Robust Code Design Over Fast Fading Channels

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Outline

- Introduction
  - System Model
  - Code Design
  - Simulation Results
  - Conclusions
Introduction

• In a quasi-static block fading environment, the channel statistics is assumed to change at a very slow rate, and hence, remains almost constant within a block transmission period. Such block-static nature of the channel coefficients facilitates their estimation.

• However, for a non-quasi-static block fading channel such as that is experienced by highly mobile devices, the channel coefficients may vary evidently during a block transmission period.

• In such case, to obtain good data transmission quality even by means of blind channel estimation technique is an engineering challenge.

• In this work, we assume:

  – The channel coefficients within a transmission block are deterministic but unknown;
  – In a static environment, these deterministic channel coefficients are the same for the entire block, while in a non-static environment, they are allowed to change several times within a block.
• In 2002, Skoglund et al proposed to introduce the computer-searched best non-linear block code to combine channel estimate, equalization and error protection.a

• In 2007, Wu et al replaced the computer-optimized non-linear code by a rule-based constructed code, and showed that the constructed code can yield comparable performance to the computer-searched best codes.b

• In this thesis, we further extend the idea of Wu et al to the design of a code that performs well not only in quasi-static block fading environment but also in non-static channels.
  
  – The only assumption in our extension is that the receiver knows exactly when the channel coefficients change within a codeword block.

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• Introduction

• **System Model**

• Code Design

• Simulation Results

• Conclusions
System Model

- The quasi-static block fading channel model can be denoted as

\[ y = \mathbb{H} b + n, \]  

where \( y = [y_1, y_2, \ldots, y_L]^T \) is the complex output vector observed at the receiver,

\[
\mathbb{H} = \begin{bmatrix}
    h_1 & 0 & 0 & 0 & \cdots & 0 \\
    h_2 & h_1 & 0 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    h_P & h_{P-1} & \vdots & h_1 & 0 \\
    0 & h_P & \vdots & \vdots & h_1 \\
    \vdots & \vdots & \vdots & h_P & h_1 \\
    0 & \cdots & 0 & h_P & \vdots \\
    0 & 0 & 0 & \cdots & 0 & h_P
\end{bmatrix}_{L \times N},
\]

is formed by \( L = N + P - 1 \) shift counterparts of the channel coefficients \( h = [h_1, h_2, \ldots, h_P]^T \), \( n = [n_1, n_2, \ldots, n_L]^T \) is the zero-mean complex Gaussian noise vector in which \( E[nn^H] = \sigma_n^2 \mathbb{I}_L \), and \( \mathbb{I}_L \) is the \( L \times L \) identity matrix.
In a non-static environment or for a large $N$, the channel coefficients however may vary within the coding block. In such case, (1) should be modified as:

$$y = H_v b + n,$$

where

$$H_v = \begin{bmatrix}
H^{(1)}_{N_1 \times N_1} & 0_{N_1 \times (N-N_1)} \\
0_{N_2 \times (N_1-(P-1))} & H^{(2)}_{N_2 \times N_2} & 0_{N_2 \times (N-N_1-N_2+(P-1))} \\
0_{N_3 \times (N_1+N_2-2(P-1))} & H^{(3)}_{N_3 \times N_3} & 0_{N_2 \times (N-N_1-N_2-N_3+2(P-1))} \\
\vdots \\
0_{N_q \times (N-N_q)} & H^{(q)}_{N_q \times N_q}
\end{bmatrix}$$

 corresponds to $(q-1)$ channel coefficient changes during the transmission of codeword $b$, and $0$ denotes the all-zero matrix whose size is indicated by its subscripts.
• In (3), $N_1 + N_2 + \cdots + N_q = L$, and

$$H_{N_1 \times N_1}^{(1)} = \begin{bmatrix} h_{1,1} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ h_{2,1} & h_{1,1} & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_{P,1} & h_{P-1,1} \vdots & h_{1,1} & 0 & \cdots & \cdots & 0 \\ 0 & h_{P,1} \vdots & h_{2,1} & h_{1,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & h_{P,1} & h_{P-1,1} & \cdots & h_{1,1} \end{bmatrix}_{N_1 \times N_1},$$

and for $1 < i < q$,

$$H_{N_i \times N_i}^{(i)} = \begin{bmatrix} h_{P,i} & h_{P-1,i} & \cdots & h_{1,i} & 0 & \cdots & \cdots & 0 \\ 0 & h_{P,i} & \cdots & h_{2,i} & h_{1,i} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & h_{P,i} & h_{P-1,i} & \cdots & h_{1,i} & 0 \\ 0 & \cdots & 0 & 0 & h_{P,i} & h_{P-1,i} & \cdots & h_{1,i} \end{bmatrix}_{N_i \times N_i}.$$
and

\[
\mathcal{H}^{(q)}_{N_q \times N_q} = \begin{bmatrix}
    h_{P,q} & h_{P-1,q} & \ldots & h_{1,q} & 0 & 0 & \ldots & 0 \\
    0 & h_{P,q} & \ldots & h_{2,q} & h_{1,q} & 0 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    0 & 0 & \ldots & 0 & h_{P,q} & h_{P-1,q} & \ldots & h_{1,q} \\
    0 & 0 & \ldots & \ldots & 0 & h_{P,q} & \ldots & h_{2,q} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    0 & \ldots & \ldots & \ldots & \ldots & 0 & h_{P,q} \\
\end{bmatrix}_{N_q \times N_q}
\]

Here we assume that $N_i \geq P$ for every $i$, and $N_i$ may be different for different $i$.

- Re-formulate (2) as

\[
y = \mathcal{B}_v \mathbf{h}_v + \mathbf{n},
\]

(4)
where \( h_v = [h_{1,1}, h_{2,1}, \ldots, h_{P,1}, h_{2,2}, \ldots, h_{P,2}, h_{3,1}, \ldots, \ldots, h_{P,q}]^T \), and

\[
B_v = \begin{bmatrix}
B_{N_1 \times P}^{(1)} & 0_{N_1 \times P} & \cdots & 0_{N_1 \times P} \\
0_{N_2 \times P} & B_{N_2 \times P}^{(2)} & \cdots & 0_{N_2 \times P} \\
\vdots & \vdots & \ddots & \vdots \\
0_{N_q \times P} & 0_{N_q \times P} & \cdots & B_{N_q \times P}^{(q)}
\end{bmatrix}_{L \times qP}.
\]

In (5),

\[
B_{N_1 \times P}^{(1)} = \begin{bmatrix}
b_1 & 0 & \cdots & 0 \\
b_2 & b_1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
b_{P-1} & b_{P-1} & \cdots & b_1 \\
b_P & b_P & \cdots & b_2 \\
\vdots & \vdots & \ddots & \vdots \\
b_{N_1-1} & b_{N_1-1} & \cdots & b_{N_1-(P-1)}
\end{bmatrix}_{N_1 \times P},
\]
and for $1 < i < q$,

$$B^{(i)}_{N_i \times P} = 
\begin{bmatrix}
 b \sum_{k=1}^{i-1} N_k + 1 & b \sum_{k=1}^{i-1} N_k & \cdots & b \sum_{k=1}^{i-1} N_k - (P-1) + 1 \\
b \sum_{k=1}^{i-1} N_k + 2 & b \sum_{k=1}^{i-1} N_k + 1 & \cdots & b \sum_{k=1}^{i-1} N_k - (P-1) + 2 \\
\vdots & \vdots & \ddots & \vdots \\
b \sum_{k=1}^{i} N_k & b \sum_{k=1}^{i} N_k - 1 & \cdots & b \sum_{k=1}^{i} N_k - (P-1)
\end{bmatrix}_{N_i \times P}$$

and

$$B^{(q)}_{N_q \times P} = 
\begin{bmatrix}
 b \sum_{k=1}^{q-1} N_k + 1 & b \sum_{k=1}^{q-1} N_k & \cdots & b \sum_{k=1}^{q-1} N_k - (P-1) + 1 \\
b \sum_{k=1}^{q-1} N_k + 2 & b \sum_{k=1}^{q-1} N_k + 1 & \cdots & b \sum_{k=1}^{q-1} N_k - (P-1) + 2 \\
\vdots & \vdots & \ddots & \vdots \\
b_N & b_{N-1} & \cdots & b_{N-(P-1)} \\
0 & b_N & \cdots & b_{N-(P-1)+1} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & b_N
\end{bmatrix}_{N_q \times P}$$
• Since \( n \) is a complex zero-mean Gaussian vector, and \( h_v \) is assumed unknown constants, the optimal decision for the transmitted codeword is:

\[
\hat{b} = \arg\min_{B_v} \min_{h_v} \| y - B_v h_v \|_2^2.
\]  

(6)

For a given \( b \), the least square (LS) estimate of \( h_v \) is given by

\[
\hat{h}_v = (B_v^T B_v)^{-1} B_v^T y.
\]

Taking the above LS estimate into (6) yields

\[
\hat{b} = \arg\min_{B_v} \| y - B_v (B_v^T B_v)^{-1} B_v^T y \|_2^2
\]

(7)

\[
= \arg\min_{P_B^\perp} \| P_B^\perp y \|_2^2,
\]

where \( P_B^\perp \triangleq I_L - P_B \) and \( P_B \triangleq B_v (B_v^T B_v)^{-1} B_v^T \).
Gauss-Markov Model

- A more general relationship between consecutive sub-block channel coefficients however is the first-order Gauss-Markov that is usually adopted in time-varying environment. Specifically,

\[ h_i = \alpha h_{i-1} + v_i \]

where \( \Pr\{h_0 = 0_{P \times 1}\} = 1 \), and \( \{v_i\}_{i=1}^q \) is zero-mean complex Gaussian distributed with \( E[v_i v_i^H] = \sigma_{v_i}^2 I_P \).
• The parameter $\alpha$ characterizes the rate of channel variation between consecutive sub-blocks. Its value lies between zero and one, and is controlled by the Doppler spread and transmission bandwidth as

$$\alpha = \exp(-\omega_d T_s) = \exp(-\pi B_d T_s) = \exp\left(-2\pi f_c \frac{v}{c} T_s\right),$$

where $B_d = \omega_d / \pi$ denotes the Doppler spread, $f_c$ is the carrier frequency, $v$ is the velocity of the transmitter, $c$ is the velocity of light, and $T_s$ is the symbol period (i.e. sub-block period in our case).

• For vehicle speed $v = 180$ km/hours, carrier frequency $f_c = 900$ MHz, and sub-block size $10^{-4}$ seconds$^a$,

$$\omega_d T_s = 2\pi f_c \frac{v}{c} T_s = 2\pi \times (900 \times 10^6 \text{ Hz}) \times \frac{180 \text{ km/hours}}{1.08 \times 10^9 \text{ km/hours}} \times 10^{-4} \text{ seconds} = 0.03\pi$$

This yields $\alpha = \exp\{-0.03\pi\} = 0.910057$. If we increase $f_c$ to 2.7 GHz and 5.4 Ghz, $\alpha$ will become $\exp\{-0.09\pi\} = 0.753713$ and $\exp\{-0.18\pi\} = 0.568084$, respectively. These $\alpha$-values will be used in our simulations.

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• Introduction

• System Model

• Code Design

• Simulation Results

• Conclusions
Self-orthogonality condition for SNR-optimized codewords

- A known inequality for the multiplication of two positive semi-definite Hermitian matrices is

\[
\text{tr}(AB) \leq \text{tr}(A) \cdot \lambda_{\text{max}}(B),
\]

and the average SNR is given by

\[
\text{SNR} = \frac{E[||H_v b||^2]}{E[||n||^2]} = \frac{E[\text{tr}(h_v^H B^T B_v h_v)\]]}{L\sigma_n^2} \leq \frac{\text{tr}(E[h_v h_v^H])}{L\sigma_n^2} \lambda_{\text{max}}(B_v^T B_v)
\]

\[
\leq \frac{N \text{tr}(E[h_v h_v^H])}{L} \frac{1}{\sigma_n^2} \lambda_{\text{max}}(\frac{1}{N} B_v^T B_v).
\]
• Inequality (9) holds with equality when \((1/N)B^T_v B_v\) is an identity matrix, namely,

\[
B^T_v B_v = NI_{qP} = \begin{bmatrix}
N & 0 & \ldots & 0 \\
0 & N & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & N
\end{bmatrix}_{qP \times qP}.
\]

As a result, the maximum SNR equals

\[
\text{SNR}_{\text{max}} = \frac{N}{L} \frac{\text{tr}(E[h_v h_v^H])}{\sigma^2_n}.
\]
Codeword Selection

• The codeword sequences satisfying (10) may not exist for certain $N$, $P$ and $q$. In such cases, one can only choose codewords that best-approximate (10), for which some examples are given below.

– **Case 1.** For $P = 2$ and $q = 1$, the codewords can be chosen according to:

$$
\mathbf{B}_v^T \mathbf{B}_v = \mathbf{B}^T \mathbf{B} = \begin{cases} 
\begin{bmatrix} N & 0 \\
0 & N 
\end{bmatrix}, & \text{for } N \text{ odd} \\
\begin{bmatrix} N & \pm 1 \\
\pm 1 & N 
\end{bmatrix}, & \text{for } N \text{ even.}
\end{cases}
$$
Case 2. For \( P = 2 \) and \( q > 1 \) with \( N_1 = N_2 = \cdots = N_{q-1} = Q \), we observe that

\[
\mathbb{B}_v = \mathbb{B}^{(1)} \oplus \mathbb{B}^{(2)} \oplus \cdots \oplus \mathbb{B}^{(q)},
\]

where “\( \oplus \)” is the direct sum operator of two matrices.\(^a\) Then, the codewords can be chosen according to:

\[
\left( \mathbb{B}^{(1)} \right)^T \mathbb{B}^{(1)} = \begin{cases} 
\begin{bmatrix} Q & 0 \\ 0 & (Q - 1) \end{bmatrix}, & \text{for } Q \text{ odd} \\
\begin{bmatrix} Q & \pm 1 \\ \pm 1 & (Q - 1) \end{bmatrix}, & \text{for } Q \text{ even}
\end{cases}
\]

\(^a\)For two matrices \( A \) and \( B \), the direct sum of \( A \) and \( B \) is defined as \( A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \).
and for $1 < i < q$, 

$$(\mathbb{B}^{(i)})^T \mathbb{B}^{(i)} = \begin{cases} 
\begin{bmatrix} Q & \pm 1 \\
\pm 1 & Q \end{bmatrix}, & \text{for } Q \text{ odd} \\
\begin{bmatrix} Q & 0 \\
0 & Q \end{bmatrix}, & \text{for } Q \text{ even}
\end{cases}$$

and 

$$(\mathbb{B}^{(q)})^T \mathbb{B}^{(q)} = \begin{cases} 
\begin{bmatrix} r & \pm 1 \\
\pm 1 & r + 1 \end{bmatrix}, & \text{for } r \text{ odd} \\
\begin{bmatrix} r & 0 \\
0 & r + 1 \end{bmatrix}, & \text{for } r \text{ even.}
\end{cases}$$
Case 3. For $P > 2$ and $q = 1$, the codewords can be chosen according to:

$$B_v B_v = B^T B = \begin{cases} 
\begin{bmatrix} 
N & 0 & \pm1 & 0 & \cdots \\
0 & N & 0 & \pm1 & \cdots \\
\pm1 & 0 & N & 0 & \cdots \\
0 & \pm1 & 0 & N & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots 
\end{bmatrix} & , \text{for } N \text{ odd} \\
\begin{bmatrix} 
N & \pm1 & 0 & \pm1 & \cdots \\
\pm1 & N & \pm1 & 0 & \cdots \\
0 & \pm1 & N & \pm1 & \cdots \\
\pm1 & 0 & \pm1 & N & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots 
\end{bmatrix} & , \text{for } N \text{ even} 
\end{cases} \quad P \times P$$
– **Case 4.** For $P = 3$ and $q = 2$ with $N_1 = Q$, the codewords can be chosen according to:

\[
\begin{align*}
\left(\mathbb{B}^{(1)}\right)^T \mathbb{B}^{(1)} &= \begin{cases}
\begin{bmatrix}
Q & 0 & \pm 1 \\
0 & Q - 1 & \pm 1 \\
\pm 1 & \pm 1 & Q - 2
\end{bmatrix}, & \text{for } Q \text{ odd} \\
\begin{bmatrix}
Q & \pm 1 & 0 \\
\pm 1 & Q - 1 & 0 \\
0 & 0 & Q - 2
\end{bmatrix}, & \text{for } Q \text{ even}
\end{cases}
\end{align*}
\]

and

\[
\begin{align*}
\left(\mathbb{B}^{(2)}\right)^T \mathbb{B}^{(2)} &= \begin{cases}
\begin{bmatrix}
N - Q & \pm 1 & \pm 1 \\
\pm 1 & N - Q + 1 & 0 \\
\pm 1 & 0 & N - Q + 2
\end{bmatrix}, & \text{for } (N - Q) \text{ odd} \\
\begin{bmatrix}
N - Q & 0 & 0 \\
0 & N - Q + 1 & \pm 1 \\
0 & \pm 1 & N - Q + 2
\end{bmatrix}, & \text{for } (N - Q) \text{ even.}
\end{cases}
\end{align*}
\]
• The number of sequences that fulfill Case 1 and \( b_1 = -1 \) is equal to
\[
(2 - (N \mod 2)) \left( N - 1 \right)_{\left\lfloor \frac{N-1}{2} \right\rfloor}.
\]

• The number of sequences that fulfill Case 2 and \( b_1 = -1 \) is equal to
\[
[2 - (Q \mod 2)] \left( \frac{Q - 1}{\left\lfloor \frac{Q-1}{2} \right\rfloor} \right) [1 + (Q \mod 2)]^{q-2} \left( \frac{Q}{\left\lfloor \frac{Q}{2} \right\rfloor} \right)^{q-2} \left[ 1 + (N - (q - 1)Q) \mod 2 \right] \left( \frac{N - (q - 1)Q}{\left\lfloor \frac{N-(q-1)Q}{2} \right\rfloor} \right).
\]

• The number of sequences that fulfill Cases 3 and 4 may not have close-form formulas, and hence, they are omitted.
The codeword selection procedure is given in the following:

**Step 1. (Initialization)** Let \( b_1 = -1 \), and let \( r_{\text{max}} \) be the total number of sequences satisfying the required \( B_v^T B_v \). Sort the \((\pm 1)\)-sequences according to their lexical order, starting from all-\((-1)\) sequence, and denote them by \( b(1), b(2), b(3), \ldots, b(r_{\text{max}}) \).

**Step 2. (Codeword Selection)** For an \((N, K)\) code, compute

\[
\Delta = \left\lfloor \frac{r_{\text{max}}}{2^K} \right\rfloor.
\]

Then, the codewords selected are \( \{b(j \times \Delta)\}_{j=1}^{2^K} \).
Decoding Criterion

- The rule for codeword selection is introduced, and only $2^K$ codewords are picked and the others are discarded. By assuming that the decoder knows $N_1, N_2, \ldots, N_q$, the optimal decision criterion in (6) are further explored.

- $P_B$ is a block matrix satisfying

$$P_B = \begin{bmatrix} P_B^{(1)} & 0 & \cdots & 0 \\ 0 & P_B^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_B^{(q)} \end{bmatrix}_{L \times L} = P_B^{(1)} \oplus P_B^{(2)} \oplus \cdots \oplus P_B^{(q)}, \quad (11)$$

where

$$P_B^{(1)} = B^{(1)} \left[ (B^{(1)})^T B^{(1)} \right]^{-1} (B^{(1)})^T.$$
The result is

\[ \hat{b} = \arg \min_{\mathbb{P}_B} \{ ||y - \mathbb{P}_B y||^2 \} \]

\[ = \arg \min_{(\mathbb{P}_B^{(1)}, \mathbb{P}_B^{(2)}, \ldots, \mathbb{P}_B^{(q)})} \{ ||L_y - (\mathbb{P}_B^{(1)} + \mathbb{P}_B^{(2)} + \cdots + \mathbb{P}_B^{(q)}) y||^2 \} \]

\[ = \arg \min_{(\mathbb{P}_B^{(1)}, \mathbb{P}_B^{(2)}, \ldots, \mathbb{P}_B^{(q)})} \{ ||(I_{N_1} - \mathbb{P}_B^{(1)}) + (I_{N_2} - \mathbb{P}_B^{(2)}) + \cdots + (I_{N_q} - \mathbb{P}_B^{(q)}) y||^2 \} \]

\[ = \arg \min_{(\mathbb{P}_B^{(1)} \perp, \mathbb{P}_B^{(2)} \perp, \ldots, \mathbb{P}_B^{(q)} \perp)} \{ ||(\mathbb{P}_B^{(1)} \perp y^{(1)}) + (\mathbb{P}_B^{(2)} \perp y^{(2)}) + \cdots ||^2 \} \]

\[ = \arg \min_{(\mathbb{P}_B^{(1)} \perp, \mathbb{P}_B^{(2)} \perp, \ldots, \mathbb{P}_B^{(q)} \perp)} \left\{ \sum_{i=1}^{q} \| (\mathbb{P}_B^{(i)} \perp y^{(i)}) \|^2 \right\}, \]

where

\[ y^{(i)} = \left[ y \sum_{j=1}^{i-1} N_j + 1, y \sum_{j=1}^{i-1} N_j + 2, \ldots, y \sum_{j=1}^{i-1} N_j + N_i \right]^T \]

represents the \( i \)-th sub-block of \( y \).
• Introduction

• System Model

• Code Design

• **Simulation Results**

• Conclusions
Simulation Results

• Several designed codes will be simulated over quasi-static Gaussian and non-static Gauss-Markov block fading channels in order to verify that the codes designed for non-static block fading channels are robust over both channels.

• As a convention, the zero-mean channel coefficients are normalized as $E[|h_{i,j}|^2] = 1/P$ for $1 \leq i \leq P$ and $1 \leq j \leq q$, and $\{h_{i,j}\}_{i=1}^{P}$ are assumed independent.

• The designed code of length $N$, which targets to be transmitted over the memory-order-$(P - 1)$ non-static fading channel whose channel coefficients change in every $Q$ symbols, is denoted by Code($N, P, Q$).

• The simulated channel, whose channel coefficients change in every $Q$ symbols, and whose memory order is $(P - 1)$, are similarly denoted as Channel($P, Q$).
Figure 1: The maximum-likelihood word error rates for Code(24, 2, 24) over Channel(2, 12) with different degree of channel variation factors $\alpha$. 
Figure 2: The maximum-likelihood word error rates for Code(24, 2, 24) over Channel(2, 6) with different degree of channel variation factors $\alpha$. 
Figure 3: The maximum-likelihood word error rates for Code(24, 2, 12) over Channel(2, 12) with different degree of channel variation factors $\alpha$. 
Figure 4: The maximum-likelihood word error rates for Code(24, 2, 12) over Channel(2, 6) with different degree of channel variation factors $\alpha$. 
Figure 5: The maximum-likelihood word error rates for Code(24, 2, 6) over Channel(2, 12) with different degree of channel variation factors $\alpha$. 
Figure 6: The maximum-likelihood word error rates for Code(24, 2, 6) over Channel(2, 6) with different degree of channel variation factors $\alpha$. 
Figure 7: The maximum-likelihood word error rates for Code(24, 2, 12) over Channel(2, Q) with channel variation factor $\alpha = 0$ and different values of $Q$. 
Figure 8: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.910057$ and different values of $Q$. 
Figure 9: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0$ and different values of $Q$. 
Figure 10: The maximum-likelihood word error rates for Code(24, 2, 6) over Channel(2, Q) with channel variation factor $\alpha = 0.910057$ and different values of $Q$. 
• Introduction
• System Model
• Code Design
• Simulation Results
• Conclusions
Conclusions

- In this work, a binary block code design for combined channel estimation and error protection, which is extended from Wu et al’s work specifically for non-static fading channels, is proposed and examined.

- Simulations hint that as long as the update rate of the channel coefficients is equal to or slower than that of the code target channel, the performance remains robust.

- However, when the channel coefficients change faster than those of the channel that the code design is presumed, the performance degrades considerably.

- The future work is to examine whether the code proposed is robust for non-stationary fading channels in which the channel coefficients change in an non-stationary non-periodic fashion.