$;

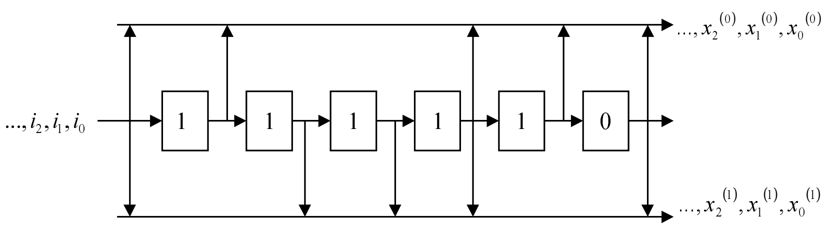
$$\Rightarrow \begin{cases} M(r_6 \mid x_6 = 11) = (0 \oplus 1)\left|r_6^{(0)}\right| + (0 \oplus 1)\left|r_6^{(1)}\right| = 0.6; \\ M(r_6 \mid x_6 = 00) = (0 \oplus 0)\left|r_6^{(0)}\right| + (0 \oplus 0)\left|r_6^{(1)}\right| = 0; \end{cases}$$

0.36
111111
7
1111111

SuccessorB

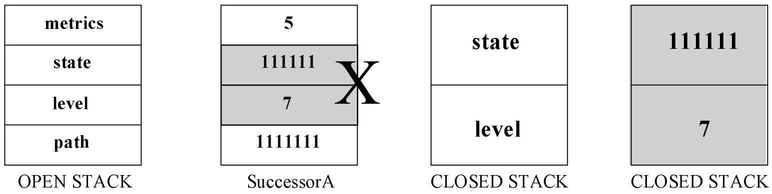
Background of MLSDA

❑ An (2,1) convolutional code encoder

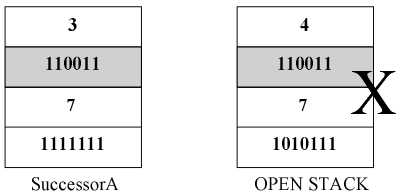


Background of MLSDA

❑ Merging for Closed Stack



❑ Merging for Open Stack



Main contributions

❑ Perform simulations on the computational complexity of trellis-based MLSDA

  – *Confirmation of the accuracy of theoretical bound derived by Chen and Han*

❑ Investigate the performance degradation of MLSDA due to Stack-size constraint.

❑ Study the performance variation of MLSDA over Rayleigh-distributed Frequency-nonselective Slowly-fading channels.

Computational complexity of MLSDA on AWGN channels

❐ **Model of Antipodal transmission of code bit and AWGN channel**

$$r = (-1)^x \sqrt{E_b} + n, \text{ where } n \equiv Normal(0, N_0/2).$$

❐ **Notations**

- *Denote by $S_{min}(l)$ the set containing all paths, which end at level l, and whose Hamming weight is the smallest among those path portions ending at the same node.*
- *Let $W_{min}(l)$ be the set of the Hamming weights of the paths $S_{min}(l)$*
- *Signal-to-Noise ratio = $\gamma = E/N_0$.*
- *Signal-to-Noise ratio per information bit = $\gamma_b = (N/KL)\gamma$.*

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Computational complexity of MLSDA on AWGN channels

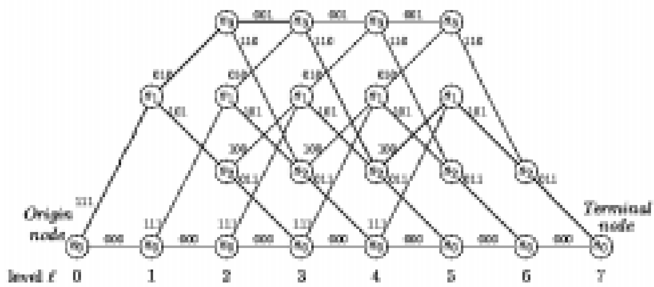

Figure 2.1: Trellis diagram for a (3, 1, 2) convolutional code with $L = 5$. In this case, $R = 1/3$ and $N = 3(5+2) = 21$. Also, the code path indicating by the thick line is labeled in sequence with 111, 010, 001, 110, 100, 101 and 011. Its corresponding codeword is $x = (1,1,1,0,1,0,0,0,1,1,1,0,1,0,0,1,0,1,0,1,1)$. The alternative representation for the same path in terms of the sequence of nodes is $(s_0, s_1, s_3, s_3, s_2, s_1, s_2, s_0)$

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Computational complexity of MLSDA on AWGN channels

- **Theorem (Chen and Han)**
  - *For an (n,k) convolutional code with input information sequence of length L, the average number of branch metric computations of trellis-based MLSDA, denoted by* $L(\gamma_b)$*, satisfies*

$$L(r_b) \le 2^k L + 2^k \sum_{l=1}^{L-1} \sum_{d \in W_{\min}(l)} N(d,l) \times B(d, N - l \times n; kL\gamma_b / N)$$

  where $N(d,l)$ *is the number of path portions in* $S_{\min}(l)$*, satisfying that the Hamming weight of the path equals to d*

  - *The function* $B(d, q; \gamma)$ *is a probability bound satisfying*

$$\Pr[X_1 + X_2 + \cdots + X_d + Z_1 + Z_2 + \cdots + Z_q \le 0] \le B(d,q;\gamma)$$

  where

$$X_i \equiv Normal(\mu, \sigma^2),\ Z_i \equiv \min[X_i, 0] \text{ and } \gamma = (1/2)\mu^2/\sigma^2.$$

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Computational complexity of MLSDA on AWGN channels

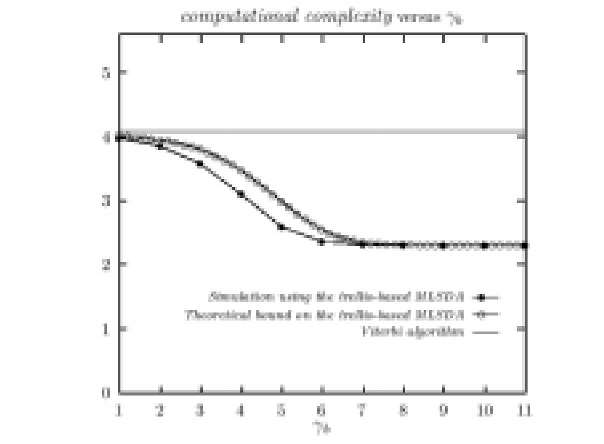

Figure 3.1: Function of average number of metric computation with respect to SNR per information bit. The convolutional code is the (2,1,6) code with generators 634, 564. The information length L is equal to 100.

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Computational complexity of MLSDA on AWGN channels

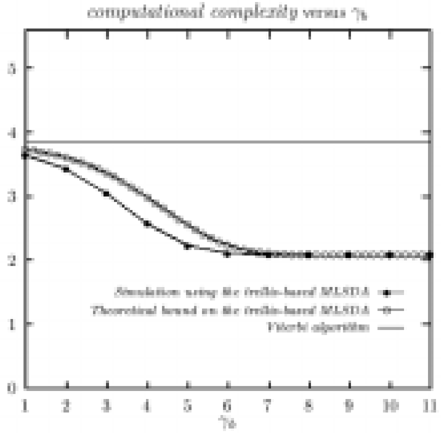

Figure 3.2: Function of average number of metric computation with respect to SNR per information bit. The convolutional code is the (2, 1, 6) code with generators 634, 564. The information length L is equal to 60.

Computational complexity of MLSDA on AWGN channels

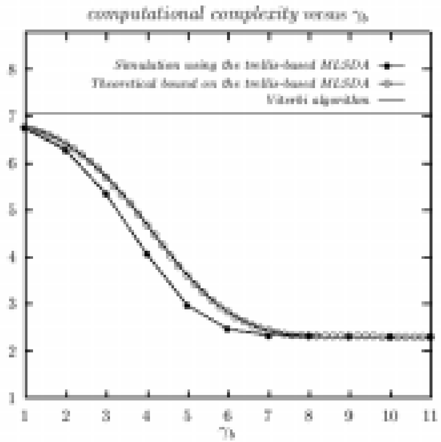

Figure 3.3: Function of average number of metric computation with respect to SNR per information bit. The convolutional code is the (2, 1, 16) code with generators 1632044, 1145734. The information length L is equal to 100.

Computational complexity of MLSDA on AWGN channels

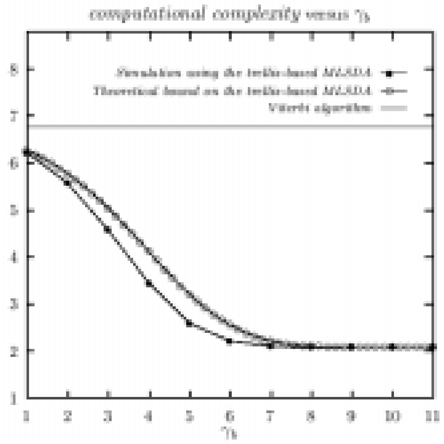

Figure 3.4: Function of average number of metric computation with respect to SNR per information bit. The convolutional code is the (2, 1, 16) code with generators 1632044, 1145734. The information length L is equal to 60.

Computational complexity of MLSDA on AWGN channels

- Observations
  - *Theoretical upper bound is indeed fairly close to the simulated result for high γ_b (above 6dB), as well as for low γ_b (below 2 dB). Even for moderate γ_b, they only differ by at most 0.67.*
  - *When γ_b > 6dB, MLSDA will have a much smaller average computational complexity than the Viterbi algorithm for all simulated codes.*
  - *When γ_b > 6dB, the average number of metric values evaluated by the MLSDA are close to 2^kL, which is exactly the lower bound of the number of metric values evaluated by any decoding algorithm.*
  - *The codes with longer constraint length, although they have a lower bit error rate (BER), will introduce more computations.However, such dilemma can be moderately released at high SNR.*

MLSDA with finite stack-size constraint on AWGN channels

- More observations from simulations
  - *The requirement of the stack size grows dramatically with the constraint length of the convolutional code.*
  - *A more dramatic growth in system memory is induced by decreasing SNR.*
  - *In practice, one needs to restrict the stack size used in the algorithm in order to limit the usage of system memory.*
  - *We make a tradeoff between BER and speed/stack-size of the MLSDA.*

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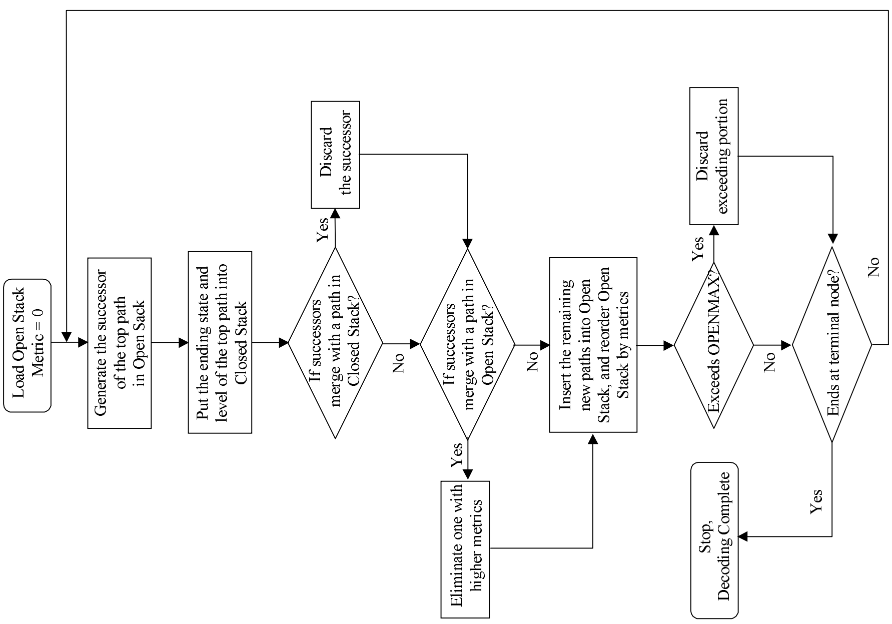
MLSDA with finite stack-size constraint



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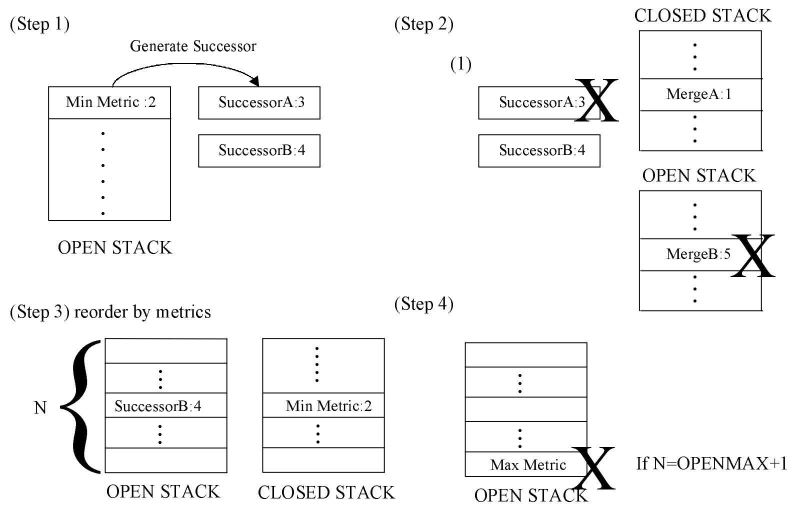
MLSDA with finite stack-size constraint on AWGN channels



MLSDA with Finite Stack-size constraint on AWGN channels

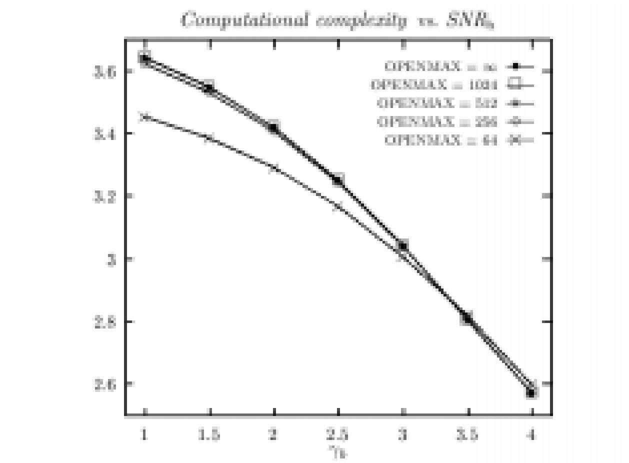


Figure 3.5: Functions of average number of metric computation (in $\log_{10}$ scale) versus signal-to-noise ratio per information bit. *OPENMAX* is the upper limit on the size of the Open Stack.

MLSDA with Finite Stack-size constraint on AWGN channels

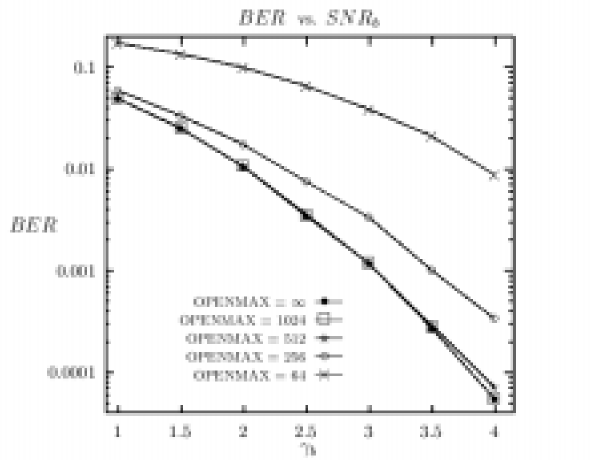


Figure 3.6: Functions of bit error rate (BER) versus signal-to-noise ratio per information bit. *OPENMAX* is the upper limit on the size of the Open Stack.

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MLSDA with finite stack-size constraint on AWGN channels

❑ Concluding remarks

- *Since the Closed Stack stores only the information of ending states and ending levels, the memory size that it requires tends to be small.(We therefore only apply stack-size constraint on Open Stack)*
- *The limitation on the stack size, although reducing the computational efforts, will unavoidably degrade the system performance.*
- *We find that there is no degradation on the MLSDA performance, when stack size is bounded above by 1024.*

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MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

- In the final section, we study the performance of MLSDA over frequency non-selective, slowly fading channels.
- *System model*

$$r = \alpha(-1)^x \sqrt{\varepsilon} + n,$$

$$\textit{where } \alpha \equiv \textit{Rayleigh with } E[\alpha^2] = 1 \textit{ and } n = \textit{Normal}(0, N_0/2).$$

- *Signal-to-noise ratio now becomes*

$$\gamma_b = \frac{\varepsilon_b}{N_0} E[\alpha^2] = \frac{\varepsilon_b}{N_0}$$

MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

- The ML metric for MLSDA on fading channel

$$M(r|x) = (y \oplus x)\left|\log_2\left[\frac{P(r|x=0)}{P(r|x=1)}\right]\right|, \textit{where } y = \begin{cases} 1, & \textit{if } \log_2\left[\frac{P(r|x=0)}{P(r|x=1)}\right] < 0; \\ 0, & \textit{otherwise.} \end{cases}$$

$$P(r|x=0) = \int_0^\infty f_\alpha(\alpha) f_n(r - \alpha\sqrt{\varepsilon})\, d\alpha$$

$$P(r|x=1) = \int_0^\infty f_\alpha(\alpha) f_n(r + \alpha\sqrt{\varepsilon})\, d\alpha$$

$$\Rightarrow \frac{P(r|x=0)}{P(r|x=1)} = \frac{\int_0^\infty \frac{\alpha}{\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\frac{1}{\sqrt{\pi N_0}}\exp\left(-\frac{(r-\alpha\sqrt{\varepsilon})^2}{N_0}\right)d\alpha}{\int_0^\infty \frac{\alpha}{\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\frac{1}{\sqrt{\pi N_0}}\exp\left(-\frac{(r+\alpha\sqrt{\varepsilon})^2}{N_0}\right)d\alpha}$$

$$= 1 + \frac{1}{\frac{1}{B}\sqrt{\frac{A}{\pi}}\exp\left(\frac{-B^2}{4A}\right) - Q\left(\frac{B}{\sqrt{2A}}\right)} \quad \text{where } A = \frac{(1+\gamma)}{E[\alpha^2]} \quad \text{B} = \frac{2\sqrt{\varepsilon}\, r}{N_0}$$

MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

- ❑ The ML metric for MLSDA on AWGN channel (for convenience, we name it AWGN metric)

$$M(r|x) = (y \oplus x)|r|, \text{where } y = \begin{cases} 1, & \text{if } r < 0; \\ 0, & \text{otherwise.} \end{cases}$$

- ❑ Question
  - *What is the influence on performance, if one mis-classify a fading channel as an AWGN channel, and hence, employs the AWGN metric over a fading channel?*
  - *Two performance indices will be investigated*
    - *Computational complexity*
    - *Probability of error*

MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

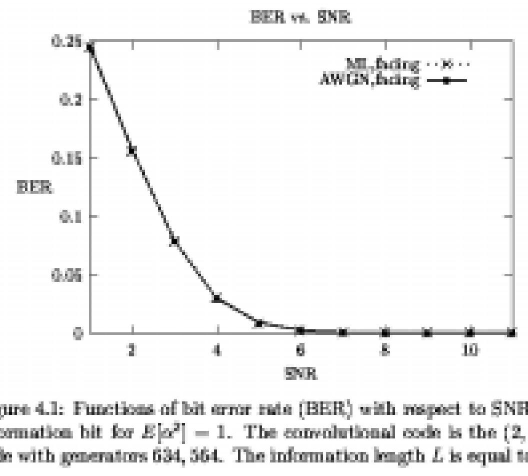

Figure 4.1: Functions of bit error rate (BER) with respect to SNR per information bit for $E[\alpha^2] = 1$. The convolutional code is the $(2, 1, 6)$ code with generators 634, 564. The information length L is equal to 60.

MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

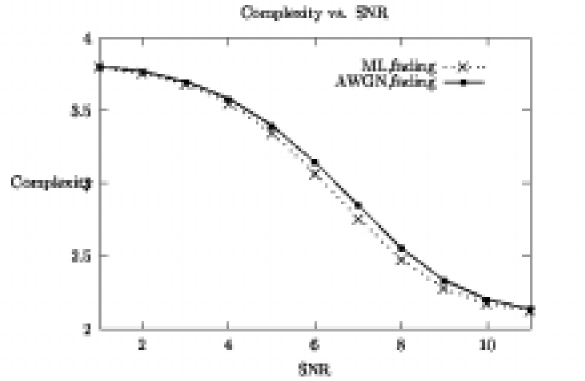

Figure 4.6: Functions of average number of metric computation with respect to SNR per information bit for $E[\alpha^2] = 1$. The convolutional code is the $(2, 1, 6)$ code with generators 634, 564. The information length L is equal to 60.

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MLSDA on Rayleigh-distributed Frequency-nonselective, Slowly-fading channels

- Concluding remarks
  - *Trellise-based SDA with ML-AWGN metric is robust in error probability under Rayleigh-distributed Frequency-nonselective Slowly-fading channels.*
  - *Trellis-based SDA with ML-AWGN metric still have superiority of computational complexity over MLSDA on fading channels. In addition, the computation of ML-AWGN metric apparently takes less time than ML-Fading metric.*

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Conclusions

- On the simulations on the computational complexity of trellis-based MLSDA to confirm the accuracy of Han-Chen theoretical bound
  - *Empirical investigation on its accuracy shows that the theoretical bound is fairly close to our simulation results.*
  - *The computational complexity of the MLSDA is much less than that the Viterbi algorithm when signal-to-noise ratio is moderately high.*
  - *Even in the worst situation where the AWGN pattern forces the MLSDA to always take the maximum number of metric operations, its computational effort is still the same as that required by the Viterbi algorithm.*

Conclusions

- On investigating the performance degradation of MLSDA due to Stack-size constraint.
  - *Under a practical consideration in system memory, a truncated convolutional code can be applied to the MLSDA.*
- On the performance variation of MLSDA over Rayleigh-distributed Frequency-nonselective Slowly-fading channels.
  - *SDA with mis-classified ML-AWGN metric is robust in error probability.*
  - *SDA with mis-classified simple ML-AWGN metric indeed have superiority on computational complexity over MLSDA with complex ML-fading-metric.*
  - *The complex ML-fading metric is a function of the noise level, while the simple ML-AWGN only depends on the received signal.*