Robust Code Design Over Fast Fading Channels

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Abstract

Nonlinear block codes that combine channel estimation, channel equalization and error protection have recently been proposed and confirmed their improvement in system performance by Skoglund et al in 2002 [10]. The design of Skoglund et al block codes, however, is based on computer search, and thus, has no efficient structure for decoding. In 2007, Wu et al have proposed a rule-constructed structural block code, and shown that the code has comparable performance to the computer-searched non-linear code of equal rate [13]. In this thesis, we extend the concept of rule-constructed self-orthogonal code design from the quasi-static fading channel to the non-static fading channel. We then examine the performance of our extension code over the first-order Gauss-Markov fading channel, and found that our extension code performs well not only in the target fast fading channel but also in the quasi-static fading channel.

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Chapter 1

Introduction

1.1 Background

In communications nowadays, high-quality data transmission usually relies on accurate knowledge of the channel characteristics. The measurement of the channel characteristics thus becomes an essential technique in real systems. Often, a training sequence that contains no data information is pre-transmitted before the data entity for the receiver to estimate the channel characteristics. Alternatively, pilot signals are inserted within the data entity to help improving the accuracy of the channel estimation [1, 8]. Recently, there is growing interest in blind channel estimation, where no training sequence or pilot signals are transmitted [12, 3]. The receiver accordingly has to retreat the data information with an implicit measurement of the channel characteristics based on the data entity [10].

In a quasi-static block fading environment [10], the channel statistics is assumed to change at a very slow rate, and hence, remains almost constant within a block transmission period. Such block-static nature of the channel coefficients facilitates their estimation. However, for a non-quasi-static block fading channel such as that is experienced by highly mobile devices, the channel coefficients may vary evidently during a block transmission period. In such case, to obtain good data transmission quality even by means of blind channel estimation technique is an engineering challenge.

In this thesis, the channel coefficients within a transmission block are assumed deterministic but unknown. In fact, this should be the case from the stand point of practical block transmission since the channel coefficients that affect the current decoding are ones that have already occurred in the past. We further assume that in a static environment, these deterministic channel coefficients are the same for the entire block, while in a non-static environment, they are allowed to change several times within a block. Both the static and non-static situations will be considered in this work. Our target is to design a robust code that can be simultaneously used in both the quasi-static block fading channels and the non-static ones.

1.2 Objective of the Research

In 2002, Skoglund et al [10] proposed to introduce the computer-searched best non-linear block code to combine channel estimate, equalization and error protection. In their work, the channel coefficients were assumed unchanged during the transmission of each codeword block, but allow to vary across blocks. Their simulations indicated that the computer-searched best code can outperform a typical training-sequence-enhanced communication system with perfect channel estimation by at least 2 dB. Nevertheless, due to the structureless nature of the computer-searched best code, the time-consuming exhaustive decoding was adopted, which limited the practical use of their code.

In 2007, Wu et al [13] replaced the computer-optimized non-linear code by a rule-based constructed code, and showed that the constructed code can yield comparable performance to the computer-searched best codes. Enforced by the structure of the rule-based construction, Wu et al subsequently derived a maximum-likelihood recursive metric for use of the priority-first sequential search decoding, and the decoding complexity when it is compared with the exhaustive decoding is greatly reduced.

In this thesis, we further extend the idea of Wu et al to the design of a code that performs well not only in quasi-static block fading environment but also in non-static channels. The only assumption in our extension is that the receiver knows exactly when the channel coefficients change within a codeword block. We will then examine the robustness of our rule-based constructed code over the Gauss-Markov fading channels [2] with different channel parameters.

1.3 Organization of thesis

The organization of the thesis is as follows. In Chapter 2, channel models for quasi-static block fading channels and their extensions to non-static block fading are described. Also introduced are the Gauss-Markov fading channels, as well as how to correspond the channel parameters to different degrees of Doppler effect. In Chapter 3, the rule used for code construction is presented, followed by the derivation of the error probability upper bound. In Chapter 4, simulation results are summarized and remarked. Chapter **??** concludes this work.

Chapter 2

System Model

2.1 Overview

Suppose the signal is transmitted through the linear time-invariant filter channel. Then, the received signal $y(t)$ can be expressed as

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau + n(t), \tag{2.1}$$

where $x(t)$ is the transmitted signal, $h(t)$ is the impulse response of the linear time-invariant channel filter, and $n(t)$ is the additive noise. The discrete equivalence of (2.1) is given by

$$y(t) = \sum_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau) + n(t). \tag{2.2}$$

In case the channel filter becomes time-variant, (2.1) is refined to

$$y(t) = \sum_{\tau=-\infty}^{\infty} h(\tau;t)x(t-\tau) + n(t),$$

where in $h(\tau;t)$, τ is the convolutional argument for filtering, and t represents the dependence of the filter on time.

Similar to [10], assume a codeword $\boldsymbol{b} = [b_1, \ldots, b_N]$ is transmitted through the so-called *quasi-static block fading channel* of which the channel coefficients remain constant within each block. Denote by P the memory order of the quasi-static fading channel. We can then re-formulate (2.2) as

$$\boldsymbol{y} = \mathbb{H}\boldsymbol{b} + \boldsymbol{n}, \tag{2.3}$$

where $\boldsymbol{y} = [y_1, y_2, \ldots, y_L]^T$ is the complex output vector observed at the receiver,

$$\mathbb{H} = \begin{bmatrix} h_1 & 0 & 0 & 0 & \ldots & 0 \\ h_2 & h_1 & 0 & & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ h_P & h_{P-1} & \vdots & h_1 & \ddots & \vdots \\ 0 & h_P & \ddots & \vdots & h_1 & 0 \\ \vdots & \vdots & \vdots & h_P & \ddots & h_1 \\ 0 & \ldots & 0 & & h_P & \vdots \\ 0 & 0 & 0 & \ldots & 0 & h_P \end{bmatrix}_{L\times N},$$

is formed by $L = N+P-1$ shift counterparts of the channel coefficients $\boldsymbol{h} = [h_1, h_2, \ldots, h_P]^T$, $\boldsymbol{n} = [n_1, n_2, \ldots, n_L]^T$ is the zero-mean complex Gaussian noise vector in which $E[\boldsymbol{n}\boldsymbol{n}^H] = \sigma_n^2\mathbb{I}_L$, and $\mathbb{I}_L$ is the $L \times L$ identity matrix. In the above notations, the superscripts "T" and "H" respectively represent the matrix transpose and Hermitian matrix transpose operations. It is assumed that the channel coefficients are flat and normalized in the sense that $\{h_i\}_{i=1}^P$ are independent and identically distributed (i.i.d.), and $E[\boldsymbol{h}^H\boldsymbol{h}] = 1$.

Formula (2.3) supposes that a codeword of length N experiences the same channel characteristics during its transmission. In a non-static environment or for a large N, the channel coefficients however may vary within the coding block. In such case, (2.3) should be modified as:

$$\boldsymbol{y} = \mathbb{H}_v\boldsymbol{b} + \boldsymbol{n}, \tag{2.4}$$

where

$$\mathbb{H}_v = \begin{bmatrix} & & \mathbb{H}^{(1)}_{N_1\times N_1} & \mathbf{0}_{N_1\times(N-N_1)} & \\ & \mathbf{0}_{N_2\times(N_1-(P-1))} & \mathbb{H}^{(2)}_{N_2\times N_2} & \mathbf{0}_{N_2\times(N-N_1-N_2+(P-1))} & \\ \mathbf{0}_{N_3\times(N_1+N_2-2(P-1))} & & & \mathbb{H}^{(3)}_{N_3\times N_3} & \mathbf{0}_{N_2\times(N-N_1-N_2-N_3+2(P-1))} \\ & & & \ddots & \\ & \mathbf{0}_{N_q\times(N-N_q)} & & \mathbb{H}^{(q)}_{N_q\times N_q} & \end{bmatrix} \tag{2.5}$$

corresponds to q channel coefficient changes during the transmission of codeword $\boldsymbol{b}$, and $\mathbf{0}$ denotes the all-zero matrix whose size is indicated by its subscripts. In (2.5), $N_1+N_2+\cdots+N_q = L$, and

$$\mathbb{H}^{(1)}_{N_1\times N_1} = \begin{bmatrix} h_{1,1} & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ h_{2,1} & h_{1,1} & 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ h_{P,1} & h_{P-1,1} & \vdots & h_{1,1} & 0 & \dots & \dots & 0 \\ 0 & h_{P,1} & \vdots & h_{2,1} & h_{1,1} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & h_{P,1} & h_{P-1,1} & \dots & \dots & h_{1,1} \end{bmatrix}_{N_1\times N_1},$$

and for $1 < i < q$,

$$\mathbb{H}^{(i)}_{N_i\times N_i} = \begin{bmatrix} h_{P,i} & h_{P-1,i} & \dots & h_{1,i} & 0 & \dots & \dots & 0 \\ 0 & h_{P,i} & \dots & h_{2,i} & h_{1,i} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & & & & \vdots \\ 0 & \dots & 0 & h_{P,i} & h_{P-1,i} & \dots & h_{1,i} & 0 \\ 0 & \dots & 0 & 0 & h_{P,i} & h_{P-1,i} & \dots & h_{1,i} \end{bmatrix}_{N_i\times N_i},$$

and

$$\mathbb{H}^{(q)}_{N_q\times N_q} = \begin{bmatrix} h_{P,q} & h_{P-1,q} & \dots & h_{1,q} & 0 & 0 & \dots & 0 \\ 0 & h_{P,q} & \dots & h_{2,q} & h_{1,q} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 0 & h_{P,q} & h_{P-1,q} & \dots & h_{1,q} \\ 0 & 0 & \dots & \dots & 0 & h_{P,q} & \dots & h_{2,q} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & h_{P,q} \end{bmatrix}_{N_q\times N_q}.$$

Here we assume that $N_i \geq P$ for every i, and N_i may be different for different i.

As similarly done in [10, 13], we can re-formulate (2.4) as

$$\boldsymbol{y} = \mathbb{B}_v \boldsymbol{h}_v + \boldsymbol{n},$$

where $\boldsymbol{h}_v = [h_{1,1}, h_{2,1}, \ldots, h_{P,1}, h_{2,1}, h_{2,2}, \ldots, h_{P,2}, h_{3,1}, \ldots\ldots, h_{P,q}]^T$, and

$$\mathbb{B}_v = \begin{bmatrix} \mathbb{B}^{(1)}_{N_1 \times P} & \mathbf{0}_{N_1 \times P} & \cdots & \mathbf{0}_{N_1 \times P} \\ \mathbf{0}_{N_2 \times P} & \mathbb{B}^{(2)}_{N_2 \times P} & \cdots & \mathbf{0}_{N_2 \times P} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N_q \times P} & \mathbf{0}_{N_q \times P} & \cdots & \mathbb{B}^{(q)}_{N_q \times P} \end{bmatrix}_{L \times qP}. \tag{2.6}$$

In (2.6),

$$\mathbb{B}^{(1)}_{N_1 \times P} = \begin{bmatrix} b_1 & 0 & \ldots & 0 \\ b_2 & b_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_P & b_{P-1} & \vdots & b_1 \\ b_{P+1} & b_P & \vdots & b_2 \\ \vdots & \vdots & & \vdots \\ b_{N_1} & b_{N_1-1} & \ldots & b_{N_1-(P-1)} \end{bmatrix}_{N_1 \times P},$$

and for $1 < i < q$,

$$\mathbb{B}^{(i)}_{N_i \times P} = \begin{bmatrix} b_{\sum_{k=1}^{i-1} N_k+1} & b_{\sum_{k=1}^{i-1} N_k} & \cdots & b_{\sum_{k=1}^{i-1} N_k-(P-1)+1} \\ b_{\sum_{k=1}^{i-1} N_k+2} & b_{\sum_{k=1}^{i-1} N_k+1} & \cdots & b_{\sum_{k=1}^{i-1} N_k-(P-1)+2} \\ \vdots & \vdots & & \vdots \\ b_{\sum_{k=1}^{i} N_k} & b_{\sum_{k=1}^{i} N_k-1} & \cdots & b_{\sum_{k=1}^{i} N_k-(P-1)} \end{bmatrix}_{N_i \times P},$$

and

$$\mathbb{B}^{(q)}_{N_q \times P} = \begin{bmatrix} b_{\sum_{k=1}^{q-1} N_k+1} & b_{\sum_{k=1}^{q-1} N_k} & \cdots & b_{\sum_{k=1}^{q-1} N_k-(P-1)+1} \\ b_{\sum_{k=1}^{q-1} N_k+2} & b_{\sum_{k=1}^{q-1} N_k+1} & \cdots & b_{\sum_{k=1}^{q-1} N_k-(P-1)+2} \\ \vdots & \vdots & & \vdots \\ b_N & b_{N-1} & \ldots & b_{N-(P-1)} \\ 0 & b_N & \ldots & b_{N-(P-1)+1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & b_N \end{bmatrix}_{N_q \times P}.$$

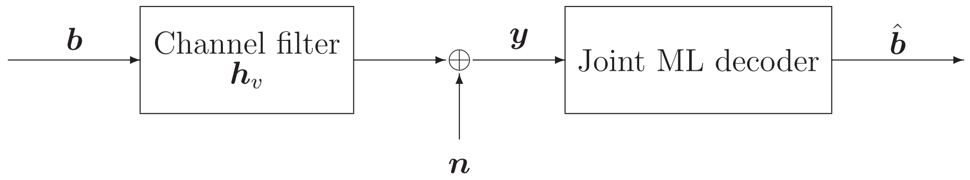

Figure 2.1: System model for combined channel estimation and error protection codes

Since $\boldsymbol{n}$ is a complex zero-mean Gaussian vector, and $\boldsymbol{h}_v$ is assumed unknown constants, the optimal decision for the transmitted codeword is:

$$\hat{\boldsymbol{b}} = \arg\min_{\mathbb{B}_v} \min_{\boldsymbol{h}_v} \|\boldsymbol{y} - \mathbb{B}_v \boldsymbol{h}_v\|^2 . \tag{2.7}$$

For a given $\boldsymbol{b}$, the least square (LS) estimate of $\boldsymbol{h}_v$ is given by

$$\hat{\boldsymbol{h}}_v = (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T \boldsymbol{y}.$$

Taking the above LS estimate into (2.7) yields

$$\begin{aligned} \hat{\boldsymbol{b}} &= \arg\min_{\mathbb{B}_v} \left\| \boldsymbol{y} - \mathbb{B}_v (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T \boldsymbol{y} \right\|^2 \\ &= \arg\min_{\mathbb{P}_B^{\perp}} \left\| \mathbb{P}_B^{\perp} \boldsymbol{y} \right\|^2 , \end{aligned} \tag{2.8}$$

where $\mathbb{P}_B^{\perp} \triangleq \mathbb{I}_L - \mathbb{P}_B$ and $\mathbb{P}_B \triangleq \mathbb{B}_v (\mathbb{B}_v^T \mathbb{B}_v)^{-1} \mathbb{B}_v^T$.

2.2 Gauss-Markov Model

In the simulations for quasi-static block fading channels in [10, 13], the channel coefficients $\boldsymbol{h}$ are generated independently in every block. We can extendedly assume that the sub-block channel coefficients $\boldsymbol{h}_i = [h_{1,i}, h_{2,i}, \cdots, h_{P,i}]^T$ in the non-static environment are also independent across sub-blocks.

A more general relationship between consecutive sub-block channel coefficients however is the first-order Gauss-Markov that is usually adopted in time-varying environment [1, 5, 11, 2]. Specifically,

$$\boldsymbol{h}_i = \alpha \boldsymbol{h}_{i-1} + \boldsymbol{v}_i = \alpha(\alpha \boldsymbol{h}_{i-2} + \boldsymbol{v}_{i-1}) + \boldsymbol{v}_i = \ldots = \alpha^i \boldsymbol{h}_0 + \sum_{j=1}^{i} \alpha^{j-1} \boldsymbol{v}_{i-j} \tag{2.9}$$

where $\Pr\{\boldsymbol{h}_0 = \boldsymbol{0}_{P\times 1}\} = 1$, and $\{\boldsymbol{v}_i\}_{i=1}^{q}$ is zero-mean complex Gaussian distributed with $E[\boldsymbol{v}_i \boldsymbol{v}_i^H] = \sigma_{v_i}^2 \boldsymbol{I}_P$. The special case is $\sigma_{h_1}^2 = \sigma_{v_1}^2$, since $\boldsymbol{h}_1$ equals $\boldsymbol{v}_1$. Notably, the parameter α charactersizes the rate of channel variation between consecutive sub-blocks. Its value lies between zero and one, and is controlled by the Doppler spread and transmission bandwidth [11] as

$$\alpha = \exp(-\omega_d T_s) = \exp\left(-\pi B_d T_s\right) = \exp\left(-2\pi f_c \frac{v}{c} T_s\right),$$

where $B_d = \omega_d/\pi$ denotes the Doppler spread, f_c is the carrier frequency, v is the velocity of the transmitter, c is the velocity of light, and T_s is the symbol period (i.e. sub-block period in our case). For vehicle speed $v = 180$ km/hours, carrier frequency $f_c = 900$ MHz, and sub-block size 10^{-4} seconds,

$$\omega_d T_s = 2\pi f_c \frac{v}{c} T_s = 2\pi \times (900 \times 10^6 \text{ Hz}) \times \frac{180 \text{ km/hours}}{1.08 \times 10^9 \text{ km/hours}} \times 10^{-4} \text{ seconds} = 0.03\pi.$$

This yields $\alpha = \exp\{-0.03\pi\} = 0.910057$ [5]. If we increase f_c to 2.7 GHz and 5.4 Ghz, α will become $\exp(-0.09\pi) = 0.753713$ and $\exp(-0.18\pi) = 0.568084$, respectively. These α-values will be used in our simulations.

Chapter 3

Code Design

In this chapter, the codeword condition under which the signal-to-noise ratio (SNR) is guaranteed maximal is derived, followed by the code construction approach proposed based on the condition. Also derived in this chapter is an error probability upper bound that will be used as the criterion to search for the best code for comparison with the constructed one.

3.1 Self-orthogonality condition for SNR-optimized codewords

A known inequality [9] for the multiplication of two positive semi-definite Hermitian matrices is

$$\mathrm{tr}(\mathbb{A}\mathbb{B}) \leq \mathrm{tr}(\mathbb{A}) \cdot \lambda_{\max}(\mathbb{B}), \tag{3.1}$$

where $\mathrm{tr}(\cdot)$ represents the matrix trace operation, and $\lambda_{\max}(\mathbb{B})$ is the maximal eigenvalue of $\mathbb{B}$. From the system model defined in (2.4), the average SNR is given by

$$\begin{aligned}\mathrm{SNR} &= \frac{E[||\mathbb{H}_v \boldsymbol{b}||^2]}{E[||\boldsymbol{n}||^2]} \\ &= \frac{E[||\mathrm{tr}(\boldsymbol{h}_v^H \mathbb{B}_v^T \mathbb{B}_v \boldsymbol{h}_v)||]}{L\sigma_n^2} \\ &= \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H] \mathbb{B}_v^T \mathbb{B}_v)}{L\sigma_n^2} \\ &\leq \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H])}{L\sigma_n^2} \lambda_{\max}(\mathbb{B}_v^T \mathbb{B}_v) \\ &= \frac{N}{L} \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H])}{\sigma_n^2} \lambda_{\max}(\frac{1}{N} \mathbb{B}_v^T \mathbb{B}_v).\end{aligned}$$

The above inequality holds with equality when $(1/N)\mathbb{B}_v^T\mathbb{B}_v$ is an identity matrix [6], namely,

$$\mathbb{B}_v^T \mathbb{B}_v = N\mathbb{I}_{qP} \triangleq \begin{bmatrix} N & 0 & \dots & 0 \\ 0 & N & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & N \end{bmatrix}_{qP \times qP} . \tag{3.2}$$

As a result, the maximum SNR equals

$$\mathrm{SNR}_{\max} = \frac{N}{L} \frac{\mathrm{tr}(E[\boldsymbol{h}_v \boldsymbol{h}_v^H])}{\sigma_n^2}. \tag{3.3}$$

3.2 Codeword Selection

The previous section established the self-orthogonal condition under which the system SNR is maximized. However, the codeword sequences satisfying (3.2) may not exist for certain N, P and q. In such cases, one can only choose codewords that best-approximate (3.2), for which some examples are given below.

Case 1. For $P = 2$ and $q = 1$, the codewords can be chosen according to:

$$\mathbb{B}_v^T \mathbb{B}_v = \mathbb{B}^T \mathbb{B} = \begin{cases} \begin{bmatrix} N & 0 \\ 0 & N \end{bmatrix}, & \text{for N odd} \\ \\ \begin{bmatrix} N & \pm 1 \\ \pm 1 & N \end{bmatrix}, & \text{for N even.} \end{cases}$$

Case 2. For $P = 2$ and $q > 1$ with $N_1 = N_2 = \cdots = N_{q-1} = Q$, we observe that

$$\mathbb{B}_v = \mathbb{B}^{(1)} \oplus \mathbb{B}^{(2)} \oplus \cdots \oplus \mathbb{B}^{(q)}, \tag{3.4}$$

where "$\oplus$" is the direct sum operator of two matrices.[1] Then, the codewords can be chosen according to:

$$\left(\mathbb{B}^{(1)}\right)^T \mathbb{B}^{(1)} = \begin{cases} \begin{bmatrix} Q & 0 \\ 0 & (Q-1) \end{bmatrix}, & \text{for } Q \text{ odd} \\ \\ \begin{bmatrix} Q & \pm 1 \\ \pm 1 & (Q-1) \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

and for $1 < i < q$,

$$\left(\mathbb{B}^{(i)}\right)^T \mathbb{B}^{(i)} = \begin{cases} \begin{bmatrix} Q & \pm 1 \\ \pm 1 & Q \end{bmatrix}, & \text{for } Q \text{ odd} \\ \\ \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

and

$$\left(\mathbb{B}^{(q)}\right)^T \mathbb{B}^{(q)} = \begin{cases} \begin{bmatrix} N-(q-1)Q & \pm 1 \\ \pm 1 & N-(q-1)Q+1 \end{bmatrix}, & \text{for } [N-(q-1)Q] \text{ odd} \\ \\ \begin{bmatrix} N-(q-1)Q & 0 \\ 0 & N-(q-1)Q+1 \end{bmatrix}, & \text{for } [N-(q-1)Q] \text{ even.} \end{cases}$$

[1]For two matrices $\mathbb{A}$ and $\mathbb{B}$, the direct sum of $\mathbb{A}$ and $\mathbb{B}$ is defined as $\mathbb{A} \oplus \mathbb{B} = \begin{bmatrix} \mathbb{A} & \mathbf{0} \\ \mathbf{0} & \mathbb{B} \end{bmatrix}$.

Case 3. For $P > 2$ and $q = 1$, the codewords can be chosen according to:

$$\mathbb{B}_v^T \mathbb{B}_v = \mathbb{B}^T \mathbb{B} = \begin{cases} \begin{bmatrix} N & 0 & \pm 1 & 0 & \pm 1 & 0 & \cdots \\ 0 & N & 0 & \pm 1 & 0 & \pm 1 & \cdots \\ \pm 1 & 0 & N & 0 & \pm 1 & 0 & \cdots \\ 0 & \pm 1 & 0 & N & 0 & \pm 1 & \cdots \\ \pm 1 & 0 & \pm 1 & 0 & N & 0 & \cdots \\ 0 & \pm 1 & 0 & \pm 1 & 0 & N & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}_{P \times P}, & \text{for N odd} \\ \begin{bmatrix} N & \pm 1 & 0 & \pm 1 & 0 & \pm 1 & \cdots \\ \pm 1 & N & \pm 1 & 0 & \pm 1 & 0 & \cdots \\ 0 & \pm 1 & N & \pm 1 & 0 & \pm 1 & \cdots \\ \pm 1 & 0 & \pm 1 & N & \pm 1 & 0 & \cdots \\ 0 & \pm 1 & 0 & \pm 1 & N & \pm 1 & \cdots \\ \pm 1 & 0 & \pm 1 & 0 & \pm 1 & N & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}_{P \times P}, & \text{for N even} \end{cases}$$

Case 4. For $P = 3$ and $q = 2$ with $N_1 = Q$, the codewords can be chosen according to:

$$\left(\mathbb{B}^{(1)}\right)^T \mathbb{B}^{(1)} = \begin{cases} \begin{bmatrix} Q & 0 & \pm 1 \\ 0 & Q-1 & \pm 1 \\ \pm 1 & \pm 1 & Q-2 \end{bmatrix}, & \text{for } Q \text{ odd} \\ \begin{bmatrix} Q & \pm 1 & 0 \\ \pm 1 & Q-1 & 0 \\ 0 & 0 & Q-2 \end{bmatrix}, & \text{for } Q \text{ even} \end{cases}$$

and

$$\left(\mathbb{B}^{(2)}\right)^T \mathbb{B}^{(2)} = \begin{cases} \begin{bmatrix} N-Q & \pm 1 & \pm 1 \\ \pm 1 & N-Q+1 & 0 \\ \pm 1 & 0 & N-Q+2 \end{bmatrix}, & \text{for } (N-Q) \text{ odd} \\ \begin{bmatrix} N-Q & 0 & 0 \\ 0 & N-Q+1 & \pm 1 \\ 0 & \pm 1 & N-Q+2 \end{bmatrix}, & \text{for } (N-Q) \text{ even.} \end{cases}$$

For convenience, the numbers of sequences that satisfy Cases 1 and 2 are listed in the lemma forms in the following.

Lemma 3.1. *The number of sequences that fulfill Case 1 and $b_1 = -1$ is equal to*

$$(2 - (N \bmod 2)) \binom{N-1}{\lfloor \frac{N-1}{2} \rfloor}. \tag{3.5}$$

Proof. The sequences must satisfy

$$c = b_1 b_2 + b_2 b_3 + \ldots + b_{N-1} b_N \tag{3.6}$$

where $c = 0$ for N odd, and $c = \pm 1$ for N even. Therefore, for N even, there must be either exactly $\lfloor (N-1)/2 \rfloor$ terms equal to -1 or exactly $\lfloor (N-1)/2 \rfloor$ terms equal to 1 among $b_1 b_2, b_2 b_3, \ldots, b_{N_1} b_N$. And for N odd, Case 1 is only satisfied when there are exactly $(N-1)/2$ terms equal to -1 among $b_1 b_2, b_2 b_3, \ldots, b_{N_1} b_N$. The lemma is then completed by noting that $(b_1 b_2, b_2 b_3, \ldots, b_{N-1} b_N)$ and $(b_1, b_2, \ldots, b_N)$ are one-to-one correspondence given that $b_1 = -1$. $\square$

Lemma 3.2. *The number of sequences that fulfill Case 2 and $b_1 = -1$ is equal to*

$$\begin{aligned} &[2 - (Q \bmod 2)] \binom{Q-1}{\lfloor \frac{Q-1}{2} \rfloor} [1 + (Q \bmod 2)]^{q-2} \binom{Q}{\lfloor \frac{Q}{2} \rfloor}^{q-2} \\ &[1 + (N - (q-1)Q) \bmod 2] \binom{N-(q-1)Q}{\lfloor \frac{N-(q-1)Q}{2} \rfloor}. \end{aligned} \tag{3.7}$$

Proof. Lemma 3.2 requires

$$\begin{cases} c_1 &= b_1 b_2 + b_2 b_3 + \ldots + b_{Q-1} b_Q \\ c_2 &= b_Q b_{Q+1} + b_{Q+1} b_{Q+2} + \ldots + b_{2Q-1} b_{2Q} \\ &\vdots \\ c_q &= b_{(q-1)Q} b_{(q-1)Q+1} + b_{(q-1)Q+1} b_{(q-1)Q+2} + \ldots + b_{N-1} b_N \end{cases} \tag{3.8}$$

where $c_1 = 0$ for Q odd and $c_1 = \pm 1$ for Q even, and for $1 < i < q$, $c_i = \pm 1$ for Q odd and $c_i = 0$ for Q even, and $c_q = \pm 1$ for $[N - (q-1)Q]$ odd and $c_q = 0$ for $[N - (q-1)Q]$ odd. By following the same reasoning as in Lemma 3.1, the numbers of sequences that make valid the equations respectively for c_1, $\{c_i\}_{i=2}^{q-1}$ and c_q are equal to

$$[2 - (Q \bmod 2)] \binom{Q-1}{\lfloor \frac{Q-1}{2} \rfloor}$$

$$[1 + (Q \bmod 2)] \binom{Q}{\lfloor \frac{Q}{2} \rfloor}$$

and

$$[1 + (N - (q-1)Q) \bmod 2] \binom{N - (q-1)Q}{\lfloor \frac{N-(q-1)Q}{2} \rfloor}.$$

□

The number of sequences that fulfill Cases 3 and 4 may not have close-form formulas, and hence, they are omitted.

For clarity, the codeword selection procedure is also given in the end.

Step 1. **(Initialization)** *Let* $b_1 = -1$, *and let* $r_{\max}$ *be the total number of sequences satisfying the required* $\mathbb{B}_v^T \mathbb{B}_v$. *Sort the* (± 1)*-sequences according to their lexical order, starting from all-*(-1) *sequence, and denote them by* $\boldsymbol{b}(1), \boldsymbol{b}(2), \boldsymbol{b}(3), \ldots, \boldsymbol{b}(r_{\max})$.

Step 2. **(Codeword Selection)** *For an* (N, K) *code, compute*

$$\Delta = \left\lfloor \frac{r_{\max}}{2^K} \right\rfloor.$$

Then, the codewords selected are $\{\boldsymbol{b}(j \times \Delta)\}_{j=1}^{2^K}$.

3.3 Decoding Criterion

In the previous section, the rule for codeword selection is introduced, and only 2^K codewords are picked and the others are discarded. By assuming that the decoder knows N_1, N_2, ..., N_q, the optimal decision criterion in (2.7) are further explored. Notably, there are at least $L^2 - \sum_{i=1}^{q} N_i^2$ zero elements in $\mathbb{P}_B^{\perp}$, which is of no use in decision, and the computation for these zero elements are accordingly vanished in the equivalent optimal decoding criterion derived in this section.

In the general non-static environment, by the fact that

$$(\mathbb{A}_1 \oplus \mathbb{A}_2 \oplus \cdots \oplus \mathbb{A}_q)(\mathbb{C}_1 \oplus \mathbb{C}_2 \oplus \cdots \oplus \mathbb{C}_q) = \mathbb{A}_1\mathbb{C}_1 \oplus \mathbb{A}_2\mathbb{C}_2 \oplus \cdots \oplus \mathbb{A}_q\mathbb{C}_q,$$

for square $\mathbb{A}_i$ and $\mathbb{C}_i$ of the same size, we have from (2.6) that

$$\begin{aligned} \mathbb{B}_v^T\mathbb{B}_v &= \begin{bmatrix} (\mathbb{B}^{(1)})^T\mathbb{B}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & (\mathbb{B}^{(2)})^T\mathbb{B}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & (\mathbb{B}^{(q)})^T\mathbb{B}^{(q)} \end{bmatrix}_{qP\times qP} \\ &= \left[(\mathbb{B}^{(1)})^T\mathbb{B}^{(1)}\right] \oplus \left[(\mathbb{B}^{(2)})^T\mathbb{B}^{(2)}\right] \oplus \cdots \oplus \left[(\mathbb{B}^{(q)})^T\mathbb{B}^{(q)}\right]. \end{aligned}$$

Similarly, $\mathbb{P}_B$ is a block matrix satisfying

$$\mathbb{P}_B = \begin{bmatrix} \mathbb{P}_B^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbb{P}_B^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbb{P}_B^{(q)} \end{bmatrix}_{L\times L} = \mathbb{P}_B^{(1)} \oplus \mathbb{P}_B^{(2)} \oplus \cdots \oplus \mathbb{P}_B^{(q)}, \tag{3.9}$$

where

$$\mathbb{P}_B^{(1)} = \mathbb{B}^{(1)}\left[(\mathbb{B}^{(1)})^T\mathbb{B}^{(1)}\right]^{-1}(\mathbb{B}^{(1)})^T.$$

Putting (3.9) into (2.8) yields that

$$\begin{aligned}
\hat{\boldsymbol{b}} &= \arg\min_{\mathbb{P}_B}\{||\boldsymbol{y}-\mathbb{P}_B\boldsymbol{y}||^2\} \\
&= \arg\min_{(\mathbb{P}_B^{(1)},\mathbb{P}_B^{(2)},\cdots,\mathbb{P}_B^{(q)})}\left\{\left\|\mathbb{I}_L\boldsymbol{y}-(\mathbb{P}_B^{(1)}\oplus\mathbb{P}_B^{(2)}\oplus\cdots\oplus\mathbb{P}_B^{(q)})\boldsymbol{y}\right\|^2\right\} \\
&= \arg\min_{(\mathbb{P}_B^{(1)},\mathbb{P}_B^{(2)},\cdots,\mathbb{P}_B^{(q)})}\left\{\left\|(\mathbb{I}_{N_1}\oplus\mathbb{I}_{N_2}\oplus\cdots\oplus\mathbb{I}_{N_q})\boldsymbol{y}-(\mathbb{P}_B^{(1)}\oplus\mathbb{P}_B^{(2)}\oplus\cdots\oplus\mathbb{P}_B^{(q)})\boldsymbol{y}\right\|^2\right\} \\
&= \arg\min_{(\mathbb{P}_B^{(1)},\mathbb{P}_B^{(2)},\cdots,\mathbb{P}_B^{(q)})}\left\{\left\|\left[(\mathbb{I}_{N_1}-\mathbb{P}_B^{(1)})\oplus(\mathbb{I}_{N_2}-\mathbb{P}_B^{(2)})\oplus\cdots\oplus(\mathbb{I}_{N_q}-\mathbb{P}_B^{(q)})\right]\boldsymbol{y}\right\|^2\right\} \\
&= \arg\min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)}\left\{\left\|((\mathbb{P}_B^{(1)})^\perp\oplus(\mathbb{P}_B^{(2)})^\perp\oplus\cdots\oplus(\mathbb{P}_B^{(q)})^\perp)\boldsymbol{y}\right\|^2\right\} \\
&= \arg\min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)}\left\{\left\|((\mathbb{P}_B^{(1)})^\perp\boldsymbol{y}^{(1)})\oplus((\mathbb{P}_B^{(2)})^\perp\boldsymbol{y}^{(2)})\oplus\cdots\oplus((\mathbb{P}_B^{(q)})^\perp\boldsymbol{y}^{(q)})\right\|^2\right\} \\
&= \arg\min_{\left((\mathbb{P}_B^{(1)})^\perp,(\mathbb{P}_B^{(2)})^\perp,\cdots,(\mathbb{P}_B^{(q)})^\perp\right)}\left\{\sum_{i=1}^{q}\left\|(\mathbb{P}_B^{(i)})^\perp\boldsymbol{y}^{(i)}\right\|^2\right\}, \qquad (3.10)
\end{aligned}$$

where

$$\boldsymbol{y}^{(i)}=\begin{bmatrix} y_{\sum_{j=1}^{i-1}N_j+1} \\ y_{\sum_{j=1}^{i-1}N_j+2} \\ \vdots \\ y_{\sum_{j=1}^{i-1}N_j+N_i}\end{bmatrix}^T$$

represents the i-th sub-block of $\boldsymbol{y}$.

3.4 Error Rate Evaluation

In this section, we derive the union bound for the error probability.

For convenience, we re-number the selected codewords as $\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{b}_3,\ldots,\boldsymbol{b}_{2^K}$ for the (N,K)

code considered. Then, the error probability P_e can be bounded above by

$$\begin{aligned} P_e &= \frac{1}{2^K}\sum_{i=1}^{2^K}\Pr(\hat{\boldsymbol{b}} \neq \boldsymbol{b}_i|\boldsymbol{b}_i \text{ transmitted}) \\ &\leq \frac{1}{2^K}\sum_{i=1}^{2^K}\Pr\left(\exists\ j \neq i \text{ such that } \left\|P_{B_j}^{\perp}\boldsymbol{y}\right\|^2 \leq \left\|P_{B_i}^{\perp}\boldsymbol{y}\right\|^2 \middle| \boldsymbol{b}_i \text{ transmitted}\right) \\ &\leq \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K}\Pr\left(\left\|P_{B_j}^{\perp}\boldsymbol{y}\right\|^2 \leq \left\|P_{B_i}^{\perp}\boldsymbol{y}\right\|^2 \middle| \boldsymbol{b}_i \text{ transmitted}\right) \\ &= \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K}\Pr\left(\boldsymbol{y}^H(P_{B_j}^{\perp})^H P_{B_j}^{\perp}\boldsymbol{y} \leq \boldsymbol{y}^H(P_{B_i}^{\perp})^H P_{B_i}^{\perp}\boldsymbol{y} \middle| \boldsymbol{b}_i \text{ transmitted}\right) \\ &= \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K}\Pr\left(\boldsymbol{y}^H\left[(P_{B_j}^{\perp})^H P_{B_j}^{\perp} - (P_{B_i}^{\perp})^H P_{B_i}^{\perp}\right]\boldsymbol{y} \leq 0 \middle| \boldsymbol{b}_i \text{ transmitted}\right), \end{aligned} \tag{3.11}$$

where $\mathbb{P}_{B_i}^{\perp}$ corresponds to the codeword $\boldsymbol{b}_i$.

Since $\mathbb{P}_{B_i}^{\perp}$ is idempotent for every $\boldsymbol{b}_i$, and $\mathbb{P}_{B_j}^{\perp} - \mathbb{P}_{B_i}^{\perp} = \mathbb{P}_{B_i} - \mathbb{P}_{B_i}$, (3.11) can be reformulated as

$$\begin{aligned} P_e &\leq \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K}\Pr\left(\boldsymbol{y}^H\left(\mathbb{P}_{B_j}^{\perp} - \mathbb{P}_{B_i}^{\perp}\right)\boldsymbol{y} \leq 0 \middle| \boldsymbol{b}_i \text{ transmitted}\right) \\ &= \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K}\Pr\left(\boldsymbol{y}^H\left(\mathbb{P}_{B_j} - \mathbb{P}_{B_i}\right)\boldsymbol{y} \geq 0 \middle| \boldsymbol{b}_i \text{ transmitted}\right). \end{aligned} \tag{3.12}$$

By following similar argument as in [10], the covariance matrix $\mathbf{S}_y(i)$ of the received vector $\boldsymbol{y}$ for given transmitted codeword $\boldsymbol{b}_i$ and zero-mean complex-Gaussian distributed $\boldsymbol{h}$ is real and symmetric, and is always positive definite for positive noise variance. We can then define $\mathbb{G}_i = \mathbf{S}_y^{1/2}(i)$, and obtain that

$$\mathbb{G}_i\left(\mathbb{P}_{B_j} - \mathbb{P}_{B_i}\right)\mathbb{G}_i = \sum_{\ell=1}^{L}\lambda_{\ell;i,j}\boldsymbol{q}_{\ell;i,j}\boldsymbol{q}_{\ell;i,j}^T,$$

where $\{\lambda_{\ell;i,j}\}_{\ell=1}^L$ and $\{\boldsymbol{q}_{\ell;i,j}\}_{\ell=1}^L$ represent the eigenvalues and eigenvectors of $\mathbb{G}_i\left(\mathbb{P}_{B_j} - \mathbb{P}_{B_i}\right)\mathbb{G}_i$.

As a result, given that $\boldsymbol{b}_i$ is transmitted,

$$\begin{aligned} \boldsymbol{y}^H \left(\mathbb{P}_{B_j} - \mathbb{P}_{B_i}\right) \boldsymbol{y} &= \left(\mathbb{G}_i^{-1}\boldsymbol{y}\right)^H \left(\mathbb{G}_i \left(\mathbb{P}_{B_j} - \mathbb{P}_{B_i}\right)\mathbb{G}_i\right)\left(\mathbb{G}_i^{-1}\boldsymbol{y}\right) \\ &= \sum_{\ell=1}^{L} \lambda_{\ell;i,j} \left|\boldsymbol{q}_{\ell;i,j}^T \mathbb{G}_i^{-1}\boldsymbol{y}\right|^2 \\ &= \sum_{\ell=1}^{L} \lambda_{\ell;i,j} |\boldsymbol{z}_{\ell;i,j}|^2, \end{aligned}$$

where $\{\boldsymbol{z}_{\ell;i,j}\}_{\ell=1}^{L}$ is independent zero-mean complex Gaussian with variance 1, and

$$P_e \le \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{j=1,j\neq i}^{2^K} \Pr\left(\sum_{\ell=1}^{L}\lambda_{\ell;i,j}|\boldsymbol{z}_{\ell;i,j}|^2 \ge 0\right). \tag{3.13}$$

Without loss of generality, we assume that $\bar{\lambda}_{1;i,j} > \bar{\lambda}_{2;i,j} > \cdots > \bar{\lambda}_{r;i,j} > 0 > \bar{\lambda}_{r+1;i,j} > \cdots > \bar{\lambda}_{L_c;i,j}$ be those eigenvalues by removing the identical ones among $\{\lambda_{\ell;i,j}\}_{\ell=1}^{L}$, and let their respective orders of multiplicity be $\{o_{\ell;i,j}\}_{\ell=1}^{L_c}$. Then, $|\boldsymbol{z}_{\ell;i,j}|^2$ is a central χ^2-random variable with $2o_{\ell;i,j}$ degree of freedom.

By elementary probabilistic theories, the cumulant distribution function of random variable $\sum_{\ell=1}^{L}\lambda_{\ell;i,j}|\boldsymbol{z}_{\ell;i,j}|^2$ is

$$F_{i,j}(\upsilon) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}\cdot \mathrm{Im}\{\phi_{i,j}(t)e^{-\mathrm{i}\,t\upsilon}\}dt \tag{3.14}$$

where $\mathrm{Im}\{\cdot\}$ represents the imaginary part, and

$$\phi_{i,j}(t) = \prod_{\ell=1}^{L_c}\left(1 - 2\,\mathrm{i}\,t\bar{\lambda}_{\ell;i,j}\right)^{-o_{\ell;i,j}}. \tag{3.15}$$

where $\mathrm{i} = \sqrt{-1}$. Finally, by denoting $d_{i,j} = \sum_{\ell=1}^{L_c} o_{\ell;i,j}$, we obtain [7] that

$$F_{i,j}(\upsilon) = 1 - \sum_{\ell=1}^{r}\frac{1}{(o_{\ell;i,j}-1)!}\left[\frac{\partial^{o_{\ell;i,j}-1}}{\partial x^{o_{\ell;i,j}-1}}\bar{F}_\ell(x,\upsilon)\right]_{x=\bar{\lambda}_{\ell;i,j}}, \tag{3.16}$$

where

$$\bar{F}_\ell(x,\upsilon) = x^{d_{i,j}-1}e^{-\upsilon/(2x)}\prod_{m=1,m\neq\ell}^{r}(x-\bar{\lambda}_{m;i,j})^{-o_{m;i,j}}. \tag{3.17}$$

Taking (3.16) and (3.17) with $\upsilon = 0$, we yield the upper bound of error probability.

Chapter 4

Simulation Results

In this chapter, we will examine the robustness of the proposed coding scheme. Specifically, several designed codes will be simulated over quasi-static Gaussian and non-static Gauss-Markov block fading channels in order to verify that the codes designed for non-static block fading channels are robust over both channels. As a convention, the zero-mean channel coefficients are normalized as $E[|h_{i,j}|^2] = 1/P$ for $1 \leq i \leq P$ and $1 \leq j \leq q$, and $\{h_{i,j}\}_{i=1}^{P}$ are assumed independent.

4.1 Codes Designed For Quasi-Static Block Fading Channels

This section summarizes the simulations over the non-static Gauss-Markov fading channels with Gaussian distributed channel coefficients. In notations, the designed code of length N, which targets to be transmitted over the memory-order-$(P-1)$ non-static fading channel whose channel coefficients change in every Q symbols, is denoted by Code(N, P, Q). The simulated channel, whose channel coefficients change in every Q symbols, and whose memory order is $(P-1)$, are similarly denoted as Channel(P, Q). Five different channel variation factors (i.e., α-values) of the first-order Gauss-Markov fading channel will be used in our simulations, which are respectively 0, 0.568084, 0.753713, 0.910057, and 1. Notably, Channel(P, Q) reduces to the quasi-static block fading channel of memory order $(P-1)$ when $\alpha = 1$.

The performance of Code$(12, 2, 12)$ over Channel$(2, 6)$ is shown in Fig. 4.1. The simulations indicate that the code designed for quasi-static fading channels performs well only over quasi-static fading environment, namely, $\alpha = 1$. As α decreases, which means that the degree of channel variations increases, the performance degrades accordingly. Similar simulations have been performed for Code(14, 2, 14), Code(16, 2, 16), Code(18, 2, 18), Code(20, 2, 20), Code(22, 2, 22), Code(24, 2, 24), Code(12, 2, 12), Code(16, 2, 16), Code(20, 2, 20), and Code(24, 2, 24) respectively over Channel(2, 7), Channel(2, 8), Channel(2, 9), Channel(2, 10), Channel(2, 11), Channel(2, 12), Channel(2, 3), Channel(2, 4), Channel(2, 5), and Channel(2, 6), and are summarized respectively in Figs. 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 4.9, 4.10, and 4.11. Same performance behaviors can be observed from these figures. As a consequence, we conclude that Code(N, P, N) performs well only over quasi-static block fading channel (namely, $\alpha = 1$), and its performance degrades considerably for moderate-to-high degree of channel variations.

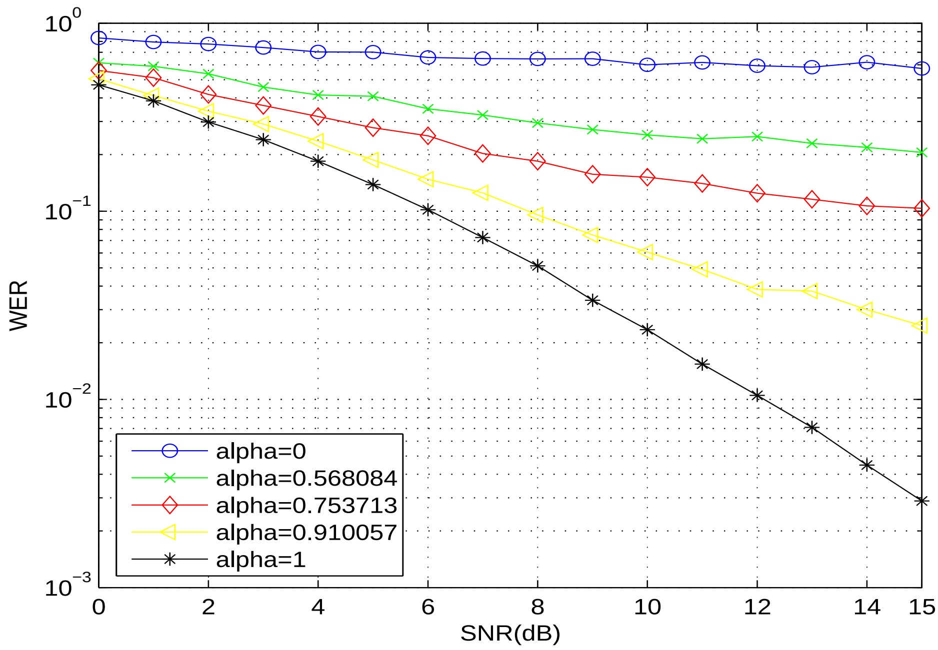

Figure 4.1: The maximum-likelihood word error rates for Code$(12, 2, 12)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

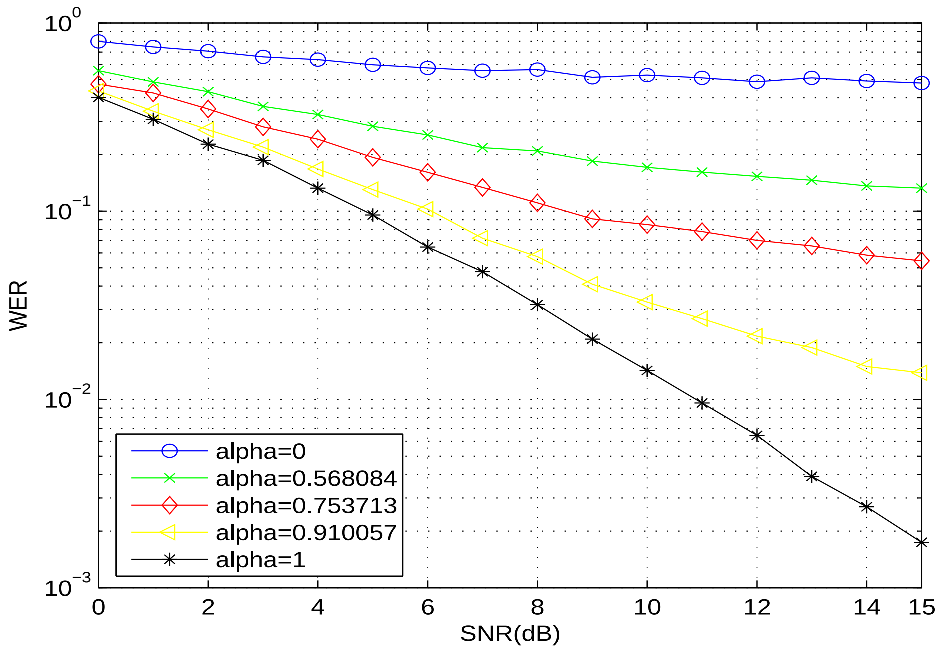

Figure 4.2: The maximum-likelihood word error rates for Code$(14, 2, 14)$ over Channel$(2, 7)$ with different degree of channel variation factors α.

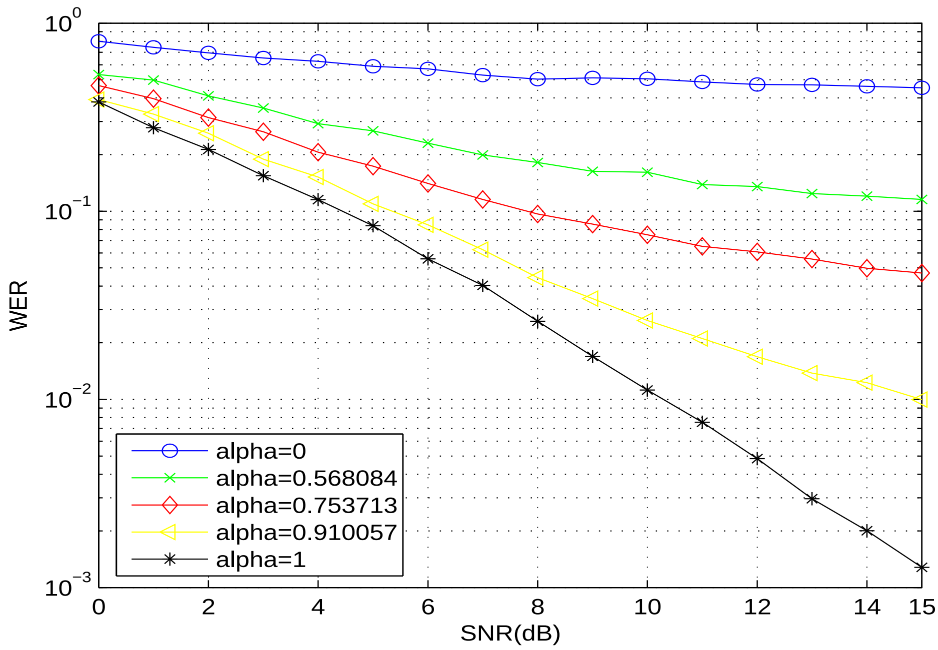

Figure 4.3: The maximum-likelihood word error rates for Code$(16, 2, 16)$ over Channel$(2, 8)$ with different degree of channel variation factors α.

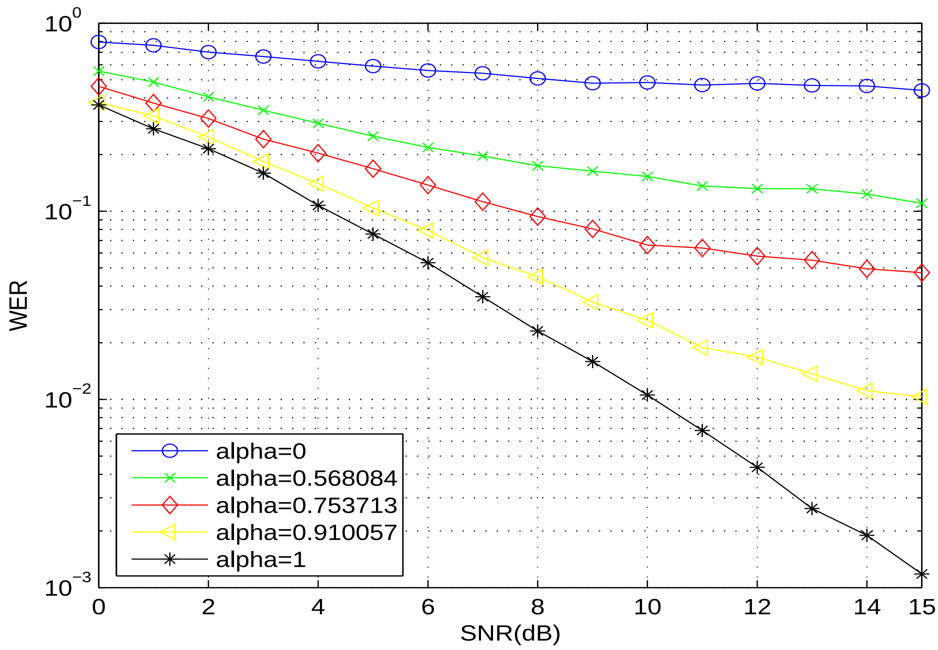

Figure 4.4: The maximum-likelihood word error rates for Code$(18, 2, 18)$ over Channel$(2, 9)$ with different degree of channel variation factors α.

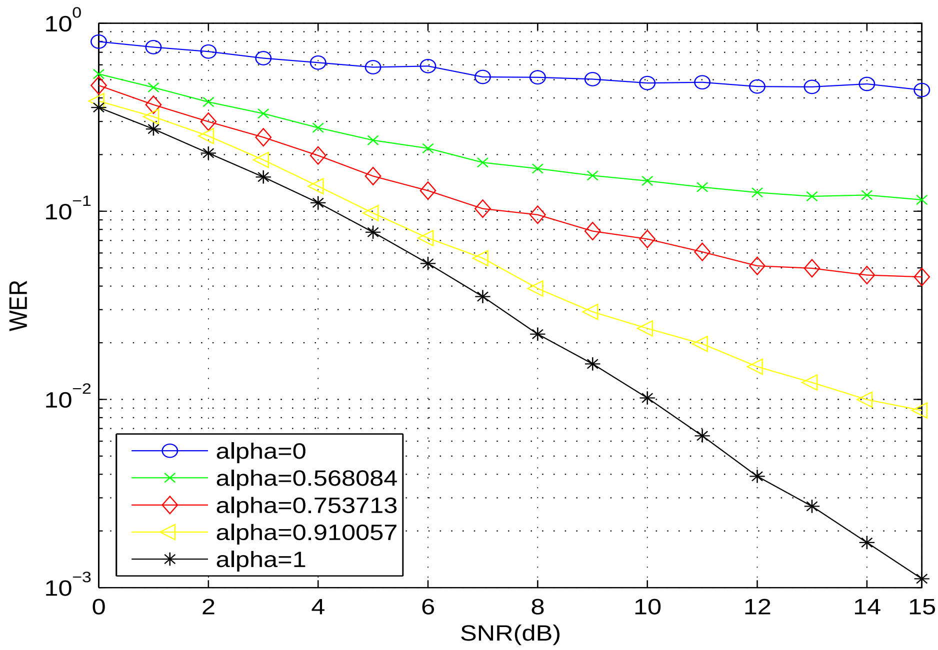

Figure 4.5: The maximum-likelihood word error rates for Code(20, 2, 20) over Channel(2, 10) with different degree of channel variation factors α.

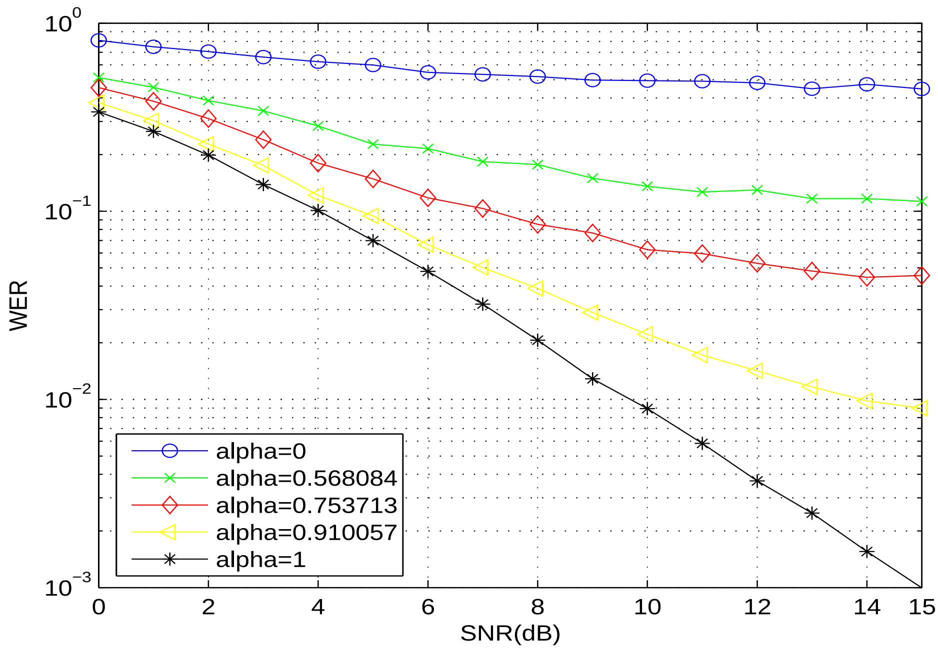

Figure 4.6: The maximum-likelihood word error rates for Code$(22, 2, 22)$ over Channel$(2, 11)$ with different degree of channel variation factors α.

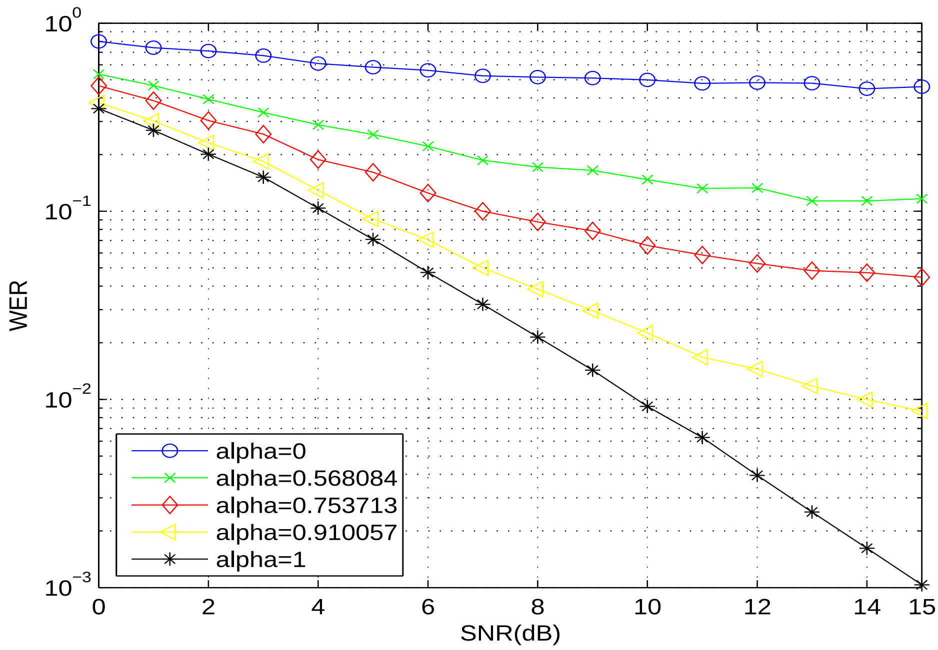

Figure 4.7: The maximum-likelihood word error rates for Code(24, 2, 24) over Channel(2, 12) with different degree of channel variation factors α.

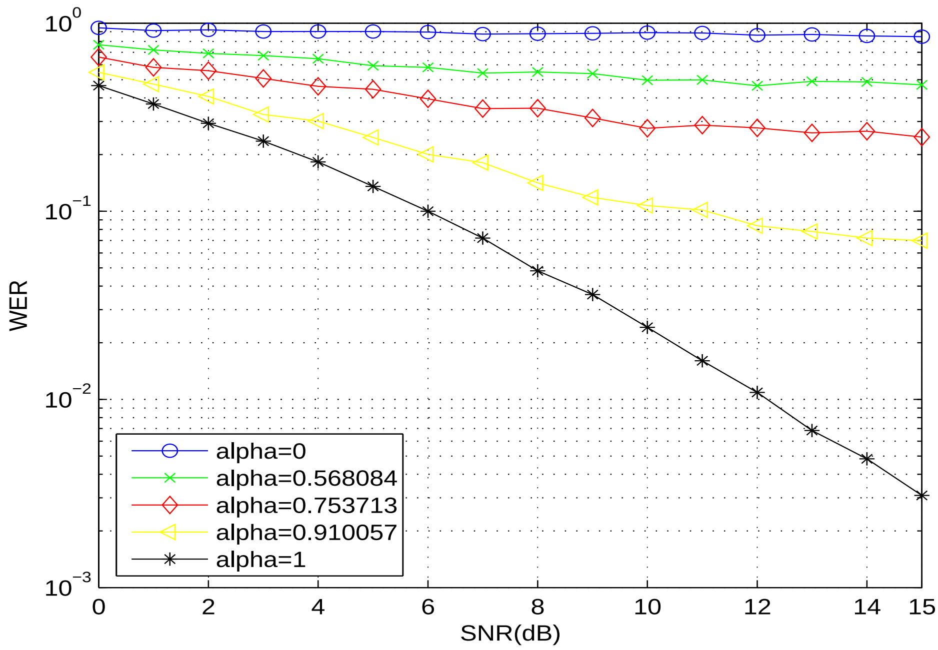

Figure 4.8: The maximum-likelihood word error rates for Code$(12, 2, 12)$ over Channel$(2, 3)$ with different degree of channel variation factors α.

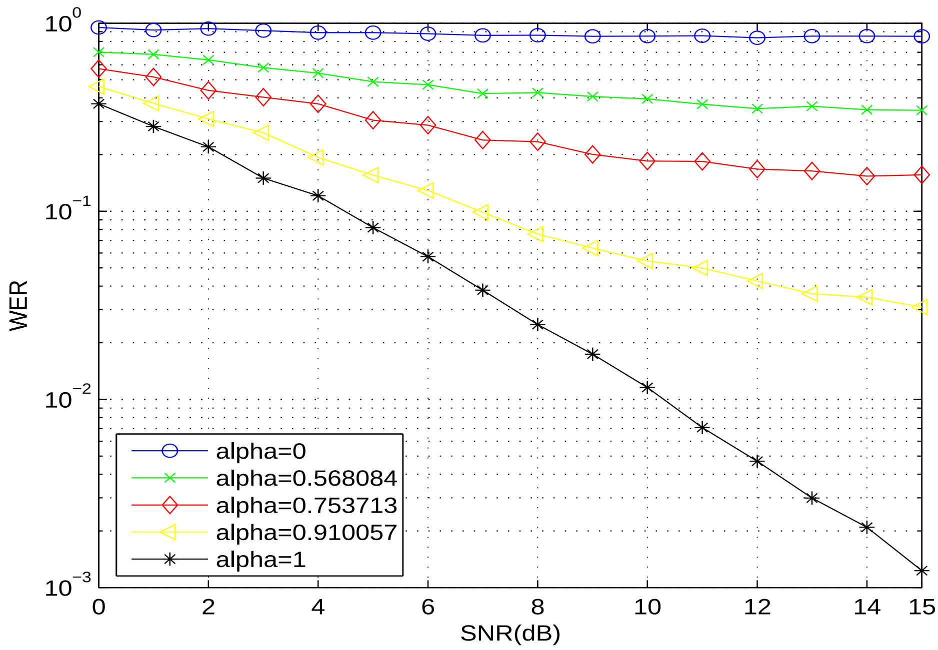

Figure 4.9: The maximum-likelihood word error rates for Code$(16, 2, 16)$ over Channel$(2, 4)$ with different degree of channel variation factors α.

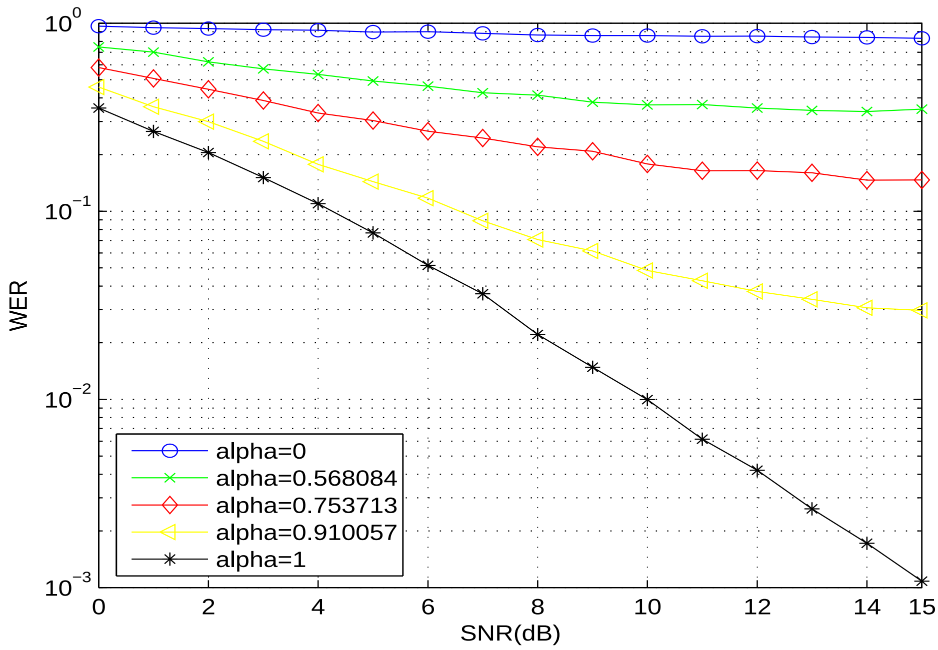

Figure 4.10: The maximum-likelihood word error rates for Code$(20, 2, 20)$ over Channel$(2, 5)$ with different degree of channel variation factors α.

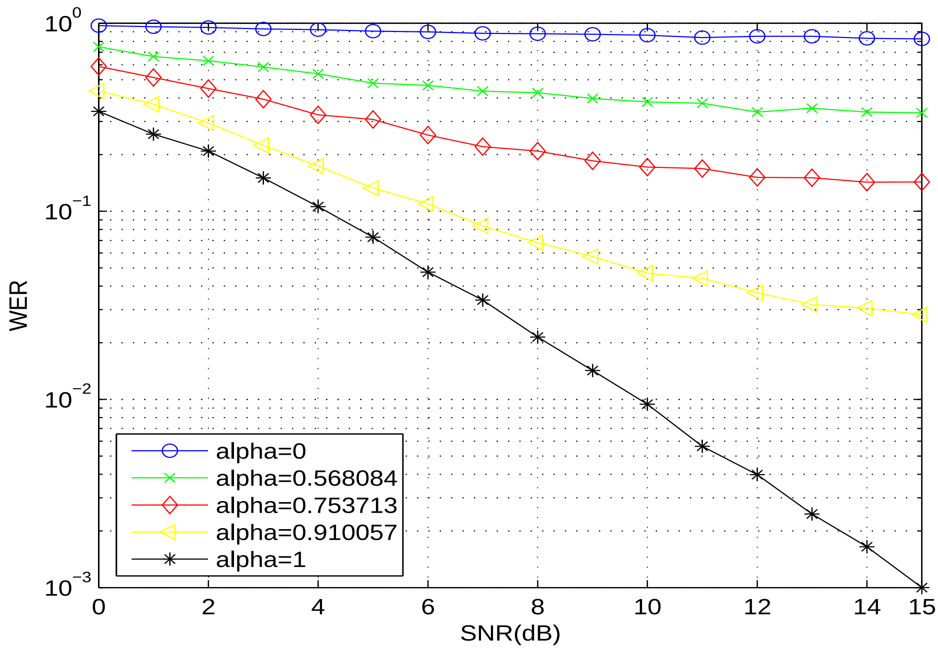

Figure 4.11: The maximum-likelihood word error rates for Code$(24, 2, 24)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

4.2 Codes Designed For Non-Static Fading Channels

In this section, we turn to the examination of codes designed for non-static fading channels.

The performance of Code$(12, 2, 6)$ over Channel$(2, 6)$ is shown in Fig. 4.12. As expected, the performances remain intact for different values of α. We however observe that for SNR larger than 5 dB, the best performance is actually obtained at $\alpha = 0$ as contrary to that observed in the previous section, and the performance degrades as α grows. Since the design of Code$(12, 2, 6)$ in fact assumes an abrupt change of channel coefficients at the middle of the codewords, thereby $[h_{1,1}, h_{2,1}]$ is allowed to be totally different from $[h_{1,2}, h_{2,2}]$ in the code derivation, it is reasonable to yield that the larger the degree of channel variations, the fitter the simulated channel model to the target one of the code design. Yet, the performance deviation between $\alpha = 0$ and $\alpha = 1$ is very small, and in certain case such as Fig. 4.13, the performance improves slightly even with larger α.

Simulations for Code(16, 2, 8), Code(18, 2, 9), Code(20, 2, 10), Code(22, 2, 11), and Code(24, 2, 12) respectively over Channel(2, 8), Channel(2, 9), Channel(2, 10), Channel(2, 11), and Channel(2, 12) are illustrated in Figs. 4.14, 4.15, 4.16, 4.17, and 4.18, respectively. Same performance behaviors as in Fig. 4.12 can be observed from these figures.

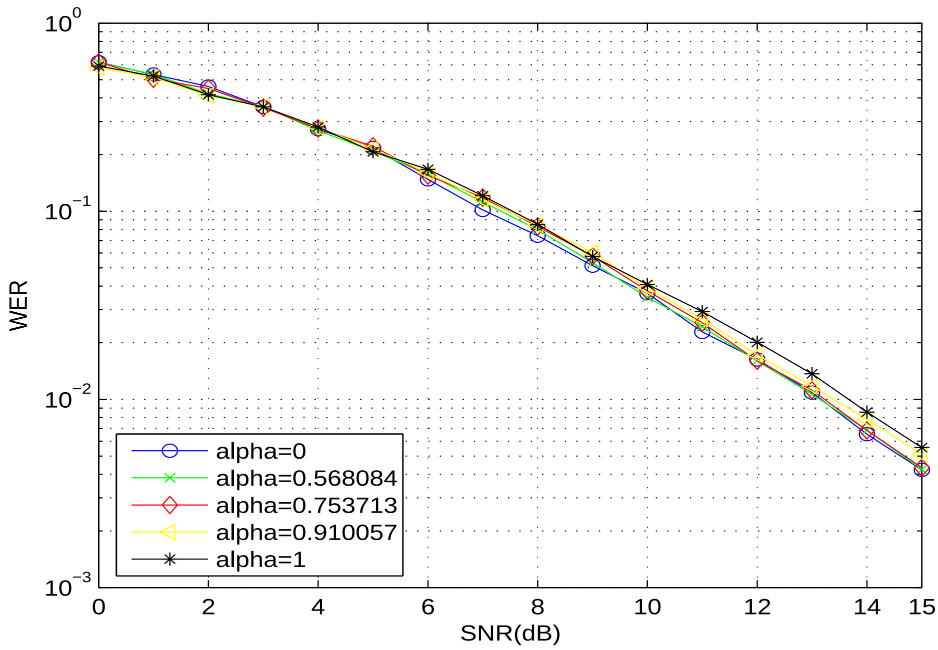

Figure 4.12: The maximum-likelihood word error rates for Code$(12, 2, 6)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

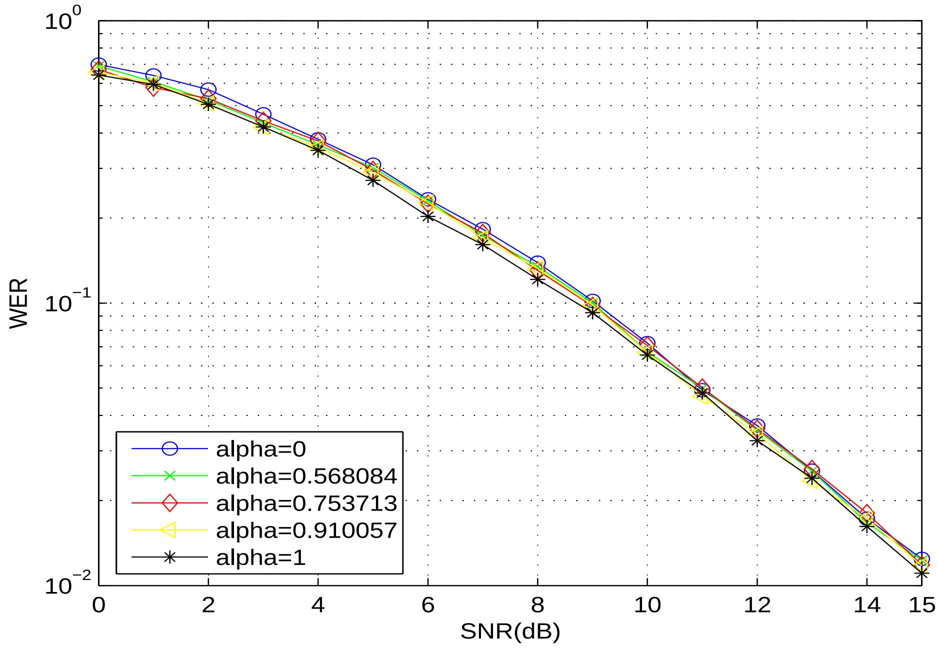

Figure 4.13: The maximum-likelihood word error rates for Code$(14, 2, 7)$ over Channel$(2, 7)$ with different degree of channel variation factors α.

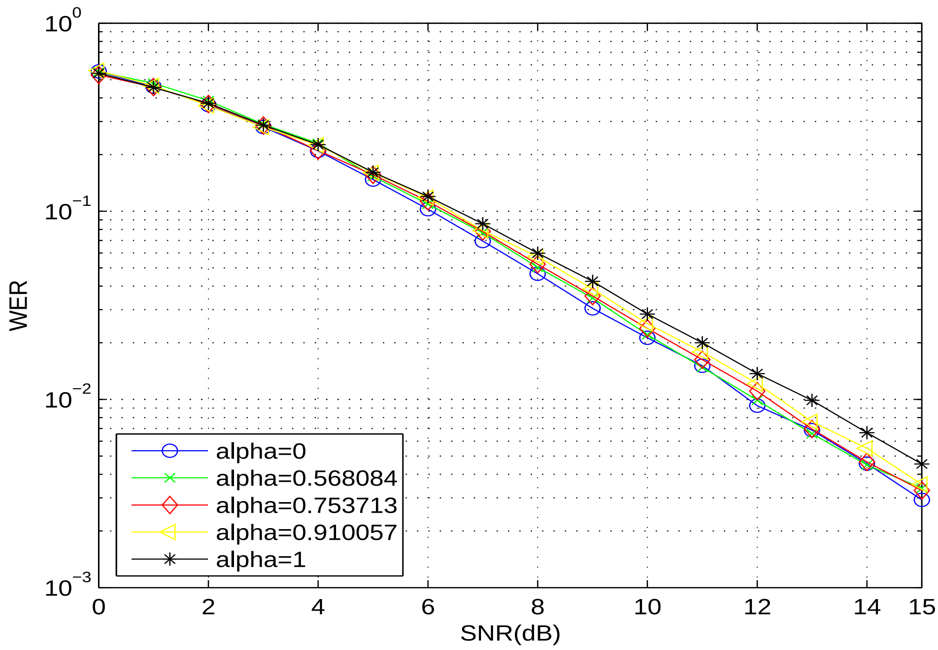

Figure 4.14: The maximum-likelihood word error rates for Code$(16, 2, 8)$ over Channel$(2, 8)$ with different degree of channel variation factors α.

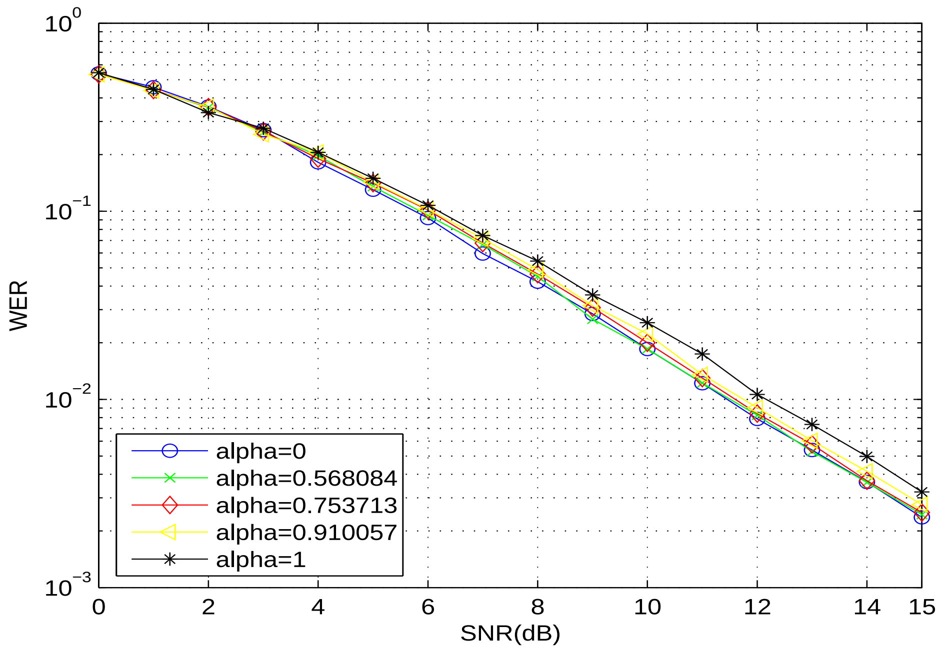

Figure 4.15: The maximum-likelihood word error rates for Code$(18, 2, 9)$ over Channel$(2, 9)$ with different degree of channel variation factors α.

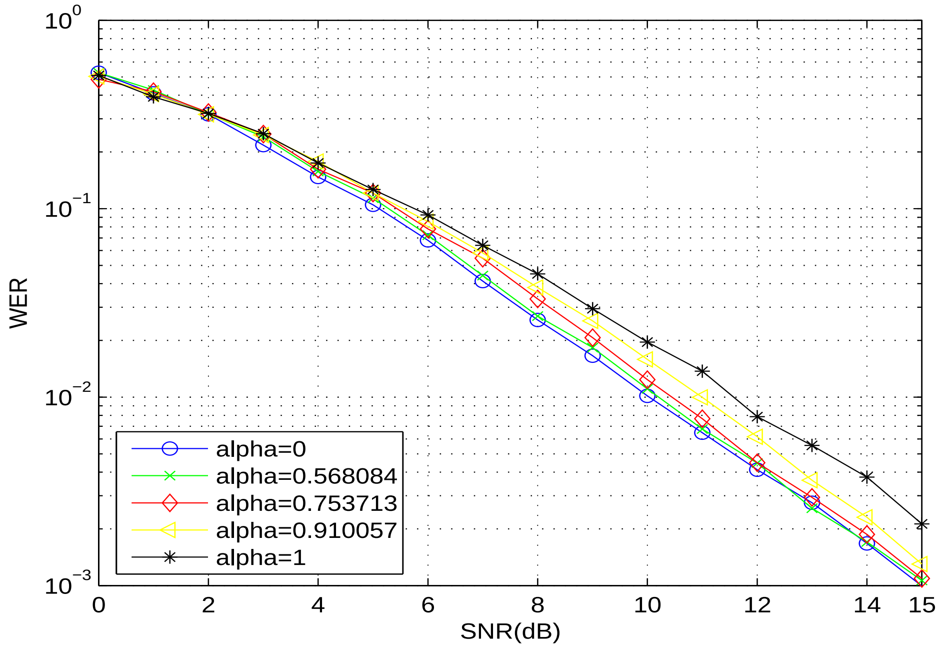

Figure 4.16: The maximum-likelihood word error rates for Code$(20, 2, 10)$ over Channel$(2, 10)$ with different degree of channel variation factors α.

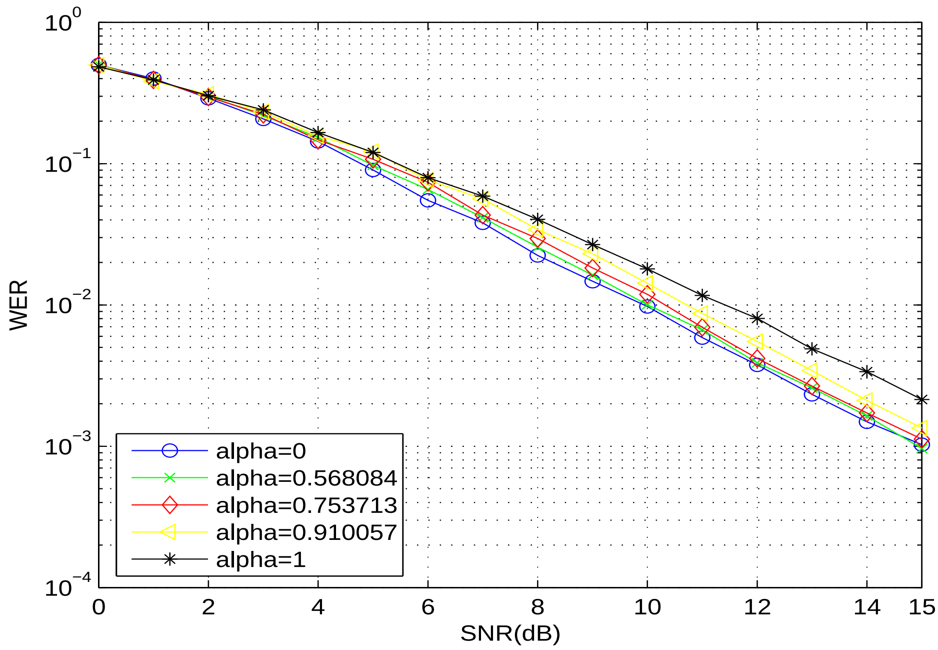

Figure 4.17: The maximum-likelihood word error rates for Code$(22, 2, 11)$ over Channel$(2, 11)$ with different degree of channel variation factors α.

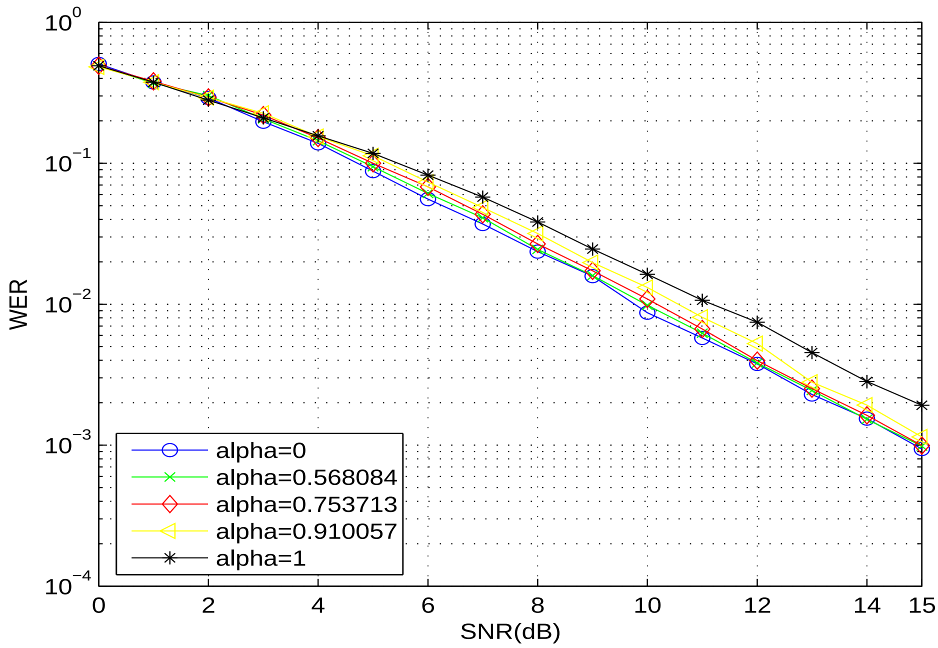

Figure 4.18: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, 12)$ with different degree of channel variation factors α.

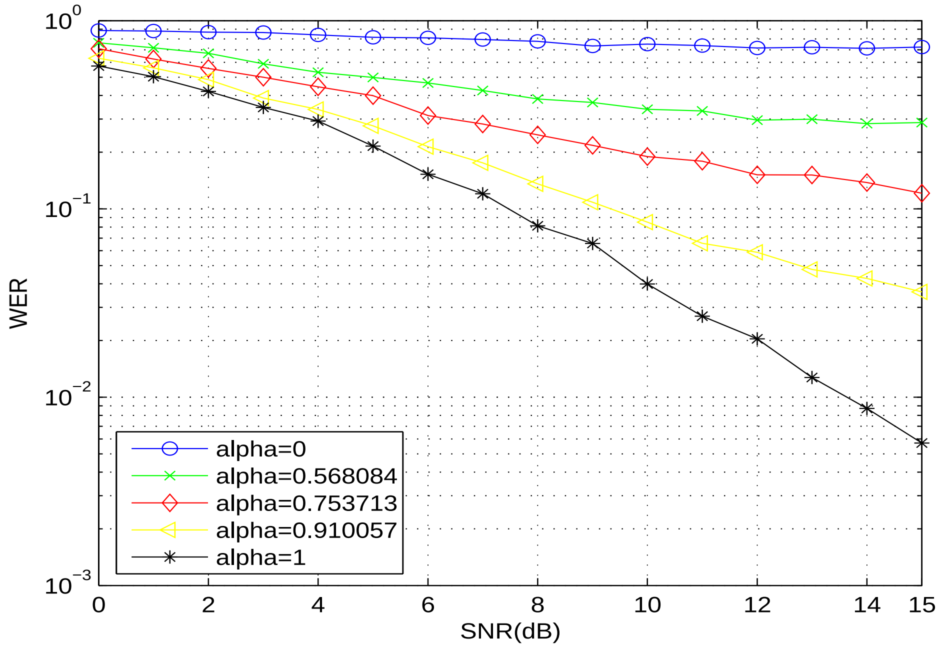

Figure 4.19: The maximum-likelihood word error rates for Code$(12, 2, 6)$ over Channel$(2, 3)$ with different degree of channel variation factors α.

Next, we examine the situation where the update rate of channel coefficients is twice of that is considered in the code design.

The performance of Code$(12, 2, 6)$ over Channel$(2, 3)$ is shown in Fig. 4.19. The simulations indicate that the code designed for Channel$(2, 6)$ performs well only over Channel$(2, 3)$ with $\alpha = 1$, which is equivalent to the code-target Channel$(2, 6)$. This is analogous to what we have obtained in Section 4.1. Similar simulations have been performed for Code(16, 2, 8), Code(20, 2, 10), and Code(24, 2, 12) respectively over Channel(2, 4), Channel(2, 5), and Channel(2, 6), and are summarized respectively in Figs. 4.20, 4.21, and 4.22. Same performance behaviors can be observed from these figures.

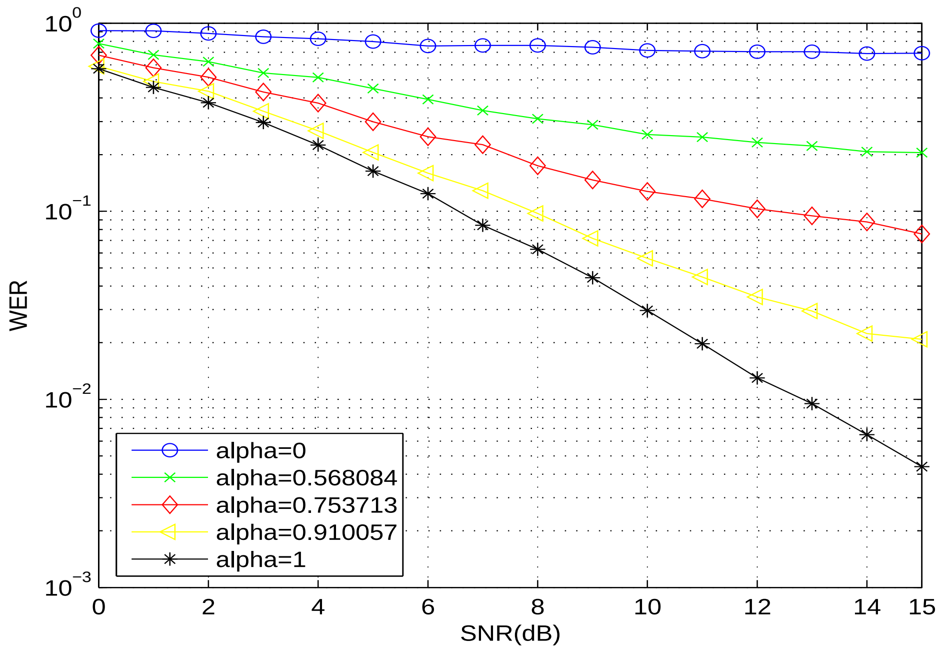

Figure 4.20: The maximum-likelihood word error rates for Code$(16, 2, 8)$ over Channel$(2, 4)$ with different degree of channel variation factors α.

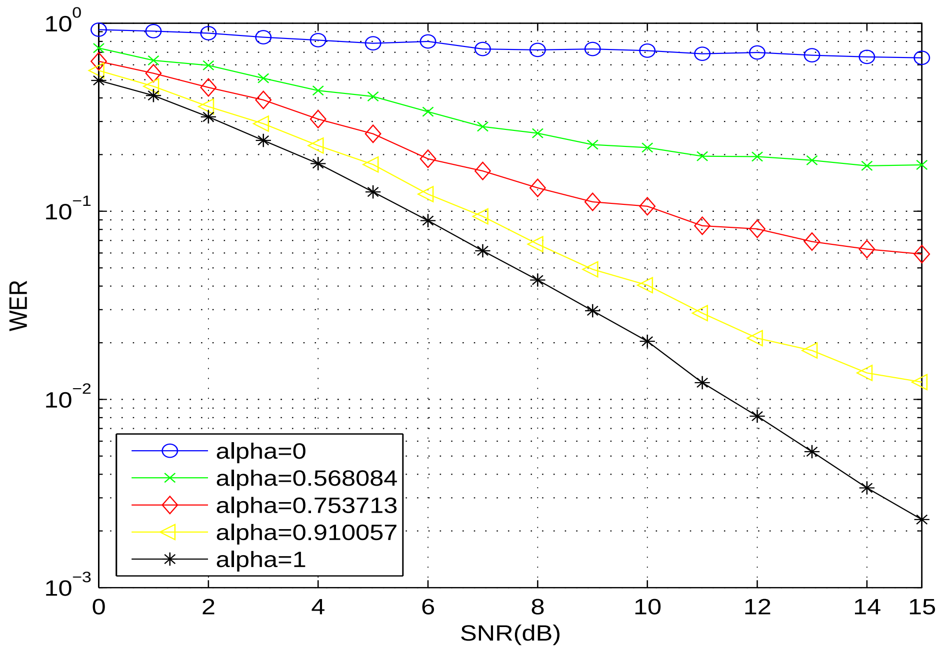

Figure 4.21: The maximum-likelihood word error rates for Code$(20, 2, 10)$ over Channel$(2, 5)$ with different degree of channel variation factors α.

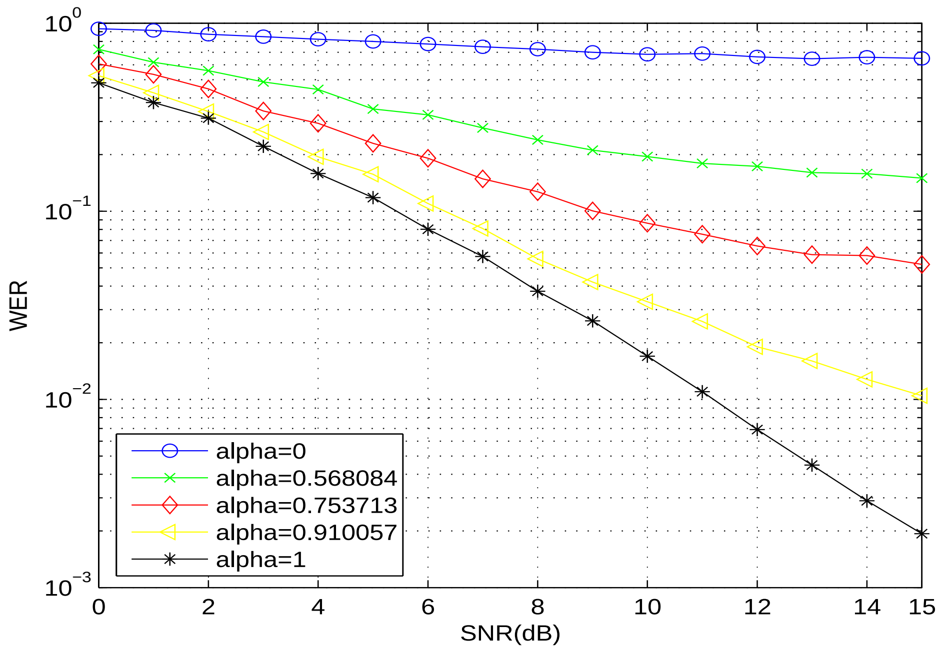

Figure 4.22: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

Now, we demonstrate the performance of Code$(12, 2, 3)$ over Channel$(2, 3)$in Fig. 4.23. Again, when the update rate of the channel coefficients fits that of the code-target channel, the performance remains intact with respect to different values of α. Simulations for Code(16, 2, 4), Code(20, 2, 5), and Code(24, 2, 6) over Channel(2, 4), Channel(2, 5), and Channel(2, 6) illustrated in Figs. 4.24, 4.25, and 4.26, respectively. We observe from these figures that the performances of these codes are the best at $\alpha = 0$, since the resultant simulated channel fits best to the code-target channel.

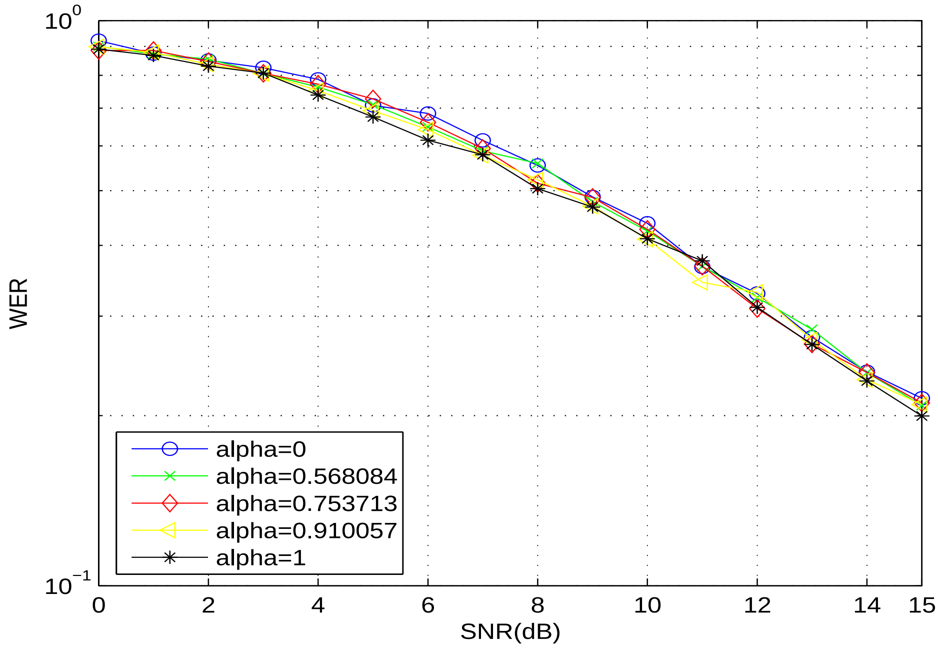

Figure 4.23: The maximum-likelihood word error rates for Code$(12, 2, 3)$ over Channel$(2, 3)$ with different degree of channel variation factors α.

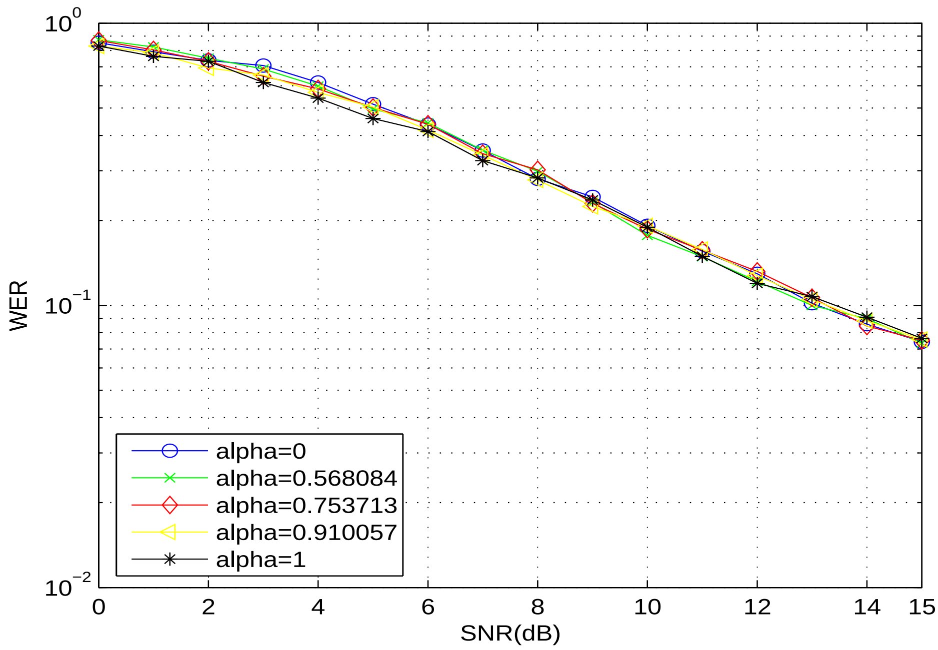

Figure 4.24: The maximum-likelihood word error rates for Code(16, 2, 4) over Channel(2, 4) with different degree of channel variation factors α.

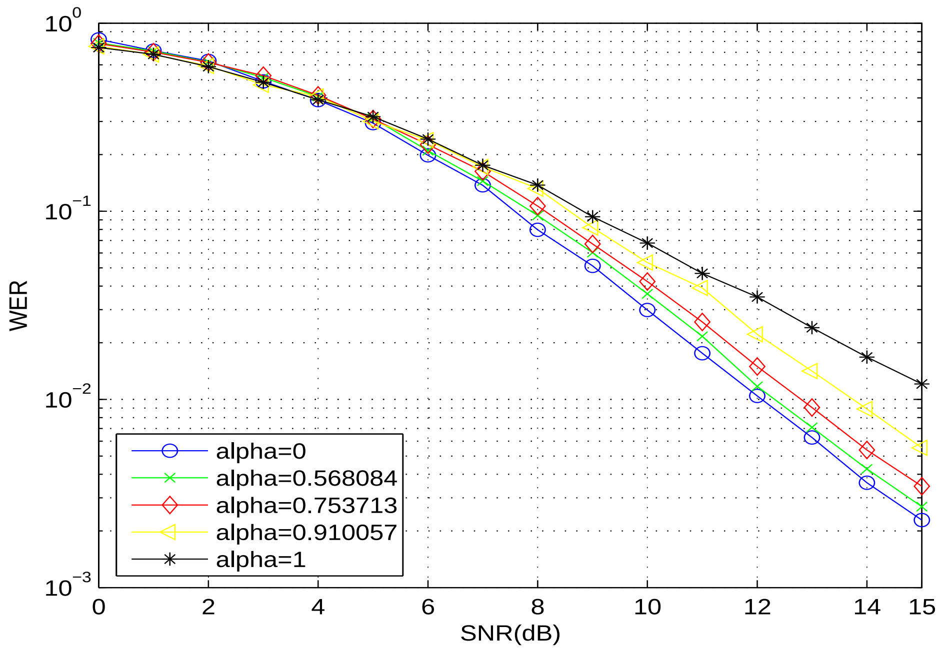

Figure 4.25: The maximum-likelihood word error rates for Code$(20, 2, 5)$ over Channel$(2, 5)$ with different degree of channel variation factors α.

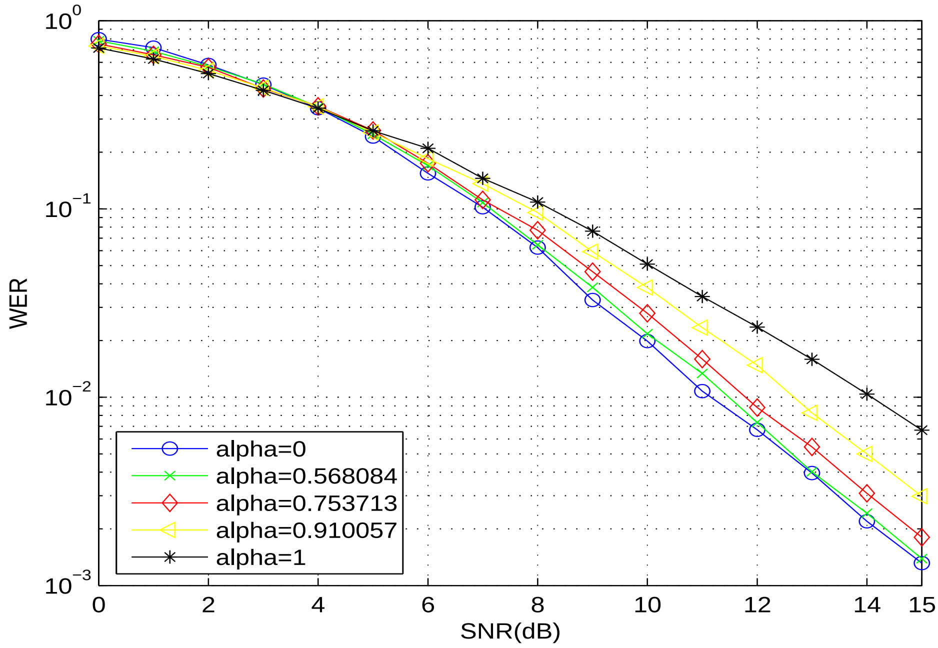

Figure 4.26: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

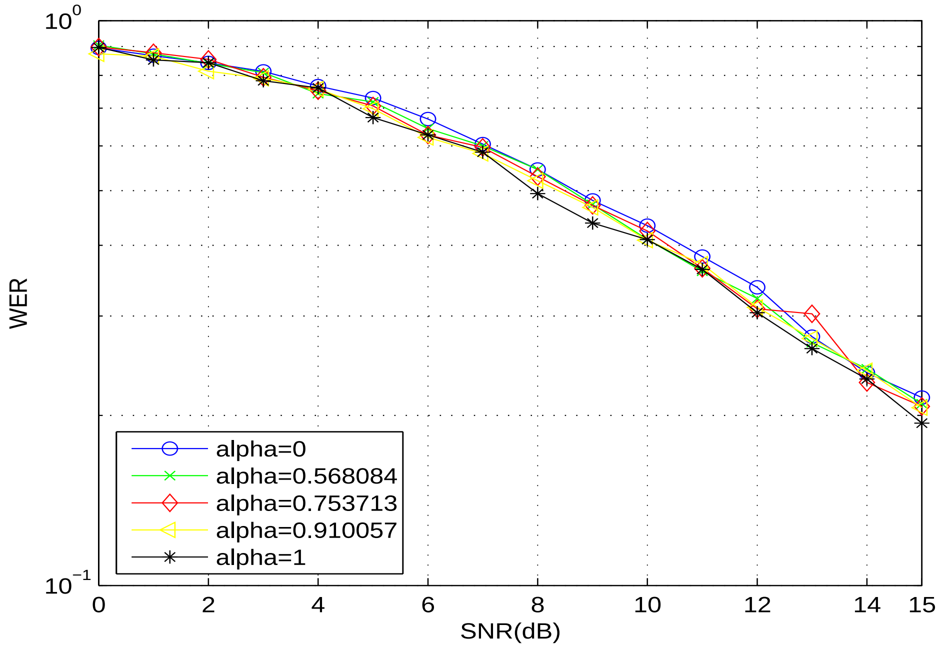

Figure 4.27: The maximum-likelihood word error rates for Code$(12, 2, 3)$ over Channel$(2, 6)$ with different degree of channel variation factors α.

Next, we simulate the case that the update rate of the channel coefficients is slower than that of the code target channel. Figure 4.27 illustrates the performance of Code$(12, 2, 3)$ over Channel$(2, 6)$. Simulation result is almost the same as that in Fig. 4.23, which indicates the robustness of the code design over simulated channels with slower coefficient change. Simulations for Code(16, 2, 4), Code(20, 2, 5), and Code(24, 2, 6) respectively over Channel(2, 8), Channel(2, 10), and Channel(2, 12) are then summarized in Figs. 4.28, 4.29, and 4.30, respectively.

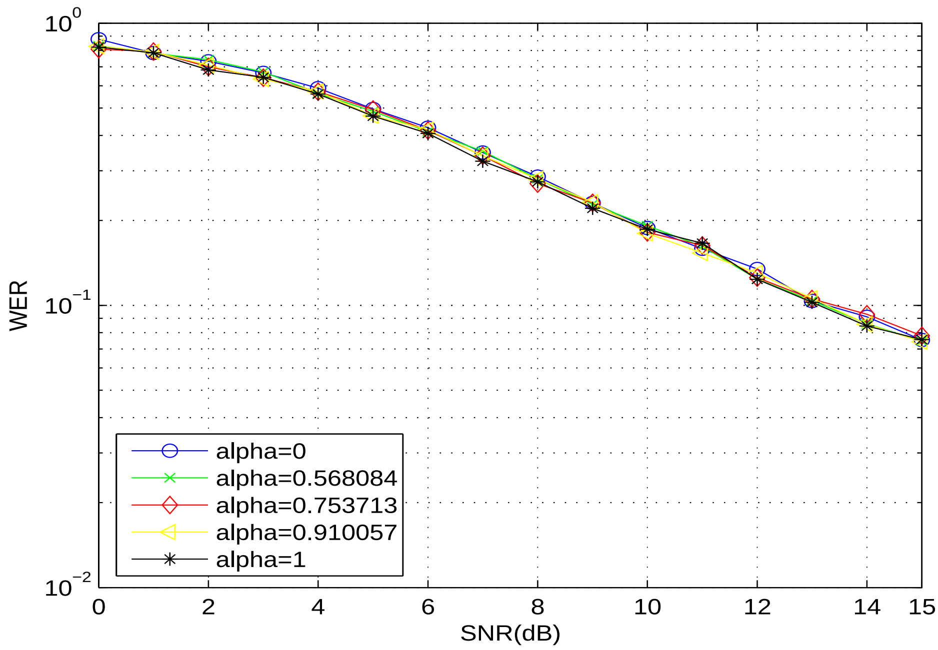

Figure 4.28: The maximum-likelihood word error rates for Code$(16, 2, 4)$ over Channel$(2, 8)$ with different degree of channel variation factors α.

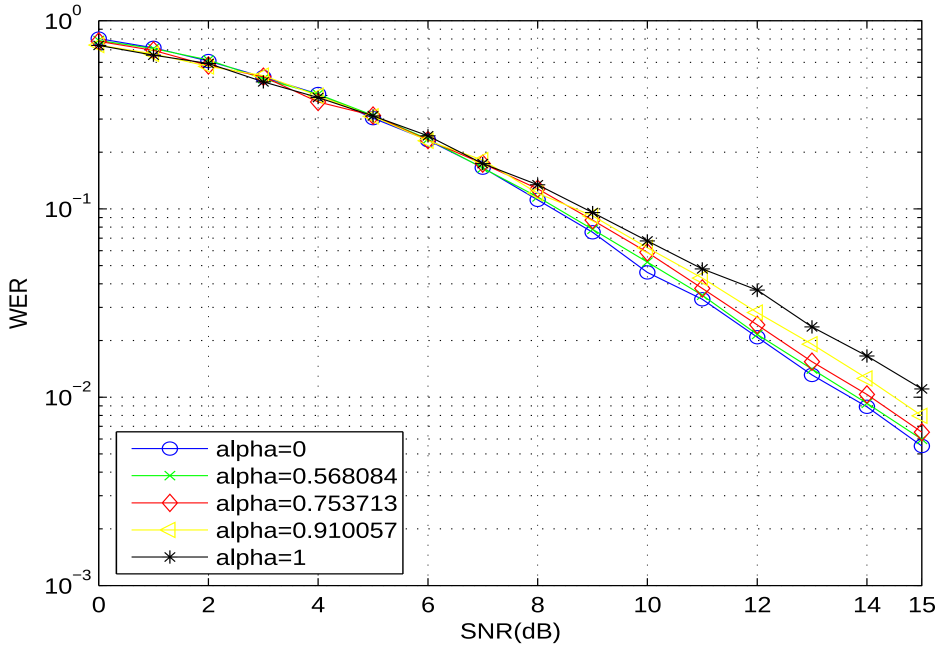

Figure 4.29: The maximum-likelihood word error rates for Code$(20, 2, 5)$ over Channel$(2, 10)$ with different degree of channel variation factors α.

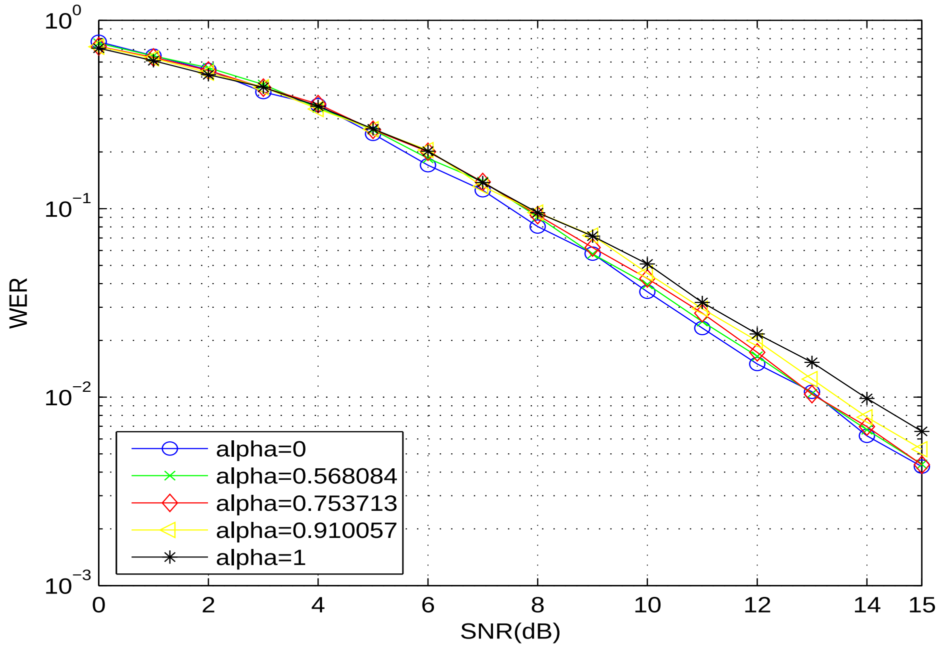

Figure 4.30: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, 12)$ with different degree of channel variation factors α.

In the end, we simulate the performance of designed codes with block length $N = 24$ over different sub-block length Q. The performance of Code$(24, 2, 12)$ over Channel$(2, Q)$ with different sub-block length Q and channel coefficient factor $\alpha = 0$ is illustrated in Fig. 4.31. Simulation results indicate that the code designed for Channel$(2, 12)$ performs well only over Channel$(2, 12)$ and Channel$(2, 24)$ (i.e., quasi-static channel). As Q differs from 12 or 24, the performance degrades considerably. Similar simulations for Code$(24, 2, 12)$ over Channel$(2, Q)$ with different sub-block length Q and three channel coefficient factors $\alpha = 0.568084$, $\alpha = 0.753713$, $\alpha = 0.910057$ are illustrated in Figs. 4.32, 4.33 and 4.34, respectively. These simulation results show that as α increases, the performance of Code$(24, 2, 12)$ over Channel$(2, Q)$ with $Q \neq 24$ tends to be closer to that over Channel$(2, 24)$.

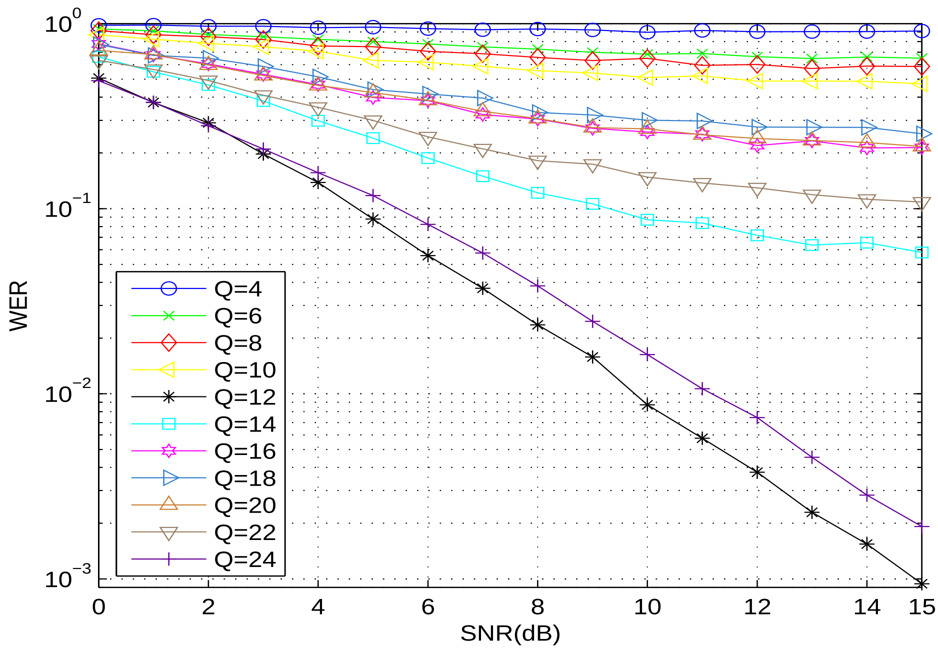

Figure 4.31: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0$ and different values of Q.

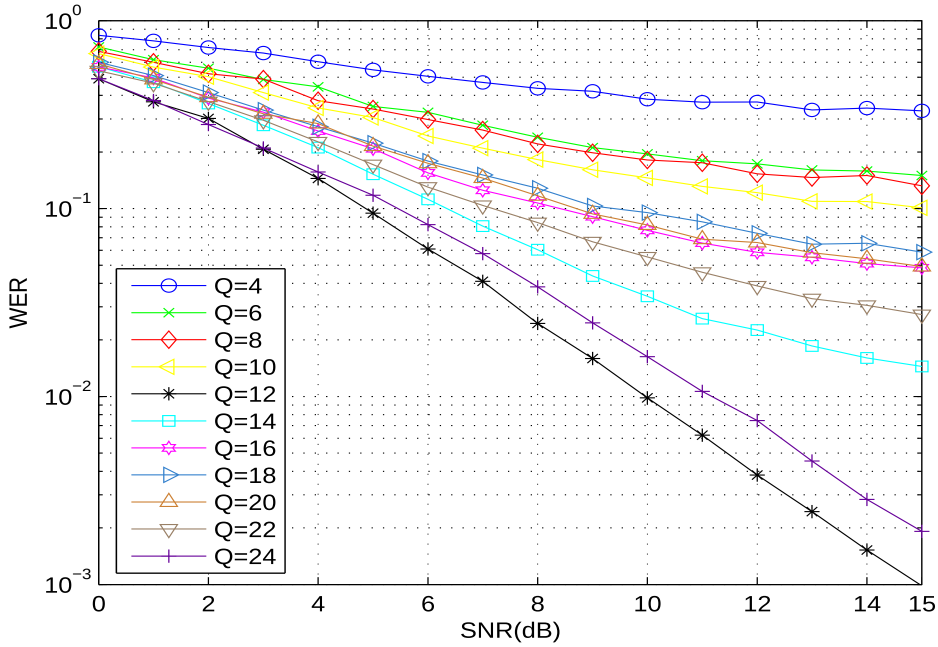

Figure 4.32: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.568084$ and different values of Q.

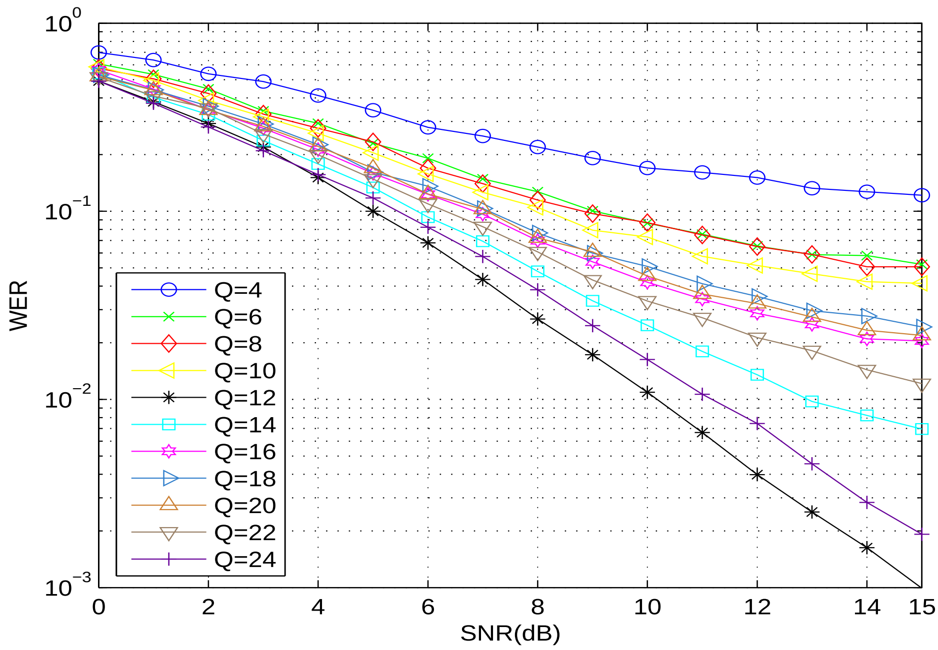

Figure 4.33: The maximum-likelihood word error rates for Code$(24, 2, 12)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.753713$ and different values of Q.

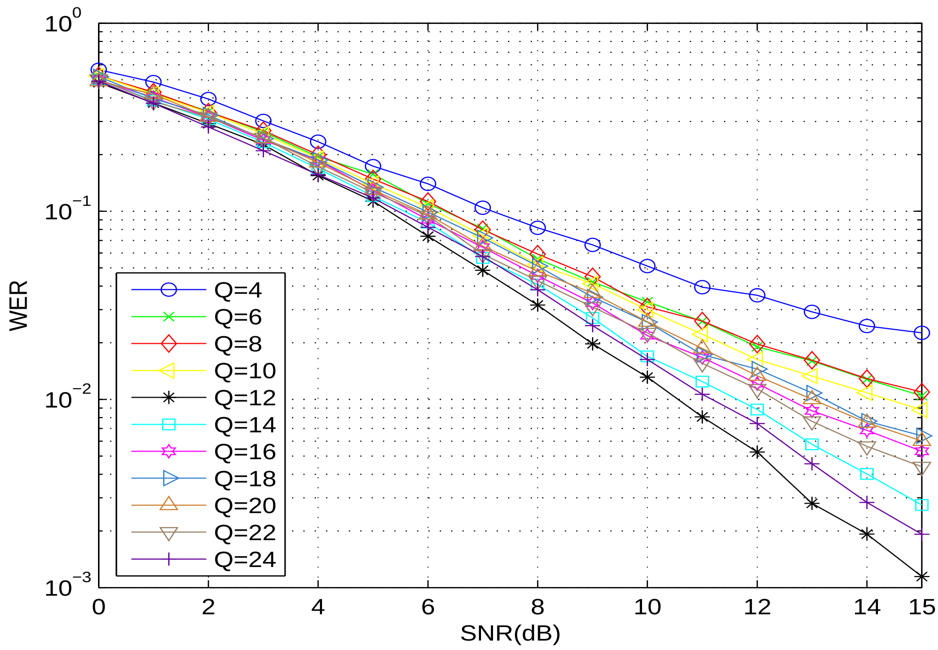

Figure 4.34: The maximum-likelihood word error rates for Code(24, 2, 12) over Channel(2, Q) with channel variation factor $\alpha = 0.910057$ and different values of Q.

The performance of Code$(24,2,6)$ over Channel$(2,Q)$ with different sub-block length Q and channel coefficient factor $\alpha = 0$ is shown in Fig. 4.35. Simulation results indicate that the code designed for Channel$(2,6)$ performs well over Channel$(2,6)$, Channel$(2,12)$, Channel$(2,18)$, and Channel$(2,24)$. The performance degrades when Q differs from 6, 12, 18 or 24. Similar simulations for Code$(24,2,6)$ over Channel$(2,Q)$ with different sub-block length Q and three channel coefficient factors $\alpha = 0.568084$, $\alpha = 0.753713$, $\alpha = 0.910057$ are illustrated in Figs. 4.36, 4.37 and 4.38. From these simulation results, we found that the performance remains well no matter what the value of α is when Q is a multiple of target subblock length for code design.

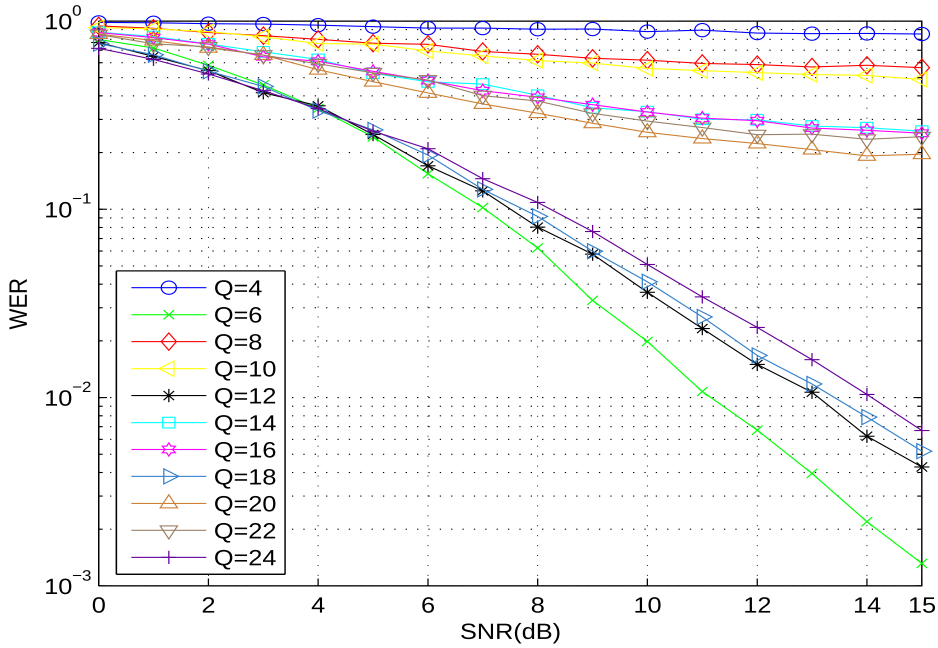

Figure 4.35: The maximum-likelihood word error rates for Code$(24,2,6)$ over Channel$(2,Q)$ with channel variation factor $\alpha = 0$ and different values of Q.

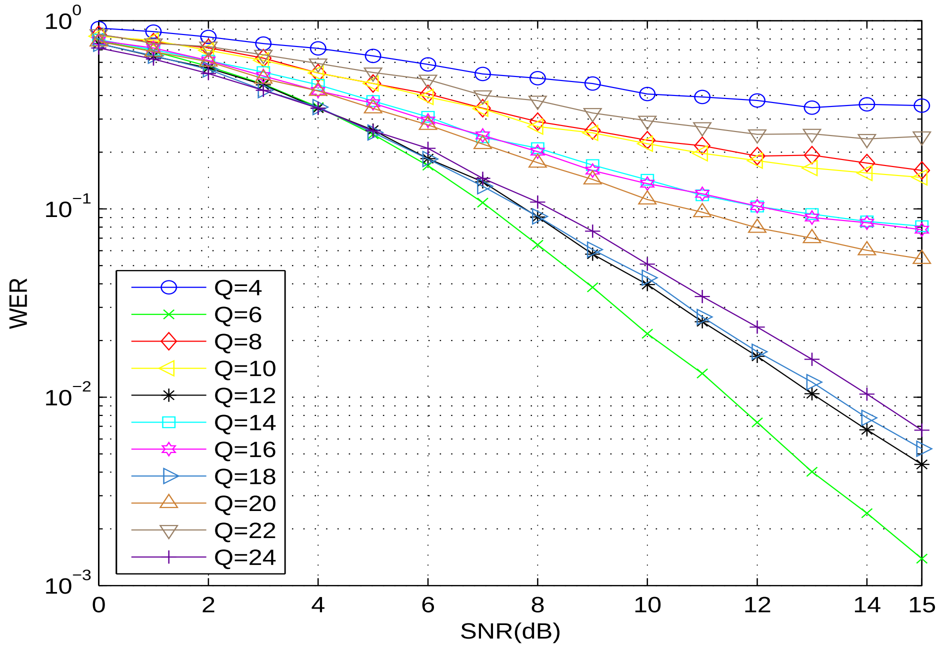

Figure 4.36: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.568084$ and different values of Q.

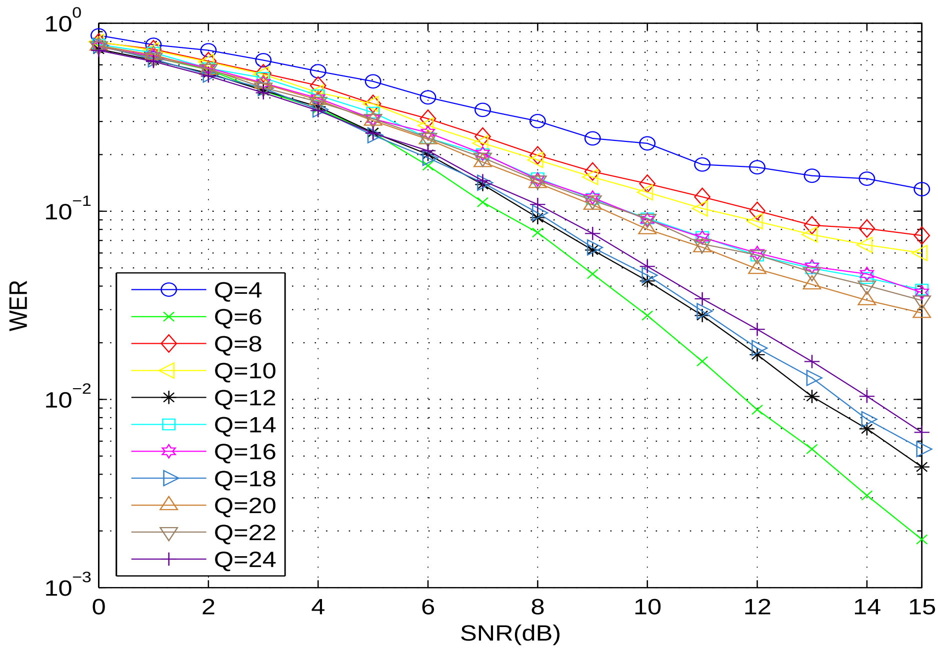

Figure 4.37: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.753713$ and different values of Q.

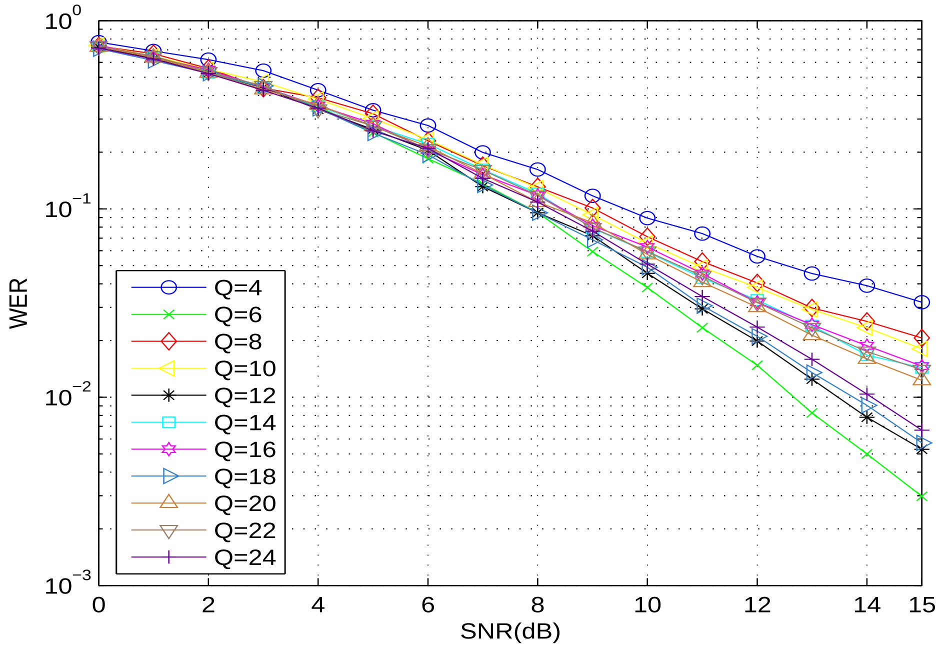

Figure 4.38: The maximum-likelihood word error rates for Code$(24, 2, 6)$ over Channel$(2, Q)$ with channel variation factor $\alpha = 0.910057$ and different values of Q.

Chapter 5

Conclusions

In this work, a binary block code design for combined channel estimation and error protection, which is extended from [13] specifically for non-static fading channels, is proposed and examined. Simulations hint that as long as the update rate of the channel coefficients is equal to or slower than that of the code target channel, the performance remains robust. However, when the channel coefficients change faster than those of the channel that the code design is presumed, the performance degrades considerably. The future work is to examine whether the code proposed is robust for non-stationary fading channels in which the channel coefficients change in an non-stationary non-periodic fashion.

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