Modified GLRT Demodulation for OFDM Signal Transmitted Over a Frequency-Selective Fading Channel

Hsin-Yu Kuo

Department of Communication Engineering
National Chiao Tung University
Hsin Chu, Taiwan 30050, R.O.C.
Outline

- Introduction
- Technical background
- Modified GLRT and Its Recursive Metric
- Simulation Result
- Conclusion and Future work
Introduction
Motivation

• A typical receiver for wireless communication system performs channel estimation, channel equalization and data detection separately.

• In recent researches, the demodulator combined channel estimation and data detection has been considered a promising approach to combat the effects of multi-path fading.

• The demodulation is regarded as blind because the channel fading coefficients are unknown to both the receiver and transmitter. The generalized likelihood ratio test (GLRT) is the best decision maker for blind demodulation.
Contributions

- We develop a blind demodulator, termed as the Modified GLRT, for an OFDM system transceived over a frequency selective channel with unknown channel coefficients.

- This demodulator jointly performs the channel estimation and data detection.

- In order to reduce the demodulation complexity, a priority-first search demodulation algorithm as well as the recursive metric formula used by this algorithm is also proposed.
Technical Background
An OFDM System with Non-Coherent Receiver

\[ s_j = \mathcal{Q}^\dagger S_j \]

\[ x_j = \mathcal{H}s_j + w_j \]

\[ x_j = \mathcal{Q}^\dagger \mathcal{Q}s_j + w_j \]

\[ X_j = \Lambda S_j + W_j \]

\[ X_j = \mathcal{Q}x_j \]
An OFDM System with Non-Coherent Receiver

- The final formation of the considered system is

\[ \mathbf{X}_j = \mathbf{S}_j \mathbf{\lambda} + \mathbf{W}_j \]  

(1)

- In (1), we transform vector \( \mathbf{S}_j \) to its equivalent diagonal matrix \( \mathbf{S}_j \) as

\[
\mathbf{S}_j = \begin{bmatrix}
S_{1,j} & 0 & \cdots & 0 \\
0 & S_{2,j} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & S_{N,j}
\end{bmatrix}_{N \times N}
\]

- The elements of vector \( \mathbf{\lambda} = [\lambda_{N-1} \cdots \lambda_1 \lambda_0]^T \) are the DFT values of the channel fading \( \mathbf{h} \), given by

\[
\Lambda = \begin{bmatrix}
\lambda_{N-1} & 0 & \cdots & 0 \\
0 & \lambda_{N-2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_0
\end{bmatrix} = \mathbf{Q} \begin{bmatrix}
h_0 & h_1 & h_2 & \cdots & h_{\nu-1} & h_{\nu} & 0 & \cdots & 0 \\
0 & h_0 & h_1 & \cdots & h_{\nu-2} & h_{\nu-1} & h_{\nu} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{\nu} \\
h_{\nu} & 0 & 0 & \cdots & 0 & 0 & h_0 & \cdots & h_{\nu-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
h_1 & h_2 & h_3 & \cdots & h_{\nu} & 0 & 0 & \cdots & h_0
\end{bmatrix} \mathbf{Q}^\dagger.
\]
An OFDM System with Non-Coherent Receiver

- It can be verified that for GLRT it requires at least two OFDM symbols to perform non-coherent detection at the receiver.

- Hence, we assume $T \geq 2T_s$, $T$ is the period that channel coefficients $h$ remain constant, and $T_s$ is the OFDM symbol duration.

- Let the two OFDM symbols received be indexed by 1 and 2. This gives

  $$X_1 = S_1 \lambda + W_1 \tag{2}$$

  and

  $$X_2 = S_2 \lambda + W_2 \tag{3}$$
An OFDM System with Non-Coherent Receiver

For convenience, we will combine (2) and (3) into

\[ \vec{X} = \vec{S}\lambda + \vec{W} \]

where

\[ \vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad \vec{S} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad \text{and} \quad \vec{W} = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}. \]

- We can of course consider a non-coherent receiver for the information \( \vec{S} = [S_1, S_2, \ldots, S_k]^T \), that is \( T \geq kT_s \). However, since the demodulation complexity dramatically grows as \( k \) increases and since a larger \( k \) implies the resulting system can be only used in a less mobile environment, this thesis only considers the case of \( k = 2 \).
We firstly give two assumptions:

(i) both transmitter and receiver knows the memory order of the channel \( \nu \)

(ii) none of them has the knowledge about channel fading \( \lambda \).

The general likelihood ratio test (GLRT) decision can be written as (4)

\[
\hat{S} = \arg \min_{\hat{S} \in S} \min_{\lambda \in \mathbb{C}^N} \| \tilde{X} - \hat{S} \lambda \|^2 = \arg \min_{\tilde{S} \in \mathbb{C}^N} \| \tilde{X} - \hat{S} \hat{\lambda} \|^2 = \arg \min_{\tilde{S} \in \mathbb{C}^N} \| \tilde{X} - P_{\hat{S}} \tilde{X} \|^2
\]

(4)

where the least square estimate of the channel coefficient \( \lambda \) for a given \( \tilde{S} \) is

\[
\hat{\lambda} = (\tilde{S}^\dagger \tilde{S})^{-1} \tilde{S}^\dagger \tilde{X}
\]

and \( P_{\tilde{S}} \triangleq \tilde{S} (\tilde{S}^\dagger \tilde{S})^{-1} \tilde{S}^\dagger \)
General Likelihood Ratio Test (GLRT) Criterion

- We can further derive that

\[
\hat{\mathbf{S}} = \arg \min_{\mathbf{s} \in \mathcal{S}} \| \mathbf{X} - \mathbb{P}_s \mathbf{X} \|^2 = \arg \max_{\mathbf{s} \in \mathcal{S}} \sum_{i=1}^{N} \frac{|S_{i,1}X_{i,1}^* + S_{i,2}X_{i,2}^*|^2}{|S_{i,1}|^2 + |S_{i,2}|^2}
\]

- \((S_{i,1}, S_{i,2}) = (1, -1)\) and \((S_{i,1}, S_{i,2}) = (-1, 1)\) are indistinguishable at the receiver; hence, one of \(S_{i,1}\) and \(S_{i,2}\) must be fixed. A GLRT-detectable transmission signal can be assigned as:

\[
\hat{\mathbf{S}} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & S_{2,1} & 0 & \cdots & 0 \\ 0 & 0 & S_{3,1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \\ S_{1,2} & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & S_{N,2} \end{bmatrix}
\]
• For demodulating $\bar{S}$, we represent all possible transmission signals by a tree. As an example for BPSK,

The fixed bits are circled.
Maximum-Likelihood Priority-First Search Algorithm

- Step 1. Load the stack with the path that ends at the original node at level 0.
- Step 2. Insert the successor paths of the current top path into the stack such that the paths in the stack are ordered according to their ascending metric values $f$; then, delete this top path from the stack.
- Step 3. If the new top path in the stack ends at a terminal node in the tree at level $N$, output the labels corresponding to the top path, and stop the algorithm; otherwise, go to Step 2.

- We quote a sufficient condition that guarantees the priority-first search algorithm expands the path with smallest metric among all paths of the same length.

**Lemma** If the metric $f$ is nondecreasing along every path in the tree, i.e.,

\[
f(\tilde{S}_\ell) \leq \min_{\{\tilde{S} \in S : \tilde{S}_\ell = \tilde{S}_\ell\}} f(\tilde{S})
\]

(5)

then the priority-first search algorithm always yields the path with the smallest metric value among all paths of the same length.
Usually, a metric $f$ is composed of two parts:

$$f(\vec{S}(\ell)) \triangleq g(\vec{S}(\ell)) + \varphi(\vec{S}(\ell)).$$

The former part $g$ is determined based on the maximum-likelihood metric and hence satisfies

$$\arg \min_{\vec{S} \in \mathcal{S}} g(\vec{S}) = \arg \min_{\vec{S} \in \mathcal{S}} ||\vec{X} - \mathbb{P}_{\vec{S}} \vec{X}||^2. \quad (6)$$

The latter part $\varphi$ is often named heuristic function. It gives the prediction of the route from the current node to an end node at level $\mathcal{N}$ so as to speed up the search process.

The heuristic function is not unique, and different designs will lead to distinct computational complexities.
Modified GLRT and Priority First Search Demodulator
The Modified GLRT

- The range of minimization in the GLRT demodulator is the entire $N$-dimensional complex domain $\mathbb{C}^N$. We however observe that the possible values of $\lambda$ should be much smaller than this $N$-dimensional complex domain.

- It can be derived for general $N$ and $\nu < N$ that
  \[ \lambda = Q_{\nu} h \]
  where
  \[ Q_{\nu} = [q_{N-\nu} \ldots q_N] . \]

- The non-coherent GLRT demodulator/decoder can be written as
  \[ \hat{S} = \arg \min_{\tilde{S} \in S} \min_{h \in C^{\nu+1}} \| \tilde{X} - \tilde{S} Q_{\nu} h \|^2 \] (7)
The Modified GLRT

- With the same two assumptions, the modified version of general likelihood ratio test (i.e., the modified GLRT) decision can be written as

\[
\hat{\tilde{S}} = \arg \min_{\tilde{S} \in \mathcal{S}} \| \tilde{X} - \tilde{S} Q_{\nu} \hat{h} \|^2 = \arg \min_{\tilde{S} \in \mathcal{S}} \| \tilde{X} - \mathbb{P}_{\tilde{S}} \tilde{X} \|^2 
\]

the least square estimate of channel fading \( h \) for given \( \hat{\tilde{S}} \) is

\[
\hat{h} = (\hat{\tilde{S}}^\dagger \hat{\tilde{S}})^{-1} \hat{\tilde{S}}^\dagger \tilde{X}
\]

here \( \hat{\tilde{S}} \triangleq \tilde{S} Q_{\nu} \), and \( \mathbb{P}_{\tilde{S}} = \tilde{S} (\hat{\tilde{S}}^\dagger \hat{\tilde{S}})^{-1} \hat{\tilde{S}}^\dagger \).
The Modified GLRT

- The estimate of $\lambda$ for the GLRT without any limitation.
- The new estimate of $h$ now lies in a $(\nu + 1)$-dimensional hyperplane, defined via $\lambda = Q_\nu h$.
- By using the modified GLRT instead of the GLRT, we expect that the probability of error will decrease.
The Modified GLRT

• The demodulator/decoder output with respect to \( \mathbf{S} \) is given by:

\[
\hat{\mathbf{S}} = \arg \min_{\mathbf{S} \in \mathcal{S}} \| \mathbf{X} - \mathbf{P}_{\mathbf{S}} \mathbf{X} \|_2 = \arg \max_{\mathbf{S} \in \mathcal{S}} \mathbf{X}^\dagger \mathbf{P}_{\mathbf{S}} \mathbf{X}
\]

• Because \( \mathbf{X}^\dagger \mathbf{P}_{\mathbf{S}} \mathbf{X} = \mathbf{X}^\dagger \mathbf{P}_{-\mathbf{S}} \mathbf{X} \), the following two OFDM symbols are indistinguishable at the receiver.

\[
\mathbf{S} = \begin{bmatrix}
-1 & 0 \\
0 & -1 \\
+1 & 0 \\
0 & -1
\end{bmatrix}
\quad \text{and} \quad
-\mathbf{S} = \begin{bmatrix}
+1 & 0 \\
0 & +1 \\
-1 & 0 \\
0 & +1
\end{bmatrix}.
\]

• We should fix at least one component of an OFDM symbol \( \tilde{\mathbf{S}} \). For instance, we can fix \( S_{1,1} = -1 \) for BPSK.
The Modified GLRT

- Half of the OFDM components should be fixed if we use the GLRT demodulator.
- The modified GLRT can reach a much higher data rate by fixing only one component in an OFDM symbol.
- In later simulations, we fix half of the components in order to compare the performance of the modified GLRT with that of the GLRT under the same data rate.
The Modified GLRT

• **Lemma**  When \( \nu = N - 1 \), the Modified GLRT is reduced to the GLRT.

**Proof:** When \( \nu = N - 1 \), we immediately have

\[
\mathbf{Q}_\nu = \begin{bmatrix} q_1 & q_2 & \cdots & q_N \end{bmatrix} = \sqrt{N} \mathbf{Q}
\]

and

\[
\mathbf{S} = \tilde{\mathbf{S}} \mathbf{Q}_\nu = \tilde{\mathbf{S}} \sqrt{N} \mathbf{Q}.
\]

The resulting \( \mathbb{P}_{\tilde{\mathbf{S}}} \) is therefore equal to

\[
\mathbb{P}_{\tilde{\mathbf{S}}} = \tilde{\mathbf{S}} \mathbf{Q} \mathbf{Q}^{-1} \tilde{\mathbf{S}}^\dagger = \tilde{\mathbf{S}} \mathbf{Q} \mathbf{Q}^{-1} \tilde{\mathbf{S}}^\dagger
\]
The Recursive Metric for the Modified GLRT

We will derive the recursive metric formulas that are used by the priority-first search algorithm under the modified GLRT criterion. Two formulas will be provided:

- The first metric $f_1$ allows the demodulation to be performed by sequentially feeding the components of an OFDM symbol,
  \[ f_1(\tilde{S}_{(\ell)}) \triangleq g(\tilde{S}_{(\ell)}) + \varphi_1(\tilde{S}_{(\ell)}), \]
- The second one $f_2$ requires parallelly using all components of an OFDM symbol at each recursion step.
  \[ f_2(\tilde{S}_{(\ell)}) \triangleq g(\tilde{S}_{(\ell)}) + \varphi_2(\tilde{S}_{(\ell)}), \]
- $f_2$ is more efficient since it expands less nodes during the priority-first search.
The ML Recursive Metric for the Modified GLRT

- From Equation (8), the ML metric $g$ can be written as,

$$\hat{S} = \arg\min_{\bar{S} \in S} \|\bar{X} - \mathbb{P}_{\hat{S}} \bar{X}\|^2$$

$$= \arg\min_{\bar{S} \in S} \left( - \sum_{m=1}^{N} \sum_{n=1}^{N} \delta_{m,n} \sum_{i=1}^{2} \sum_{j=1}^{2} S_{m,i}^* X_{m,i} S_{n,j} X_{n,j}^* \right)$$

$$= \arg\min_{\bar{S} \in S} -2 \sum_{m=1}^{N} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right)$$

where

$$w_{m,n} = \sum_{i=1}^{2} \sum_{j=1}^{2} \text{Re}\{\delta_{m,n} S_{m,i}^* X_{m,i} S_{n,j} X_{n,j}^*\}$$

and $\delta_{m,n}$ is the $(m, n)$ th entry of $(Q_{\nu} Q_{\nu}^\dagger)^T$
The ML Recursive Metric for the Modified GLRT

To satisfy the sufficient condition (5), we add a constant $u_{m,n}$. According to (9), a convenient choice that satisfies the need is $u_{m,n} = |w_{m,n}|$. By this, we know that for BPSK-modulated OFDM symbols,

$$u_{m,n} = \sum_{i=1}^{2} \sum_{j=1}^{2} |\text{Re}\{\delta_{m,n}X_{m,i}X_{n,j}^*}\}|$$

and for QPSK-modulated OFDM symbols,

$$u_{m,n} = \sum_{i=1}^{2} \sum_{j=1}^{2} |\delta_{m,n}X_{m,i}X_{n,j}^*|$$

- The recursive ML metric $g$ for the Modified GLRT is

$$g(\tilde{S}_{(\ell+1)}) = g(\tilde{S}_{(\ell)}) + \left( \sum_{n=1}^{\ell+1} u_{\ell+1,n} - \frac{1}{2} w_{\ell+1,\ell+1} \right) - \left( \sum_{n=1}^{\ell+1} w_{\ell+1,n} - \frac{1}{2} w_{\ell+1,\ell+1} \right).$$

(10)
Heuristic Function $\varphi_1$ for the Modified GLRT

We next derive the heuristic function that validates (5). The heuristic function should satisfy

$$\varphi(\tilde{S}(\ell)) \leq \min_{\tilde{\mathbf{s}} \in S : \tilde{\mathbf{s}} = \bar{\mathbf{s}}(\ell)} \left[ \sum_{m=1}^{N} \left( \sum_{n=1}^{m} u_{m,n} - \frac{1}{2} u_{m,m} \right) - \sum_{m=1}^{N} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right) \right]$$

$$- \left[ \sum_{m=1}^{\ell} \left( \sum_{n=1}^{m} u_{m,n} - \frac{1}{2} u_{m,m} \right) - \sum_{m=1}^{\ell} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right) \right]$$

(11)

- Hence, we choose $\varphi_1(\bar{\mathbf{s}}(\ell)) = 0$ for every $\bar{\mathbf{s}}(\ell)$. It is obvious that the all-zero heuristic function is the largest one that satisfies (11) and is independent of future routes.
Heuristic Function $\varphi_2$ for the Modified GLRT

If we drop the requirement that the metric cannot depend on the future routes, based on (11), we can define

$$\varphi_2(\vec{S}(\ell)) \triangleq \left[ \sum_{m=1}^{N} \left( \sum_{n=1}^{m} u_{m,n} - \frac{1}{2} u_{m,m} \right) - \sum_{m=1}^{N} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right) \right] - \left[ \sum_{m=1}^{\ell} \left( \sum_{n=1}^{m} u_{m,n} - \frac{1}{2} u_{m,m} \right) - \sum_{m=1}^{\ell} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right) \right]$$

(12)

We can simplify (12) to:

$$\varphi_2(\vec{S}(\ell)) \triangleq \sum_{m=\ell+1}^{N} \left( \sum_{n=1}^{m} u_{m,n} - \frac{1}{2} u_{m,m} \right) - \sum_{m=\ell+1}^{N} \left( \sum_{n=1}^{m} w_{m,n} - \frac{1}{2} w_{m,m} \right)$$

(13)

The computational complexity can be further reduced.
The Recursive Demodulation Metric for the GLRT

- Its recursive ML metric $g$ is given by

$$g(\tilde{S}_{(\ell+1)}) = g(\tilde{S}_{(\ell)}) + \sum_{i=1}^{2} \sum_{j=1}^{2} |X_{\ell+1,i}X_{\ell+1,j}^*| - \sum_{i=1}^{2} \sum_{j=1}^{2} S_{\ell+1,i}^*X_{\ell+1,i}S_{\ell+1,j}X_{\ell+1,j}^*$$

- Its heuristic functions are

$$\varphi_1(\tilde{S}_{(\ell)}) = 0$$

$$\varphi_2(\tilde{S}_{(\ell)}) = \sum_{m=\ell+1}^{N} \sum_{i=1}^{2} \sum_{j=1}^{2} |X_{m,i}X_{m,j}^*| - \sum_{m=\ell+1}^{N} \sum_{i=1}^{2} \sum_{j=1}^{2} S_{m,i}^*X_{m,i}S_{m,j}X_{m,j}^*.$$

- The above formula is suitable for both BPSK- and QPSK-modulated OFDM symbols.
Simulation Results
Figure 1: The BERs of the BPSK-OFDM systems demodulated by the Modified GLRT and the GLRT, respectively. The lengths examined are $N = 4, 6$. 
Figure 2: The BERs of the BPSK-OFDM systems demodulated by the Modified GLRT and the GLRT, respectively. The lengths examined are $N = 10, 14$. 

Simulation Result
Simulation Result

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Figure 3: The demodulation complexities corresponding to Figure 1, using the demodulation metric $f_1$
Figure 4: The demodulation complexities corresponding to Figure 2, using the demodulation metric $f_1$. 
Figure 5: The demodulation complexities corresponding to Figure 1, using the demodulation metric $f_2$. 
Figure 6: The demodulation complexities corresponding to Figure 2, using the demodulation metric $f_2$
Simulation Result

Figure 7: The BERs of the QPSK-OFDM systems demodulated by the Modified GLRT and the GLRT, respectively. The lengths examined are $N = 4, 6$. 
Figure 8: The BERs of the QPSK-OFDM systems demodulated by the Modified GLRT and the GLRT, respectively. The length examined is $N = 10$. 
Figure 9: The demodulation complexities corresponding to Figure 7, using the demodulation metrics $f_1$.
Figure 10: The demodulation complexities corresponding to Figure 7, using the demodulation metrics $f_2$.
Figure 11: The demodulation complexities corresponding to Figure 8, using the demodulation metrics $f_2$
Figure 12: The BERs of the BPSK-OFDM and QPSK-OFDM systems demodulated by the Modified GLRT. The lengths examined are \( N = 4, 6 \).
Conclusion and Future Work
Conclusion

- We proposed the Modified GLRT criterion for blind demodulation of the OFDM signals transmitted over a frequency-selective channel. As anticipated, the proposed Modified GLRT demodulator can jointly perform channel estimation and data detection.

- By simulations, we found that the BER performance can be improved by using the Modified GLRT demodulator when it is compared with the GLRT demodulator.

- By deriving the recursive formula corresponding to the Modified GLRT criterion, we can apply the priority first search algorithm to the Modified GLRT, and hence the demodulation complexity is significantly reduced in comparison with the exhaustive demodulator.
Future Work

- It should be interesting to examine our Modified GLRT demodulation for the QAM-modulation-based OFDM system, which is of more common use in OFDM system and which can provide a higher data rate.

- Although the demodulation complexity of the Modified GLRT is reduced by introducing the priority-first search, the complexity still grows exponentially with respect to the symbol length $N$. Efforts should be placed to further reduce the demodulation complexity without sacrifice much of the good performance of the Modified GLRT.
Thank You :)