Robust Code Design Over Fast Fading Channels

Prepared by Ming-Hsin Kuo

Advisory by Prof. Po-Ning Chen

Institute of Communication Engineering

National Chiao-Tung University

Hsinchu, Taiwan 300, R.O.C.

June 20, 2008

Outline

- **Introduction**
- System Model
- Code Design
- Simulation Results
- Conclusions

Introduction

- In a quasi-static block fading environment, the channel statistics is assumed to change at a very slow rate, and hence, remains almost constant within a block transmission period. Such block-static nature of the channel coefficients facilitates their estimation.
- However, for a non-quasi-static block fading channel such as that is experienced by highly mobile devices, the channel coefficients may vary evidently during a block transmission period.
- In such case, to obtain good data transmission quality even by means of blind channel estimation technique is an engineering challenge.
- In this work, we assume:
  - The channel coefficients within a transmission block are deterministic but unknown;
  - In a static environment, these deterministic channel coefficients are the same for the entire block, while in a non-static environment, they are allowed to change several times within a block.

- In 2002, Skoglund *et al* proposed to introduce the computer-searched best non-linear block code to combine channel estimate, equalization and error protection.[a]
- In 2007, Wu *et al* replaced the computer-optimized non-linear code by a rule-based constructed code, and showed that the constructed code can yield comparable performance to the computer-searched best codes.[b]
- In this thesis, we further extend the idea of Wu *et al* to the design of a code that performs well not only in quasi-static block fading environment but also in non-static channels.
  - The only assumption in our extension is that the receiver knows exactly when the channel coefficients change within a codeword block.

[a] M. Skoglund, J. Giese, and S. Parkvall, "Code design for combined channel estimation and error protection," *IEEE Trans. Inform. Theory*, vol. 48, no. 5, pp. 1162-1171, May 2002.

[b] C. L. Wu, P. N. Chen and Y. S. Han, "Maximum-likelihood priority-first search decodable codes for combined channel estimation and error protection," submitted to *IEEE Trans. Inform. Theory*, 2007.

- Introduction
- **System Model**
- Code Design
- Simulation Results
- Conclusions

System Model

- The *quasi-static block fading channel* model can be denoted as

$$\boldsymbol{y} = \mathbb{H}\boldsymbol{b} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{y} = [y_1, y_2, \ldots, y_L]^T$ is the complex output vector observed at the receiver,

$$\mathbb{H} = \begin{bmatrix} h_1 & 0 & 0 & 0 & \ldots & 0 \\ h_2 & h_1 & 0 & & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ h_P & h_{P-1} & \vdots & h_1 & \ddots & \vdots \\ 0 & h_P & \ddots & \vdots & h_1 & 0 \\ \vdots & \vdots & \vdots & h_P & \ddots & h_1 \\ 0 & \ldots & 0 & & h_P & \vdots \\ 0 & 0 & 0 & \ldots & 0 & h_P \end{bmatrix}_{L \times N},$$

is formed by $L = N + P - 1$ shift counterparts of the channel coefficients $\boldsymbol{h} = [h_1, h_2, \ldots, h_P]^T$, $\boldsymbol{n} = [n_1, n_2, \ldots, n_L]^T$ is the zero-mean complex Gaussian noise vector in which $E[\boldsymbol{n}\boldsymbol{n}^H] = \sigma_n^2 \mathbb{I}_L$, and $\mathbb{I}_L$ is the $L \times L$ identity matrix.

- In a non-static environment or for a large N, the channel coefficients however may vary within the coding block. In such case, (1) should be modified as:

$$\boldsymbol{y} = \mathbb{H}_v \boldsymbol{b} + \boldsymbol{n}, \tag{2}$$

where

$$\mathbb{H}_v = \begin{bmatrix} & \mathbb{H}^{(1)}_{N_1 \times N_1} & \mathbf{0}_{N_1 \times (N-N_1)} & \\ \mathbf{0}_{N_2 \times (N_1-(P-1))} & \mathbb{H}^{(2)}_{N_2 \times N_2} & \mathbf{0}_{N_2 \times (N-N_1-N_2+(P-1))} & \\ \mathbf{0}_{N_3 \times (N_1+N_2-2(P-1))} & & \mathbb{H}^{(3)}_{N_3 \times N_3} & \mathbf{0}_{N_2 \times (N-N_1-N_2-N_3+2(P-1))} \\ & & \ddots & \\ & \mathbf{0}_{N_q \times (N-N_q)} & & \mathbb{H}^{(q)}_{N_q \times N_q} \end{bmatrix} \tag{3}$$

corresponds to $(q-1)$ channel coefficient changes during the transmission of codeword $\boldsymbol{b}$, and $\mathbf{0}$ denotes the all-zero matrix whose size is indicated by its subscripts.

- In (3), $N_1 + N_2 + \cdots + N_q = L$, and

$$\mathbb{H}^{(1)}_{N_1\times N_1} = \begin{bmatrix} h_{1,1} & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ h_{2,1} & h_{1,1} & 0 & \ldots & \ldots & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ h_{P,1} & h_{P-1,1} & \vdots & h_{1,1} & 0 & \ldots & \ldots & 0 \\ 0 & h_{P,1} & \vdots & h_{2,1} & h_{1,1} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & h_{P,1} & h_{P-1,1} & \ldots & \ldots & h_{1,1} \end{bmatrix}_{N_1\times N_1},$$

and for $1 < i < q$,

$$\mathbb{H}^{(i)}_{N_i\times N_i} = \begin{bmatrix} h_{P,i} & h_{P-1,i} & \ldots & h_{1,i} & 0 & \ldots & \ldots & 0 \\ 0 & h_{P,i} & \ldots & h_{2,i} & h_{1,i} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & & & & \vdots \\ 0 & \ldots & 0 & h_{P,i} & h_{P-1,i} & \ldots & h_{1,i} & 0 \\ 0 & \ldots & 0 & 0 & h_{P,i} & h_{P-1,i} & \ldots & h_{1,i} \end{bmatrix}_{N_i\times N_i},$$

and

$$\mathbb{H}^{(q)}_{N_q \times N_q} = \begin{bmatrix} h_{P,q} & h_{P-1,q} & \ldots & h_{1,q} & 0 & 0 & \ldots & 0 \\ 0 & h_{P,q} & \ldots & h_{2,q} & h_{1,q} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 0 & h_{P,q} & h_{P-1,q} & \ldots & h_{1,q} \\ 0 & 0 & \ldots & \ldots & 0 & h_{P,q} & \ldots & h_{2,q} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & \ldots & \ldots & \ldots & 0 & h_{P,q} \end{bmatrix}_{N_q \times N_q} .$$

Here we assume that $N_i \geq P$ for every i, and N_i may be different for different i.

- Re-formulate (2) as

$$\boldsymbol{y} = \mathbb{B}_v \boldsymbol{h}_v + \boldsymbol{n}, \tag{4}$$

where $\boldsymbol{h}_v = [h_{1,1}, h_{2,1}, \ldots, h_{P,1}, h_{2,1}, h_{2,2}, \ldots, h_{P,2}, h_{3,1}, \ldots\ldots, h_{P,q}]^T$, and

$$\mathbb{B}_v = \begin{bmatrix} \mathbb{B}^{(1)}_{N_1\times P} & \mathbf{0}_{N_1\times P} & \cdots & \mathbf{0}_{N_1\times P} \\ \mathbf{0}_{N_2\times P} & \mathbb{B}^{(2)}_{N_2\times P} & \cdots & \mathbf{0}_{N_2\times P} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N_q\times P} & \mathbf{0}_{N_q\times P} & \cdots & \mathbb{B}^{(q)}_{N_q\times P} \end{bmatrix}_{L\times qP} . \tag{5}$$

In (5),

$$\mathbb{B}^{(1)}_{N_1\times P} = \begin{bmatrix} b_1 & 0 & \ldots & 0 \\ b_2 & b_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_P & b_{P-1} & \vdots & b_1 \\ b_{P+1} & b_P & \vdots & b_2 \\ \vdots & \vdots & & \vdots \\ b_{N_1} & b_{N_1-1} & \ldots & b_{N_1-(P-1)} \end{bmatrix}_{N_1\times P} ,$$

and for $1 < i < q$,

$$\mathbb{B}^{(i)}_{N_i \times P} = \begin{bmatrix} b_{\sum_{k=1}^{i-1} N_k+1} & b_{\sum_{k=1}^{i-1} N_k} & \cdots & b_{\sum_{k=1}^{i-1} N_k-(P-1)+1} \\ b_{\sum_{k=1}^{i-1} N_k+2} & b_{\sum_{k=1}^{i-1} N_k+1} & \cdots & b_{\sum_{k=1}^{i-1} N_k-(P-1)+2} \\ \vdots & \vdots & & \vdots \\ b_{\sum_{k=1}^{i} N_k} & b_{\sum_{k=1}^{i} N_k-1} & \cdots & b_{\sum_{k=1}^{i} N_k-(P-1)} \end{bmatrix}_{N_i \times P},$$

and

$$\mathbb{B}^{(q)}_{N_q \times P} = \begin{bmatrix} b_{\sum_{k=1}^{q-1} N_k+1} & b_{\sum_{k=1}^{q-1} N_k} & \cdots & b_{\sum_{k=1}^{q-1} N_k-(P-1)+1} \\ b_{\sum_{k=1}^{q-1} N_k+2} & b_{\sum_{k=1}^{q-1} N_k+1} & \cdots & b_{\sum_{k=1}^{q-1} N_k-(P-1)+2} \\ \vdots & \vdots & & \vdots \\ b_N & b_{N-1} & \cdots & b_{N-(P-1)} \\ 0 & b_N & \cdots & b_{N-(P-1)+1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_N \end{bmatrix}_{N_q \times P}.$$

- Since $\boldsymbol{n}$ is a complex zero-mean Gaussian vector, and $\boldsymbol{h}_v$ is assumed unknown constants, the optimal decision for the transmitted codeword is:

$$\hat{\boldsymbol{b}} = \arg\min_{\mathbb{B}_v} \min_{\boldsymbol{h}_v} \|\boldsymbol{y} - \mathbb{B}_v \boldsymbol{h}_v\|^2 . \tag{6}$$

For a given $\boldsymbol{b}$, the least square (LS) estimate of $\boldsymbol{h}_v$ is given by

$$\hat{\boldsymbol{h}}_v = (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T \boldsymbol{y}.$$

Taking the above LS estimate into (6) yields

$$\begin{aligned} \hat{\boldsymbol{b}} &= \arg\min_{\mathbb{B}_v} \left\|\boldsymbol{y} - \mathbb{B}_v (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T \boldsymbol{y}\right\|^2 \qquad (7) \\ &= \arg\min_{\mathbb{P}_B^{\perp}} \left\|\mathbb{P}_B^{\perp} \boldsymbol{y}\right\|^2 , \end{aligned}$$

where $\mathbb{P}_B^{\perp} \triangleq \mathbb{I}_L - \mathbb{P}_B$ and $\mathbb{P}_B \triangleq \mathbb{B}_v (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T$.

Gauss-Markov Model

- A more general relationship between consecutive sub-block channel coefficients however is the first-order Gauss-Markov that is usually adopted in time-varying environment. Specifically,

$$\boldsymbol{h}_i = \alpha \boldsymbol{h}_{i-1} + \boldsymbol{v}_i$$

where $\Pr\{\boldsymbol{h}_0 = \mathbf{0}_{P\times 1}\} = 1$, and $\{\boldsymbol{v}_i\}_{i=1}^{q}$ is zero-mean complex Gaussian distributed with $E[\boldsymbol{v}_i \boldsymbol{v}_i^H] = \sigma_{v_i}^2 \boldsymbol{I}_P$.

- The parameter α charactersizes the rate of channel variation between consecutive sub-blocks. Its value lies between zero and one, and is controlled by the Doppler spread and transmission bandwidth as

$$\alpha = \exp(-\omega_d T_s) = \exp\left(-\pi B_d T_s\right) = \exp\left(-2\pi f_c \frac{v}{c} T_s\right),$$

where $B_d = \omega_d/\pi$ denotes the Doppler spread, f_c is the carrier frequency, v is the velocity of the transmitter, c is the velocity of light, and T_s is the symbol period (i.e. sub-block period in our case).

- For vehicle speed $v = 180$ km/hours, carrier frequency $f_c = 900$ MHz, and sub-block size 10^{-4} seconds[a],

$$\omega_d T_s = 2\pi f_c \frac{v}{c} T_s = 2\pi \times (900 \times 10^6 \text{ Hz}) \times \frac{180 \text{ km/hours}}{1.08 \times 10^9 \text{ km/hours}} \times 10^{-4} \text{ seconds} = 0.03\pi.$$

This yields $\alpha = \exp\{-0.03\pi\} = 0.910057$. If we increase f_c to 2.7 GHz and 5.4 Ghz, α will become $\exp(-0.09\pi) = 0.753713$ and $\exp(-0.18\pi) = 0.568084$, respectively. These α-values will be used in our simulations.

[a] R. Haeb, "A comparison of coherent and differentially coherent detection schemes for fading channels," *Vehicular Technology Conference, 1988 IEEE 38th*, pp. 364-370, June 1988.

- Introduction
- System Model
- **Code Design**
- Simulation Results
- Conclusions

Self-orthogonality condition for SNR-optimized codewords

- A known inequality for the multiplication of two positive semi-definite Hermitian matrices is

$$\mathrm{tr}(\mathbb{A}\mathbb{B}) \leq \mathrm{tr}(\mathbb{A}) \cdot \lambda_{\max}(\mathbb{B}), \tag{8}$$

and the average SNR is given by

$$\begin{aligned} \mathrm{SNR} &= \frac{E[||\mathbb{H}_v \boldsymbol{b}||^2]}{E[||\boldsymbol{n}||^2]} \\ &= \frac{E[||\mathrm{tr}(\boldsymbol{h}_v^H \mathbb{B}_v^T \mathbb{B}_v \boldsymbol{h}_v)||]}{L\sigma_n^2} \\ &= \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H]\mathbb{B}_v^T \mathbb{B}_v)}{L\sigma_n^2} \\ &\leq \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H])}{L\sigma_n^2}\lambda_{\max}(\mathbb{B}_v^T \mathbb{B}_v) \\ &= \frac{N}{L}\frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H])}{\sigma_n^2}\lambda_{\max}(\frac{1}{N}\mathbb{B}_v^T \mathbb{B}_v). \end{aligned} \tag{9}$$

- Inequality (9) holds with equality when $(1/N)\mathbb{B}_v^T\mathbb{B}_v$ is an identity matrix, namely,

$$\mathbb{B}_v^T\mathbb{B}_v = N\mathbb{I}_{qP} \triangleq \begin{bmatrix} N & 0 & \dots & 0 \\ 0 & N & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & N \end{bmatrix}_{qP\times qP} . \tag{10}$$

As a result, the maximum SNR equals

$$\mathrm{SNR}_{\max} = \frac{N}{L}\frac{\mathrm{tr}(E[\boldsymbol{h}_v\boldsymbol{h}_v^H])}{\sigma_n^2}.$$

Codeword Selection

- The codeword sequences satisfying (10) may not exist for certain N, P and q. In such cases, one can only choose codewords that best-approximate (10), for which some examples are given below.
  - **Case 1.** For $P = 2$ and $q = 1$, the codewords can be chosen according to:

$$\mathbb{B}_v^T \mathbb{B}_v = \mathbb{B}^T \mathbb{B} = \begin{cases} \begin{bmatrix} N & 0 \\ 0 & N \end{bmatrix}, & \text{for N odd} \\ \\ \begin{bmatrix} N & \pm 1 \\ \pm 1 & N \end{bmatrix}, & \text{for N even.} \end{cases}$$

– **Case 2.** For $P = 2$ and $q > 1$ with $N_1 = N_2 = \cdots = N_{q-1} = Q$, we observe that

$$\mathbb{B}_v = \mathbb{B}^{(1)} \oplus \mathbb{B}^{(2)} \oplus \cdots \oplus \mathbb{B}^{(q)},$$

where "$\oplus$" is the direct sum operator of two matrices.[a] Then, the codewords can be chosen according to:

$$\left(\mathbb{B}^{(1)}\right)^T \mathbb{B}^{(1)} = \begin{cases} \begin{bmatrix} Q & 0 \\ 0 & (Q-1) \end{bmatrix}, & \text{for } Q \text{ odd} \\ \\ \begin{bmatrix} Q & \pm 1 \\ \pm 1 & (Q-1) \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

[a] For two matrices $\mathbb{A}$ and $\mathbb{B}$, the direct sum of $\mathbb{A}$ and $\mathbb{B}$ is defined as $\mathbb{A} \oplus \mathbb{B} = \begin{bmatrix} \mathbb{A} & \mathbf{0} \\ \mathbf{0} & \mathbb{B} \end{bmatrix}$.

and for $1 < i < q$,

$$\left(\mathbb{B}^{(i)}\right)^T \mathbb{B}^{(i)} = \begin{cases} \begin{bmatrix} Q & \pm 1 \\ \pm 1 & Q \end{bmatrix}, & \text{for } Q \text{ odd} \\ \\ \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

and

$$\left(\mathbb{B}^{(q)}\right)^T \mathbb{B}^{(q)} = \begin{cases} \begin{bmatrix} r & \pm 1 \\ \pm 1 & r+1 \end{bmatrix}, & \text{for } r \text{ odd} \\ \\ \begin{bmatrix} r & 0 \\ 0 & r+1 \end{bmatrix}, & \text{for } r \text{ even.} \end{cases} \qquad (r = N - (q-1)Q)$$

– **Case 3.** For $P > 2$ and $q = 1$, the codewords can be chosen according to:

$$\mathbb{B}_v^T \mathbb{B}_v = \mathbb{B}^T \mathbb{B} = \begin{cases} \begin{bmatrix} N & 0 & \pm 1 & 0 & \cdots \\ 0 & N & 0 & \pm 1 & \cdots \\ \pm 1 & 0 & N & 0 & \cdots \\ 0 & \pm 1 & 0 & N & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}_{P \times P} & , \quad \text{for N odd} \\ \\ \begin{bmatrix} N & \pm 1 & 0 & \pm 1 & \cdots \\ \pm 1 & N & \pm 1 & 0 & \cdots \\ 0 & \pm 1 & N & \pm 1 & \cdots \\ \pm 1 & 0 & \pm 1 & N & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}_{P \times P} & , \quad \text{for N even} \end{cases}$$

– **Case 4.** For $P = 3$ and $q = 2$ with $N_1 = Q$, the codewords can be chosen according to:

$$\left(\mathbb{B}^{(1)}\right)^T \mathbb{B}^{(1)} = \begin{cases} \begin{bmatrix} Q & 0 & \pm 1 \\ 0 & Q-1 & \pm 1 \\ \pm 1 & \pm 1 & Q-2 \end{bmatrix}, & \text{for } Q \text{ odd} \\ \\ \begin{bmatrix} Q & \pm 1 & 0 \\ \pm 1 & Q-1 & 0 \\ 0 & 0 & Q-2 \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

and

$$\left(\mathbb{B}^{(2)}\right)^T \mathbb{B}^{(2)} = \begin{cases} \begin{bmatrix} N-Q & \pm 1 & \pm 1 \\ \pm 1 & N-Q+1 & 0 \\ \pm 1 & 0 & N-Q+2 \end{bmatrix}, & \text{for } (N-Q) \text{ odd} \\ \\ \begin{bmatrix} N-Q & 0 & 0 \\ 0 & N-Q+1 & \pm 1 \\ 0 & \pm 1 & N-Q+2 \end{bmatrix}, & \text{for } (N-Q) \text{ even.} \end{cases}$$

- The number of sequences that fulfill Case 1 and $b_1 = -1$ is equal to

$$(2-(N \bmod 2))\binom{N-1}{\lfloor\frac{N-1}{2}\rfloor}.$$

- The number of sequences that fulfill Case 2 and $b_1 = -1$ is equal to

$$[2-(Q \bmod 2)]\binom{Q-1}{\lfloor\frac{Q-1}{2}\rfloor}[1+(Q \bmod 2)]^{q-2}\binom{Q}{\lfloor\frac{Q}{2}\rfloor}^{q-2}$$

$$[1+(N-(q-1)Q) \bmod 2]\binom{N-(q-1)Q}{\lfloor\frac{N-(q-1)Q}{2}\rfloor}.$$

- The number of sequences that fulfill Cases 3 and 4 may not have close-form formulas, and hence, they are omitted.

- The codeword selection procedure is given in the following:

  Step 1. **(Initialization)** *Let* $b_1 = -1$, *and let* $r_{\max}$ *be the total number of sequences satisfying the required* $\mathbb{B}_v^T \mathbb{B}_v$. *Sort the* (± 1)*-sequences according to their lexical order, starting from all-*(-1) *sequence, and denote them by* $\boldsymbol{b}(1), \boldsymbol{b}(2), \boldsymbol{b}(3), \ldots, \boldsymbol{b}(r_{\max})$.

  Step 2. **(Codeword Selection)** *For an* (N, K) *code, compute*

$$\Delta = \left\lfloor \frac{r_{\max}}{2^K} \right\rfloor .$$

  Then, the codewords selected are $\{\boldsymbol{b}(j \times \Delta)\}_{j=1}^{2^K}$.

Decoding Criterion

- The rule for codeword selection is introduced, and only 2^K codewords are picked and the others are discarded. By assuming that the decoder knows N_1, N_2, ..., N_q, the optimal decision criterion in (6) are further explored.

- $\mathbb{P}_B$ is a block matrix satisfying

$$\mathbb{P}_B = \begin{bmatrix} \mathbb{P}_B^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbb{P}_B^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbb{P}_B^{(q)} \end{bmatrix}_{L\times L} = \mathbb{P}_B^{(1)} \oplus \mathbb{P}_B^{(2)} \oplus \cdots \oplus \mathbb{P}_B^{(q)}, \qquad (11)$$

where

$$\mathbb{P}_B^{(1)} = \mathbb{B}^{(1)} \left[(\mathbb{B}^{(1)})^T \mathbb{B}^{(1)} \right]^{-1} (\mathbb{B}^{(1)})^T.$$

- The result is

$$\hat{\boldsymbol{b}} = \arg\min_{\mathbb{P}_B}\{||\boldsymbol{y} - \mathbb{P}_B\boldsymbol{y}||^2\}$$

$$= \arg \min_{(\mathbb{P}_B^{(1)},\mathbb{P}_B^{(2)},\cdots,\mathbb{P}_B^{(q)})} \left\{ \left\| \mathbb{I}_L\boldsymbol{y} - (\mathbb{P}_B^{(1)} \oplus \mathbb{P}_B^{(2)} \oplus \cdots \oplus \mathbb{P}_B^{(q)})\boldsymbol{y} \right\|^2 \right\}$$

$$= \arg \min_{(\mathbb{P}_B^{(1)},\mathbb{P}_B^{(2)},\cdots,\mathbb{P}_B^{(q)})} \left\{ \left\| \left[(\mathbb{I}_{N_1} - \mathbb{P}_B^{(1)}) \oplus (\mathbb{I}_{N_2} - \mathbb{P}_B^{(2)}) \oplus \cdots \oplus (\mathbb{I}_{N_q} - \mathbb{P}_B^{(q)}) \right] \boldsymbol{y} \right\|^2 \right\}$$

$$= \arg \min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)} \left\{ \left\| ((\mathbb{P}_B^{(1)})^\perp \oplus (\mathbb{P}_B^{(2)})^\perp \oplus \cdots \oplus (\mathbb{P}_B^{(q)})^\perp)\boldsymbol{y} \right\|^2 \right\}$$

$$= \arg \min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)} \left\{ \left\| ((\mathbb{P}_B^{(1)})^\perp\boldsymbol{y}^{(1)}) \oplus ((\mathbb{P}_B^{(2)})^\perp\boldsymbol{y}^{(2)}) \oplus \cdots \right\|^2 \right\}$$

$$= \arg \min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)} \left\{ \sum_{i=1}^{q} \left\| (\mathbb{P}_B^{(i)})^\perp\boldsymbol{y}^{(i)} \right\|^2 \right\},$$

where

$$\boldsymbol{y}^{(i)} = \left[y_{\sum_{j=1}^{i-1} N_j+1}, y_{\sum_{j=1}^{i-1} N_j+2}, \cdots, y_{\sum_{j=1}^{i-1} N_j+N_i} \right]^T$$

represents the i-th sub-block of $\boldsymbol{y}$.

- Introduction
- System Model
- Code Design
- **Simulation Results**
- Conclusions

Simulation Results

- Several designed codes will be simulated over quasi-static Gaussian and non-static Gauss-Markov block fading channels in order to verify that the codes designed for non-static block fading channels are robust over both channels.
- As a convention, the zero-mean channel coefficients are normalized as $E[|h_{i,j}|^2] = 1/P$ for $1 \leq i \leq P$ and $1 \leq j \leq q$, and $\{h_{i,j}\}_{i=1}^{P}$ are assumed independent.
- The designed code of length N, which targets to be transmitted over the memory-order-$(P-1)$ non-static fading channel whose channel coefficients change in every Q symbols, is denoted by $\text{Code}(N, P, Q)$.
- The simulated channel, whose channel coefficients change in every Q symbols, and whose memory order is $(P-1)$, are similarly denoted as $\text{Channel}(P, Q)$.

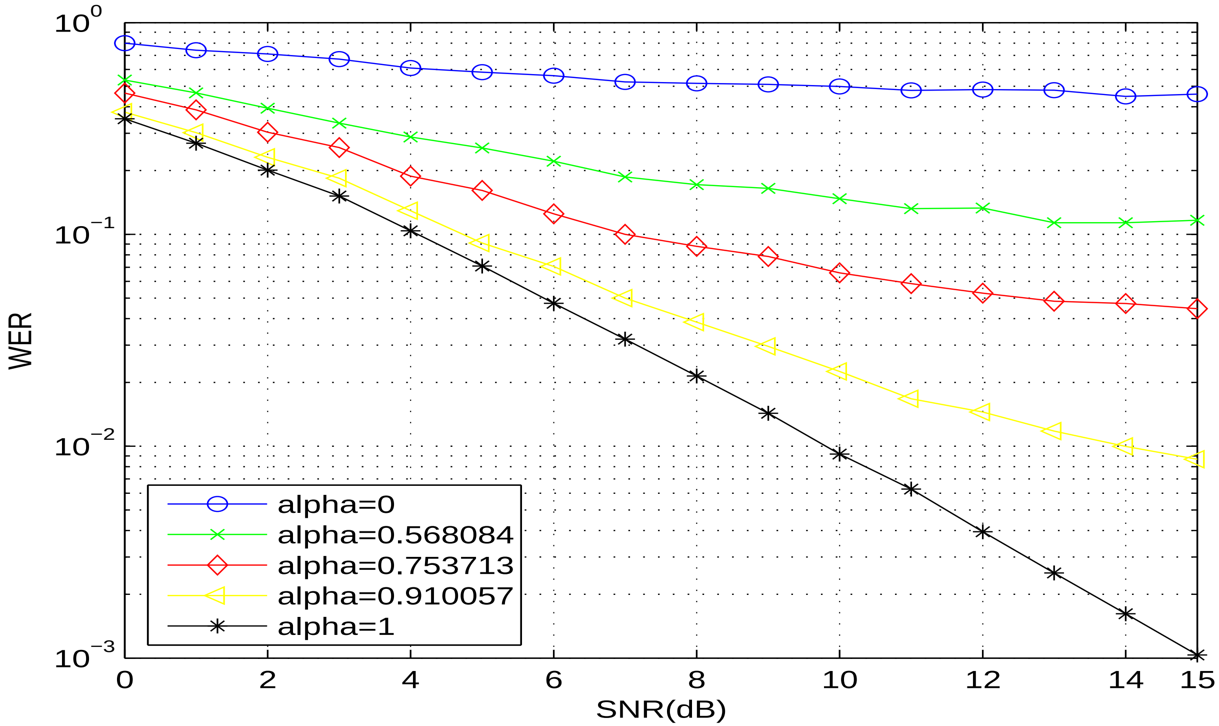

Figure 1: The maximum-likelihood word error rates for Code$(24, 2, 24)$ over Channel$(2, 12)$ with different degree of channel variation factors α.

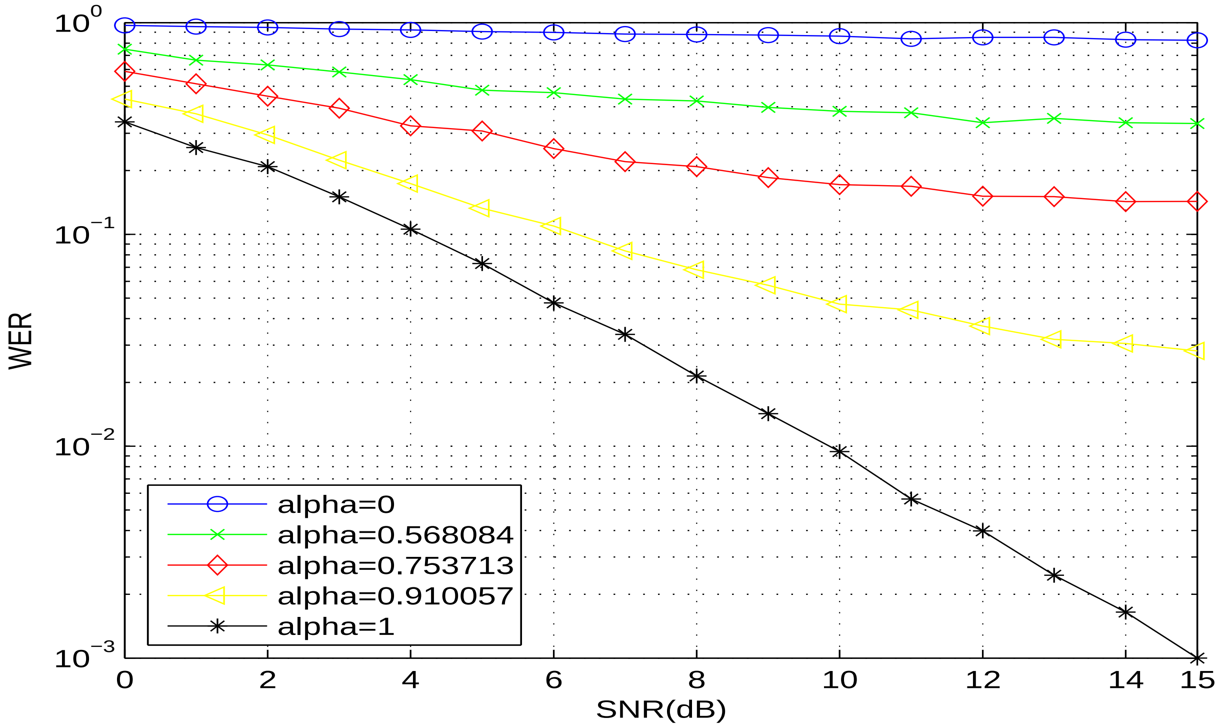

Figure 2: The maximum-likelihood word error rates for Code$(24, 2, 24)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

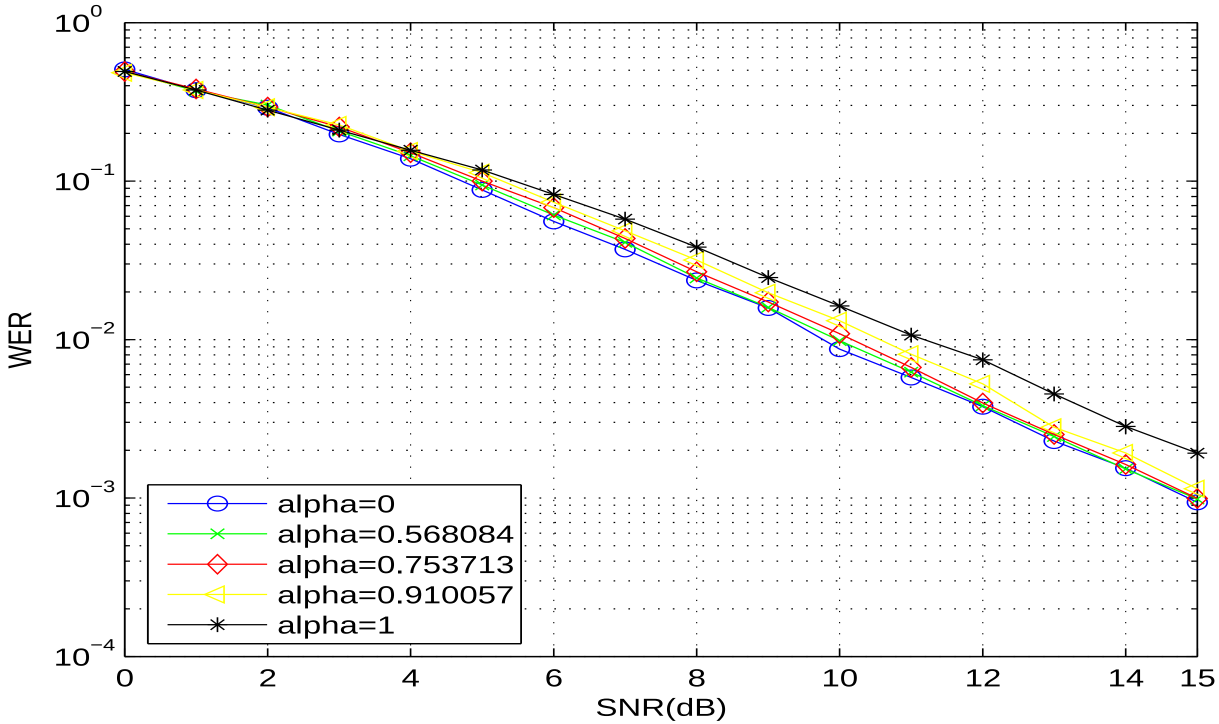

Figure 3: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, 12)$ with different degree of channel variation factors α.

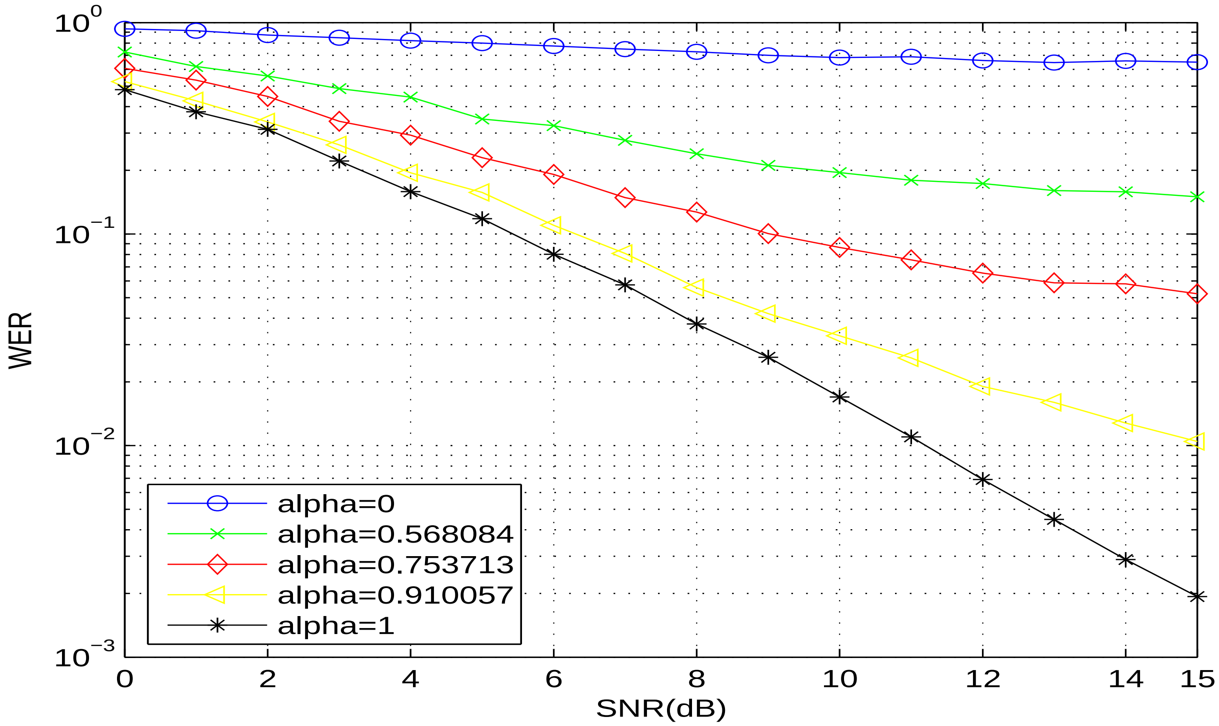

Figure 4: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

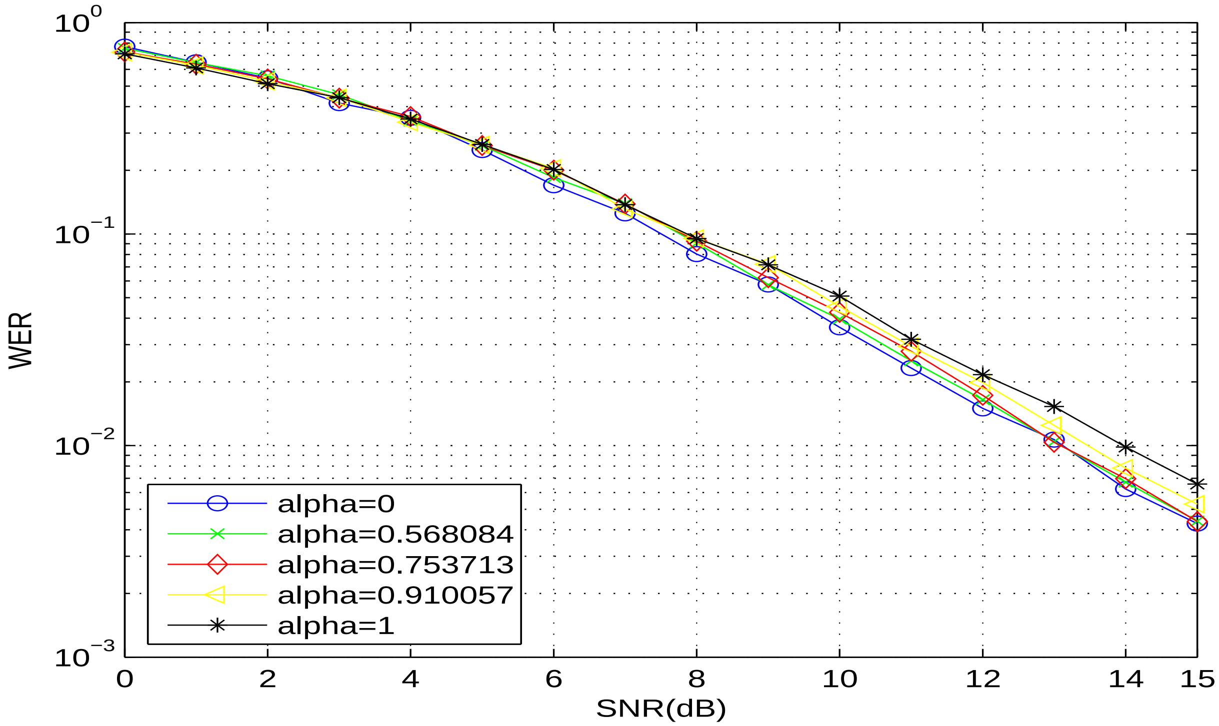

Figure 5: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, 12)$ with different degree of channel variation factors α.

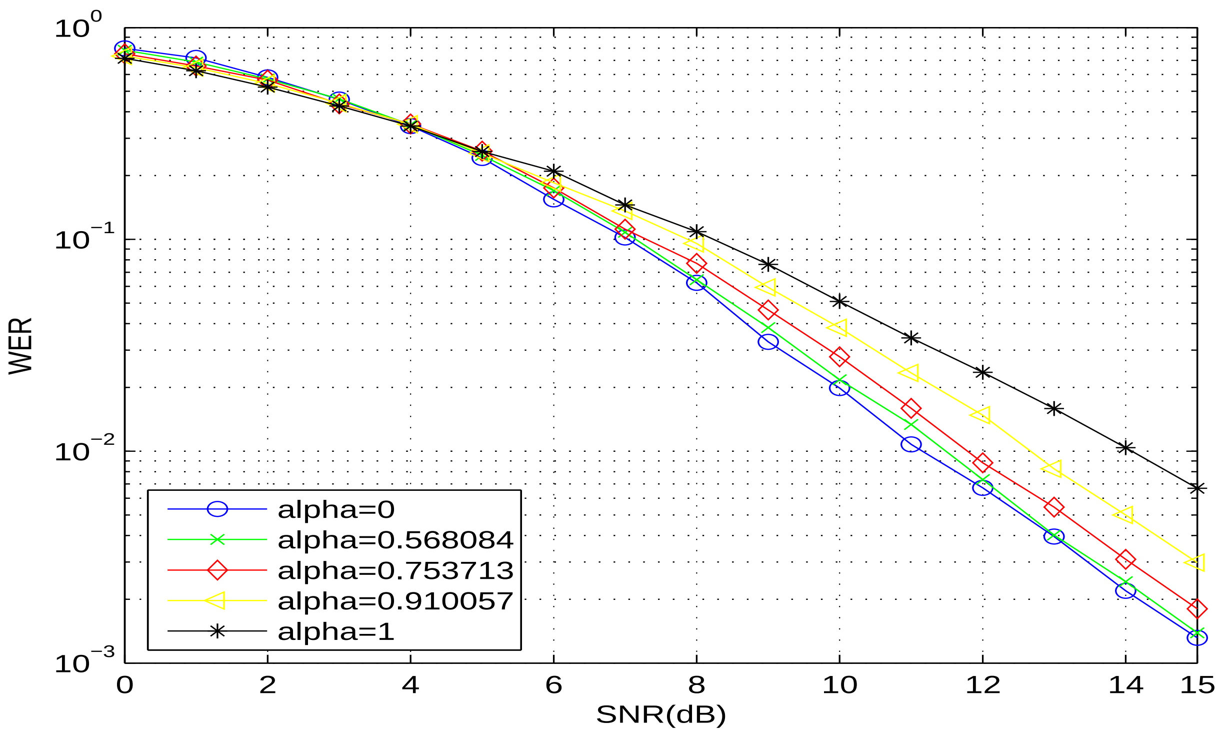

Figure 6: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

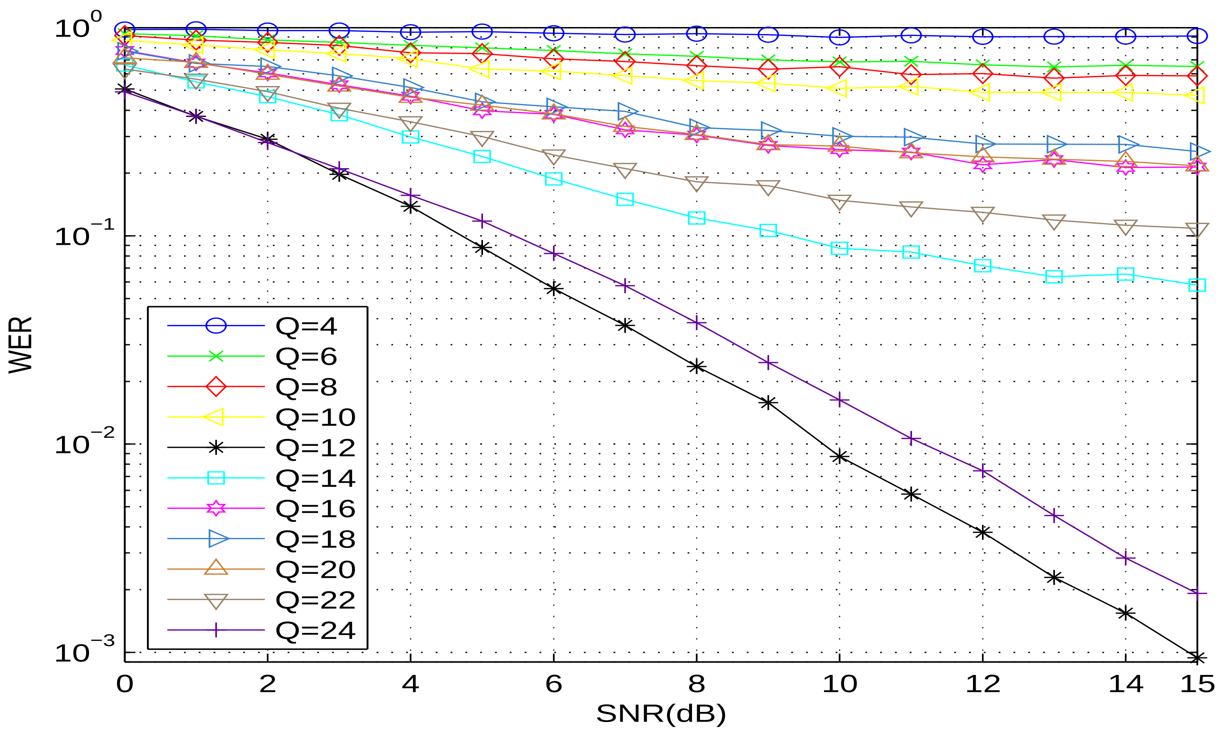

Figure 7: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0$ and different values of Q.

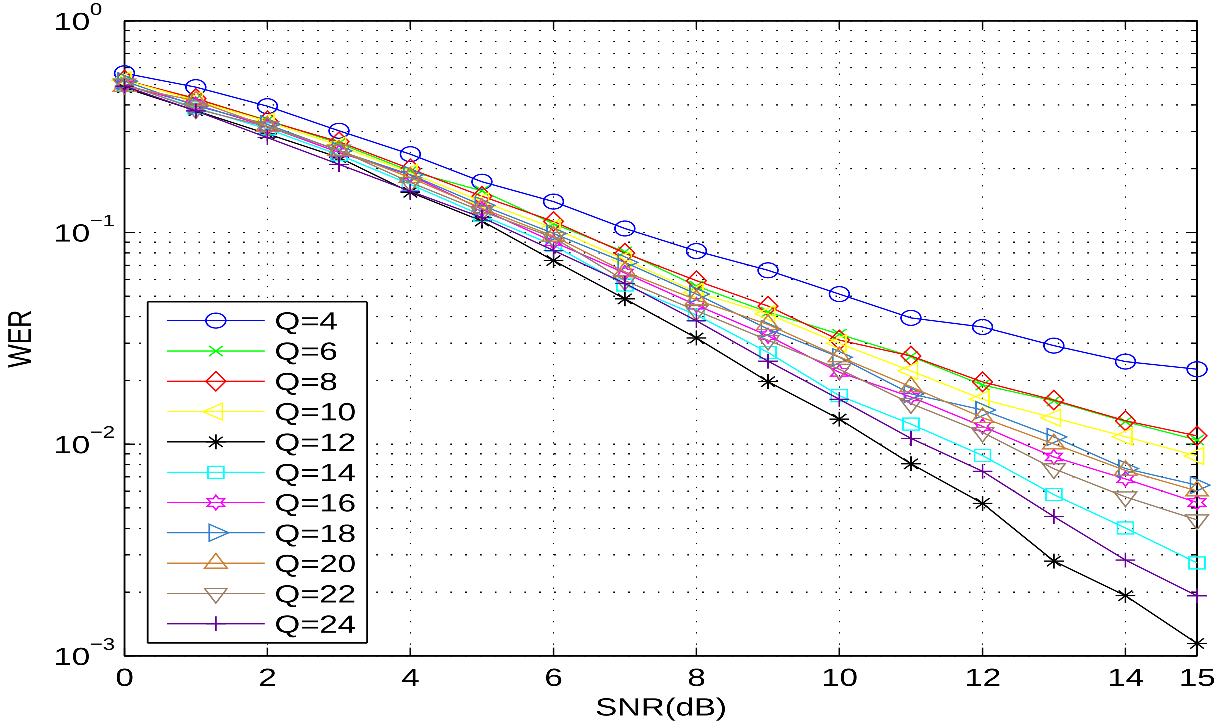

Figure 8: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.910057$ and different values of Q.

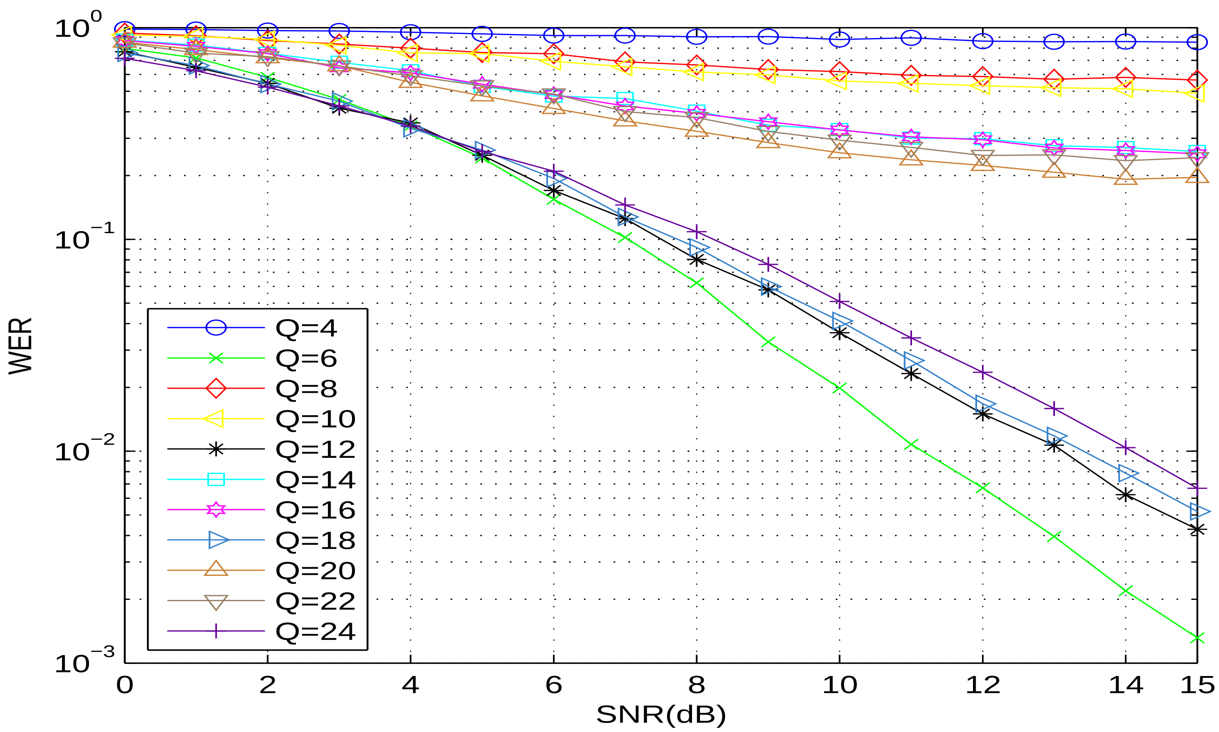

Figure 9: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0$ and different values of Q.

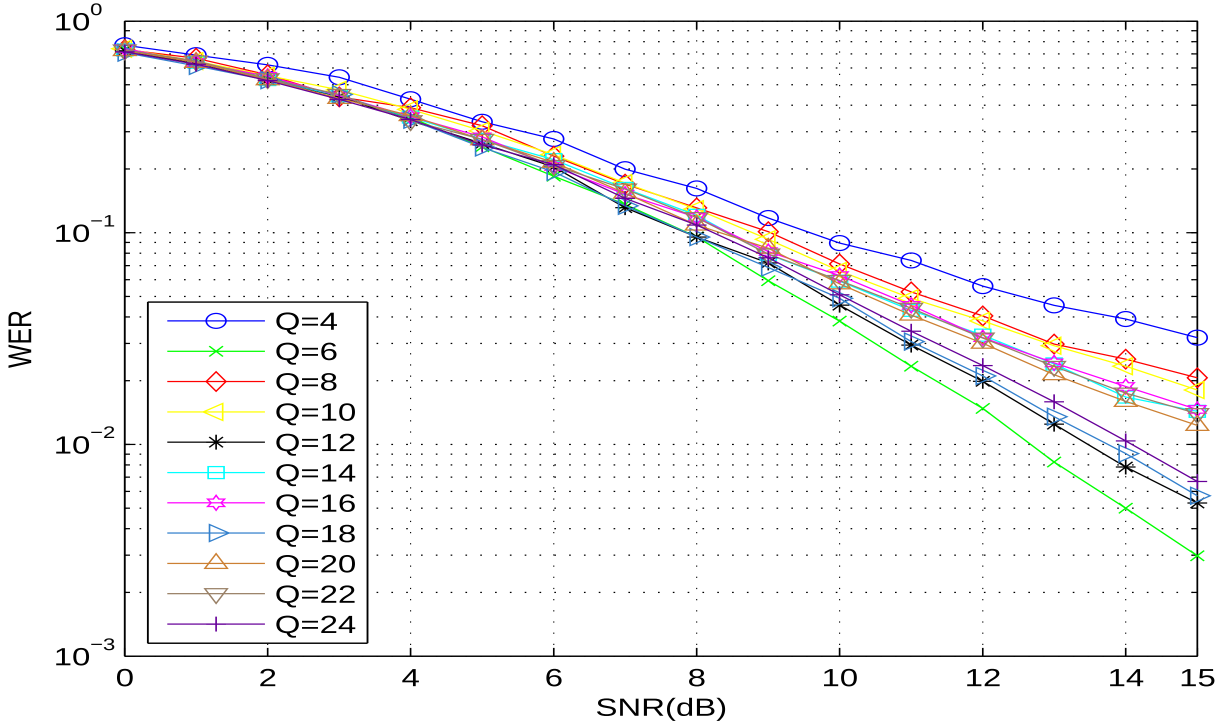

Figure 10: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.910057$ and different values of Q.

- Introduction
- System Model
- Code Design
- Simulation Results
- **Conclusions**

Conclusions

- In this work, a binary block code design for combined channel estimation and error protection, which is extended from Wu *et al*'s work specifically for non-static fading channels, is proposed and examined.
- Simulations hint that as long as the update rate of the channel coefficients is equal to or slower than that of the code target channel, the performance remains robust.
- However, when the channel coefficients change faster than those of the channel that the code design is presumed, the performance degrades considerably.
- The future work is to examine whether the code proposed is robust for non-stationary fading channels in which the channel coefficients change in an non-stationary non-periodic fashion.