Similarity of Discrete Gilbert-Elliot and Polya Channel Models to Continuous Rayleigh Fading Channel Model

Sun, Pen-Ting

Advisor: Prof. Po-Ning Chen

Department of Communications Engineering
National Chiao Tung University, Taiwan 30050, R.O.C.

June, 2002

OUTLINE

- Introduction
- Channel Similarity: Analysis Based on Divergence
- Channel Similarity: Analysis Based on Error Free Run
- Simulation Results and Discussions
- Conclusions and Future Works

Introduction

- When the transmission rate for wireless communication is prohibitively low, the channel is sufficiently modelled as an additive white Gaussian noise (AWGN) channel.
- After being sampled, the AWGN channel is transformed to a discrete-in-time additive Gaussian channel, for which the additive noise at the present time instance is not statistically affected by the noises of other time instances. This noise phenomenon is often termed as *memoryless*.
- As the transmission rate is getting higher due to the demand of simultaneous transmission of voice and data, the channel noise is no longer *memoryless* in time, which results in a channel that exhibits *memory*. Thus, the classical *memoryless* binary symmetric channel is no longer effective in characterizing the coding transmission behavior for such channels.
- A need for developing a new channel model that well approximates a true channel with memory is therefore arose.

Channel Models with Memory

- Three well-known channel models with memory have already been proposed in the literature:
  - *Gilbert model*,
  - *Gilbert-Elliot model*, and
  - *Fritchman model*.

  These three channel models are in fact variations of hidden Markov models or finite-state channel models.
- Due to their simplicity, these three channel models are extensively used in mathematical analysis and computer simulations.

The Polya Model

- In 1994, F. Alajaji and T. Fuja re-visited a channel model with memory, named *Polya channel*. Originally, the Polya model was used to model the spread of a contagion.
- Alajaji and Fuja noticed that this model is apt to approximate those channels whose errors, once they occur, increase the probability of the future errors. This provides an alternative channel model with memory for use of computer evaluation of coding performance.

Motivations

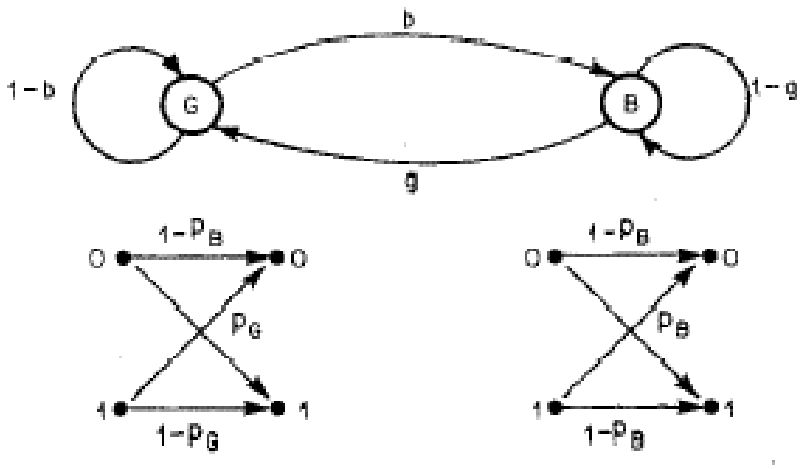
- Recently, some European researchers adopted the Polya model for simulating their joint source-channel coding scheme (Iordache, Tabus and Astola 2001). They have raised the question that "Is the binary Polya model more suitable to be treated as a binary-quantized simplification of a discrete-in-time continuous channel such as Rayleigh than the Gilbert-type model?"
- Their question motivates our research on the similarity of the Polya and Gilbert-Elliot channels to the Rayleigh fading channel.

Channel Similarity: Analysis Based on Divergence

We begin with a brief review of these channel models.

The Gilbert-Elliott Model

- The Gilbert-Elliott model is basically a hidden Markov chain in its nature.
- A simple description of a hidden Markov chain can be given as follows.
  - A Markov chain is a stochastic process with a countable state space.
  - The Markov chain resides in one of the states at each time instance.
  - The probability of making a transition to another state is a (time-stationary) function of the present state.
  - The statistics of the observation process is determined by the present state of the associated Markov process.



The State-Transition Diagram of the Gilbert-Elliott model.
(Source: Mordechai 1989)

- As shown in the above figure, the Gilbert-Elliott model is a two-state hidden Markov chain, in which the two states are denoted by G (good state) and B (bad state). The state transition probability satisfies $\Pr(B|G) = b$ and $\Pr(G|B) = g$.
- In each state, the Gilbert-Elliott model acts exactly like a binary symmetric channel. The crossover probabilities for the good state and the bad state are denoted by P_G and P_B, respectively.

- Usually, p_G should be made relatively small to p_B as it is assumed that state G is a good and favored state. Also, g is usually small; hence, the bad state tends to persist, which emulates the *burst error state* in practical computation system.

- As long as one of g and b is positive, the Gilbert-Elliott channel model has memory. The state process for the Gilbert-Elliott model can be made stationary by assuming the initial state probability

$$\Pr(S_0 = G) = \frac{g}{g+b} \quad \text{and} \quad \Pr(S_0 = B) = \frac{b}{g+b}.$$

- Accordingly, the Gilbert-Elliott model is a model with four parameters, i.e., g, b, p_G and p_B.

The Polya Model

- The Polya model is a binary additive communication channel with memory. Specifically,

$$Y_i = X_i \oplus Z_i \qquad \text{for} \quad i = 1, 2, 3, ...$$

  where the ith channel output Y_i is a modulo-2 sum of the ith channel input $X_i \in \{0, 1\}$ and the ith noise symbol $Z_i \in \{0, 1\}$. The input sequence $\{X_i\}_{i=1}^{\infty}$ and the noise sequence $\{Z_i\}_{i=1}^{\infty}$ are independent of each other.

- The error spread in the Polya model is similar to the spread of a contagious disease through a population, where the occurrence of an error in the present time increases the probability of error occurrence in the future.

The binary noise sequence $\{Z_i\}_{i=1}^{\infty}$ is drawn as follows:

- An urn originally contains R red balls (sick persons) and S black balls (healthy persons).
- The balls are drawn successively from the urn. After each draw, $1 + \Delta$ balls of the same color as was just drawn are returned to the urn.
- Then the noise sequence $\{Z_i\}_{i=1}^{\infty}$ is given by

$$Z_i = \begin{cases} 1, & \text{if the } i\text{th ball drawn is red} \\ 0, & \text{if the } i\text{th ball drawn is black.} \end{cases}$$

- It can be seen from the above description that the Polya model is determined by three parameters, R, S and Δ. Hence, the number of parameters for the Polya model is less than that of the Gilbert-Elliott model.

- Often, it is assumed that $\Delta > 0$, which reflects the contagion condition.
- Also, the number of red ball R is usually less than the number of black ball S initially.
- According to [Alajaji & Fuja 1994], the block transition probability of the Polya model is given by:

$$P(Y^n = y^n \mid X^n = x^n) = \frac{\Gamma(1/\delta)\Gamma(\rho/\delta + d)\Gamma(\sigma/\delta + n - d)}{\Gamma(\rho/\delta)\Gamma(\sigma/\delta)\Gamma(1/\delta + n)},$$

where

$$\delta = \Delta/(R+S), \quad \rho = 1 - \sigma = R/(R+S),$$

and d is the Hamming distance between x^n and y^n, and $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$, defined for $x > 0$, is the well-known Gamma function.

Channel Similarities

- A reasonable quantitative index of our channel similarity should be defined based on the output error probability due to a transmission scheme.
- Specifically, if two channel models are completely *similar*, then their resultant error probabilities due to the same transmission scheme should be close.
- Also, a channel model can resemble or replace the other channel model, if for any model parameters chosen for the second channel model, there exists some model parameters for the first channel model such that the two channels yield the same error probability due to a transmission scheme.
- Based on these conceptual guideline, the similarity of the Gilbert-Elliott channel model and the Polya channel model is partially derived.

Preliminaries

- **Definition 1 (probability of error)** *The probability of error for an* (f_n, g_n) *code with a prior distribution over the codebook* P_{X^n}*, transmitted via channel* $Q_{Y^n|X^n}$*, is defined as*

$$P_e(f_n, Q_{Y^n|X^n}, g_n) = \sum_{i=1}^{M_n} P_{X^n}(f_n(i)) \cdot \lambda_i(f_n, Q_{Y^n|X^n}, g_n),$$

where

$$\lambda_i(f_n, Q_{Y^n|X^n}, g_n) \stackrel{\triangle}{=} \sum_{\left\{y^n \in \mathcal{Y}^n \,:\, g_n(y^n) \neq i\right\}} Q_{Y^n|X^n}(y^n|f_n(i)).$$

Furthermore,

$$P_e^*(f_n, Q_{Y^n|X^n}) \stackrel{\triangle}{=} \min_{g_n} P_e(f_n, Q_{Y^n|X^n}, g_n),$$

where the minimum probability of error corresponding to channel encoder $f_n(\cdot)$ *and* $P_{X^n} Q_{Y_n|X^n}$ *is achieved by the MAP decoder.*

- Although the error probability is the performance index that a system designer concerns most, its derivation often much involves with the encoding function taken.
- Instead, we found that if the Kullback-Leiber divergence of the two channel statistics is small, the error probability difference for a transmission scheme over the two channels shall be small.

- **Theorem 1** *Fix a channel encoder $f_n(\cdot)$ and a distribution over the codebook P_{X^n}. For any two channels respectively defined through transition probabilities $P_{Y^n|X^n}$ and $Q_{Y^n|X^n}$,*

$$\left|P_e^*(f_n, P_{Y^n|X^n}) - P_e^*(f_n, Q_{Y^n|X^n})\right| \leq \frac{1}{\sqrt{2}} \cdot D^{1/2}\left(P_{Y^n|X^n} \left\| Q_{Y^n|X^n} \right| P_{X^n}\right),$$

where $D\left(P_{Y^n|X^n} \left\| Q_{Y^n|X^n} \right| P_{X^n}\right)$ is the conditional Kullback-Leibler divergence between $P_{Y^n|X^n}$ and $Q_{Y^n|X^n}$.

Parameterized Channel Model

- We denote the channel statistics W^n (i.e., transition probability of channel output due to channel input), together with the model parameters $\vec{\lambda}$ associated with it, by $W^n(\vec{\lambda})$.

- Now for a true channel W^n_{true} and a given computer-resolvable channel model $W^n(\vec{\lambda})$ with model parameter vector $\vec{\lambda}$, we say that the channel model **"fits"** to the true channel model, if there exists parameters $\vec{\lambda}$ such that the resultant bit error rates for a transmission scheme f_n under test due to W^n_{true} and $W^n(\vec{\lambda})$ are close, i.e.,

$$\min_{\vec{\lambda}} \left| P_e^*(f_n, W^n_{\text{true}}) - P_e^*(f_n, W^n(\vec{\lambda})) \right| < \varepsilon$$

  for ε small, which is ensured by

$$\min_{\vec{\lambda}} D(W^n_{\text{true}} \| W^n(\vec{\lambda}) | X^n) < 2\varepsilon^2,$$

  where X^n places equal weight on the codewords.

- Hence, $\min_{\vec{\lambda}} D(W^n_{\text{true}} \| W^n(\vec{\lambda}) | X^n)$ may be served as a good index of the **"fitness"** for the respective channel model to the true channel.

Channel Resemblance

- Suppose that there is a channel model $W_1^n(\vec{\lambda})$ that is widely accepted (because of its fitness to the true channel model).
- In case a person proposes another channel model $W_2^n(\vec{\rho})$ with, say, less complexity.
- If for any $\vec{\lambda}$ satisfying $\left|P_e^*(f_n, W_{\text{true}}^n) - P_e^*(f_n, W_1^n(\vec{\lambda}))\right| < \gamma$,

$$\min_{\vec{\rho}} \left|P_e^*(f_n, W_1^n(\vec{\lambda})) - P_e^*(f_n, W_2^n(\vec{\rho}))\right| < \varepsilon \text{ for some } \varepsilon > 0 \text{ fixed,}$$

then

$$\min_{\vec{\rho}} |P_e^*(f_n, W_{\text{true}}^n) - P_e^*(f_n, W_2^n(\vec{\rho}))| < \varepsilon + \gamma.$$

- The above result can be interpreted as follows. Since $W_1^n(\vec{\lambda})$ is believed to be a good model, γ should be small (at last for one proper choice of $\vec{\lambda}$).
- If $W_2^n(\vec{\rho})$ is shown to be close to $W_1^n(\vec{\lambda})$, i.e., ε is small, then $W_2^n(\vec{\rho})$ is $(\varepsilon+\gamma)$-close to the true channel.
- Consequently, $\min_{\vec{\rho}} \left| P_e^*(f_n, W_1^n(\vec{\lambda})) - P_e^*(f_n, W_2^n(\vec{\rho})) \right|$ can be served as a guideline for how "good" the model $W_2^n(\vec{\rho})$ can **resemble** (or replace) the model $W_1^n(\vec{\lambda})$.
- In extreme case, if model $W_2^n(\vec{\rho})$ can completely resemble model $W_1^n(\vec{\lambda})$ (i.e., $\min_{\vec{\rho}} \left| P_e^*(f_n, W_1^n(\vec{\lambda})) - P_e^*(f_n, W_2^n(\vec{\rho})) \right|$ can be made arbitrarily small) and its number of model parameters is smaller (or has less complexity), then one can surely use the less complex model in simulations.
- Note that "model $W_2^n(\vec{\rho})$ well-resembles model $W_1^n(\vec{\lambda})$" does not necessarily implies "model $W_1^n(\vec{\rho})$ well-resembles model $W_2^n(\vec{\lambda})$." In other words, the implication is **not** symmetric.

- Based on the above discussions, we may define the **"resemblance"** of model $W_2^n(\vec{\rho})$ to model $W_1^n(\vec{\lambda})$ as the quantity:

$$\max_{\vec{\lambda}} \min_{\vec{\rho}} \left| P_e^*(f_n, W_1^n(\vec{\lambda})) - P_e^*(f_n, W_2^n(\vec{\rho})) \right|,$$

  For simplicity of derivation, the above index is replaced by:

$$\max_{\vec{\lambda}} \min_{\vec{\rho}} D(W_1^n(\vec{\lambda}))\|W_2^n(\vec{\rho}))|X^n).$$

- The above quantity in situation that model $W_1^n(\vec{\lambda})$ is the Gilbert-Elliott model and model $W_2^n(\vec{\rho})$ is the Polya model will be examined. As aforementioned, the quantity can serve as a guide on how well the Polya model resembles the Gilbert-Elliott model.

Resemblance of Gilbert-Elliott and Polya Models

- The Gilbert-Elliott and Polya models can be expressed as

$$Y_i = X_i \oplus Z_i \quad \text{and} \quad Y_i = X_i \oplus \tilde{Z}_i,$$

where $Z^n = (Z_1, Z_2, \ldots, Z_n)$ and $\tilde{Z}^n = (\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_n)$ are the corresponding Gilbert-Elliott and Polya urn drawing sequences, respectively. A known consequence of the additive binary channel with independence between channel input and noise sequences is that

$$P_{Y^n|X^n}(y^n|x^n) = P_{Z^n}(x^n \oplus y^n)$$

for any additive binary noise sequence Z^n.

- We can therefore re-write the divergence between the Gilbert-Elliott model and the Polya model as

$$\begin{aligned} & D(W_1^n || W_2^n | X_n) = D(Z^n \| \tilde{Z}^n) \\ = \quad & E_{Z^n}\left[\log \frac{P_{Z^n}(Z^n)}{P_{\tilde{Z}^n}(Z^n)}\right] = E\left[\log P_{Z^n}(Z^n)\right] - E\left[\log P_{\tilde{Z}^n}(Z^n)\right]. \end{aligned}$$

- By denoting $s^n = (s_1, s_2, \ldots, s_n) \in \{G, B\}^n$ the states of the Gilbert-Elliott noise process Z^n, we obtain

$$E\left[\log P_{Z^n}(Z^n)\right] = -\sum_{k=0}^{n} \Pr\{\ell_G(S^n) = k\}\left[k \cdot H_b(P_G) + (n-k)H_b(P_B)\right],$$

where $\ell_G(s^n)$ is the number of $s_i = G$ for $1 \le i \le n$, and $H_b(x) = -x\log(x) - (1-x)\log(1-x)$ is the binary Entropy function.

- On the other hand,

$$\begin{aligned}\frac{1}{n}E\left[\log P_{\tilde{Z}^n}(Z^n)\right] &= \frac{1}{n}\log\frac{\Gamma(1/\delta)}{\Gamma(\rho/\delta)\Gamma(\sigma/\delta)} \\ &\quad + \frac{1}{n}E\left[\log\frac{\Gamma(\rho/\delta + D)\Gamma(\sigma/\delta + n - D)}{\Gamma(1/\delta + n)}\right],\end{aligned}$$

where D is the number of 1 in Z^n.

- Here, the general solution for $\Pr\{\ell_G(S^n) = k\}$ may be complicated in its expression. So we reduce to the simplest case, as a trial, of $g = b = 1/2$ and $p_G = p_B = 1/2$, which gives us $\Pr\{\ell_G(S^n) = k\} = \binom{n}{k}2^{-n}$. As a result,

$$\begin{aligned}
\frac{1}{n}E\left[\log P_{Z^n}(Z^n)\right] &= -\sum_{k=0}^{n}\binom{n}{k}2^{-n}\left[\frac{k}{n}\cdot H_b(P_G) + \left(1-\frac{k}{n}\right)H_b(P_B)\right] \\
&= -H_b(p_G)\sum_{k=0}^{n}\frac{k}{n}\binom{n}{k}2^{-n} - H_b(p_B)\sum_{k=0}^{n}\frac{n-k}{n}\binom{n}{k}2^{-n} \\
&= -\frac{H_b(p_G)+H_b(P_B)}{2} = -\log(2).
\end{aligned}$$

- In addition, by letting $R = S = \Delta$,

$$\begin{aligned}
\frac{1}{n}E\left[\log P_{\tilde{Z}^n}(Z^n)\right] &= \frac{1}{n}\log\frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} + \frac{1}{n}E\left[\log\frac{\Gamma(1+D)\Gamma(1+n-D)}{\Gamma(2+n)}\right] \\
&= \frac{1}{n}\log\frac{1}{n} - \sum_{k=0}^{n}\binom{n}{k}2^{-n}\log\binom{n}{k}^{1/n}.
\end{aligned}$$

- Hence, for this specific case,

$$\begin{aligned}
\lim_{n\to\infty}\frac{1}{n}D(W_1^n||W_2^n|X_n) &= \lim_{n\to\infty}\sum_{k=0}^{n}\binom{n}{k}2^{-n}H_b(k/n)-\log(2)\\
&> \lim_{n\to\infty}2\sum_{k=0}^{\lfloor n/2\rfloor}\binom{n}{k}2^{-n}\cdot 2\log(2)\frac{k}{n}-\log(2)\\
&= \lim_{n\to\infty}2\log(2)\sum_{k=1}^{\lfloor n/2\rfloor}\binom{n-1}{k-1}2^{-(n-1)}-\log(2)=0.
\end{aligned}$$

- Consequently, **at least for this specific case, the Polya channel does not resemble the Gilbert-Elliott channel**.

- But here, we did not optimize the model parameters of the Polya channel to fit the Gilbert-Elliott channel. Still, a lot of future work needs to be done along this research line.

Channel Similarity: Analysis Based on Error Free Run

- Now we choose another (possible) similarity index, fitness of *Error Free Run* (EFR).
- Again, the concept behind channel similarity is that when two channels are similar, they should exhibit almost identical error behavior at the receiver end for the same transmission sequence.
- A channel model is similar to another channel model, if they give similar EFR due to the same transmission sequence.
- Our goal is **to examine which one of the Gilbert-Elliott and the Polya models is more "similar" to the binary-quantized multi-path Rayleigh fading channel**.

The Multi-Path Rayleigh Fading Channel Model

- The impulse response of a simple multi-path Rayleigh fading channel model can be written as:

$$h(\tau;t) = \sum_{n=0}^{L-1} \alpha_n(t) e^{-j2\pi f_c \tau_n} \delta(\tau - \tau_n), \tag{1}$$

- By employing some techniques, equation (1) can be simplified to:

$$h(\tau;t) = \sum_{n=0}^{L-1} \alpha_n(t) \delta(\tau - \tau_n). \tag{2}$$

Both (1) and (2) will be used for resemblance examination of the Gilbert-Elliott and the Polya models.

Error Free Run

- To facilitate and simplify our analysis, we do not adopt the conventional EFR definition, and define the *stationary EFR* specifically for our concerned hidden Markov channel model.

  Definition 2 (Stationary Error Free Run for Hidden Markov) *Fix an additive binary channel, in which the binary output is the modulo-2 sum of the binary input and a binary noise sample. Suppose the binary noise samples are defined by a hidden Markov model. Then the* stationary error free run *is defined as the probability of receiving at least consecutive m error free bits, given that an error has occurred, and is denoted by* $\mathrm{EFR}(m)$*, provided that the Markov state transition is stationary.*

- From Definition 2, the EFR is **the probability of receiving an error bit, followed by consecutive m correct bits** under *stationarity*.

EFR of the Gilbert-Elliott model

- Stationary EFR of the Gilbert-Elliott model:

$$\mathrm{EFR}_{\mathrm{GE}}(m) = \left(\frac{g}{g+b}\right) p_G(1-p_G)^m + \left(\frac{b}{g+b}\right) p_B(1-p_B)^m.$$

- From the above formula, the complexity of EFR(m) increases exponentially as the error free run length m grows.

EFR of the Polya model

- Alajaji and Fuja in their 1994 paper suggested to define the state of the Polya process as the total number of red balls drawn after n trials. With the state definition, they showed that the Polya channel can be made stationary by treating R and S as pre-specified deterministic model parameters.
- Hence, the stationary EFR of the Polya model is

$$\mathrm{EFR}_{\mathrm{Polya}}(m) = \left(\frac{R}{R+S}\right)\prod_{j=1}^{m}\left(\frac{S+(j-1)\Delta}{R+S+j\Delta}\right).$$

Criterion for Optimal Model Parameters

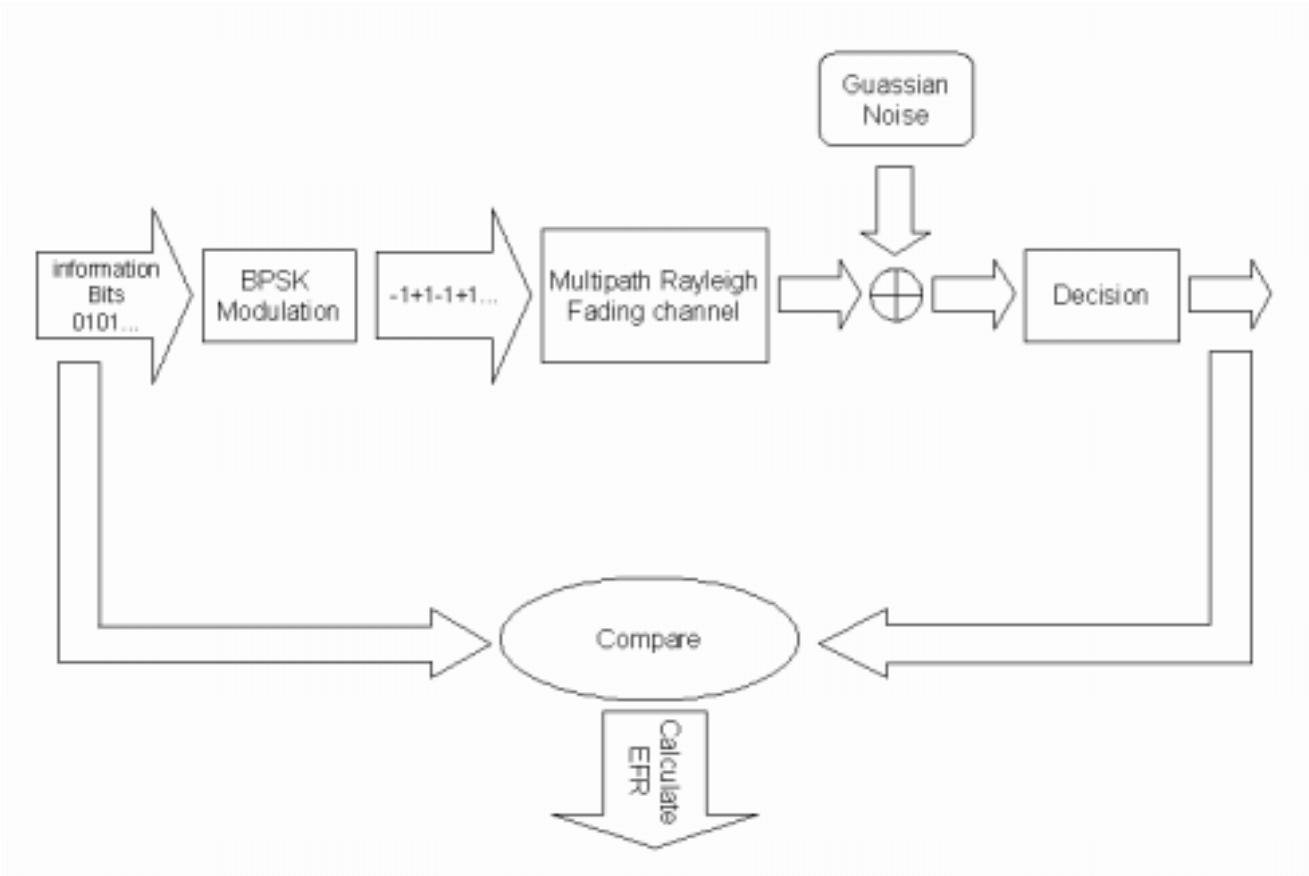
- Total square error

$$\sum_{j=1}^{m} [\mathrm{EFR}(j) - a_j]^2$$

  where $a_1, a_2, \ldots, a_m$ are the simulated EFR values of the binary-quantized multi-path Rayleigh fading channel.

- Unfortunately, no analytical optimality can be obtained based on the expressions of $\mathrm{EFR}_{\mathrm{GE}}(\cdot)$ and $\mathrm{EFR}_{\mathrm{Polya}}(\cdot)$. Therefore, the best-bit model parameters are selected though a **simulation-based** optimization.

Simulation System



Channel parameters for multipath Rayleigh fading channels

Condition	Number of Path	SNR(dB)	Phase Rotate
1	1	5	X
2	1	10	X
3	1	15	X
4	2	5	X
5	2	10	X
6	2	15	X

Condition	Number of Path	SNR(dB)	Phase Rotate
7	6	5	X
8	6	10	X
9	6	15	X
10	10	5	X
11	10	10	X
12	10	15	X
13	2	5	O
14	2	10	O
15	2	15	O

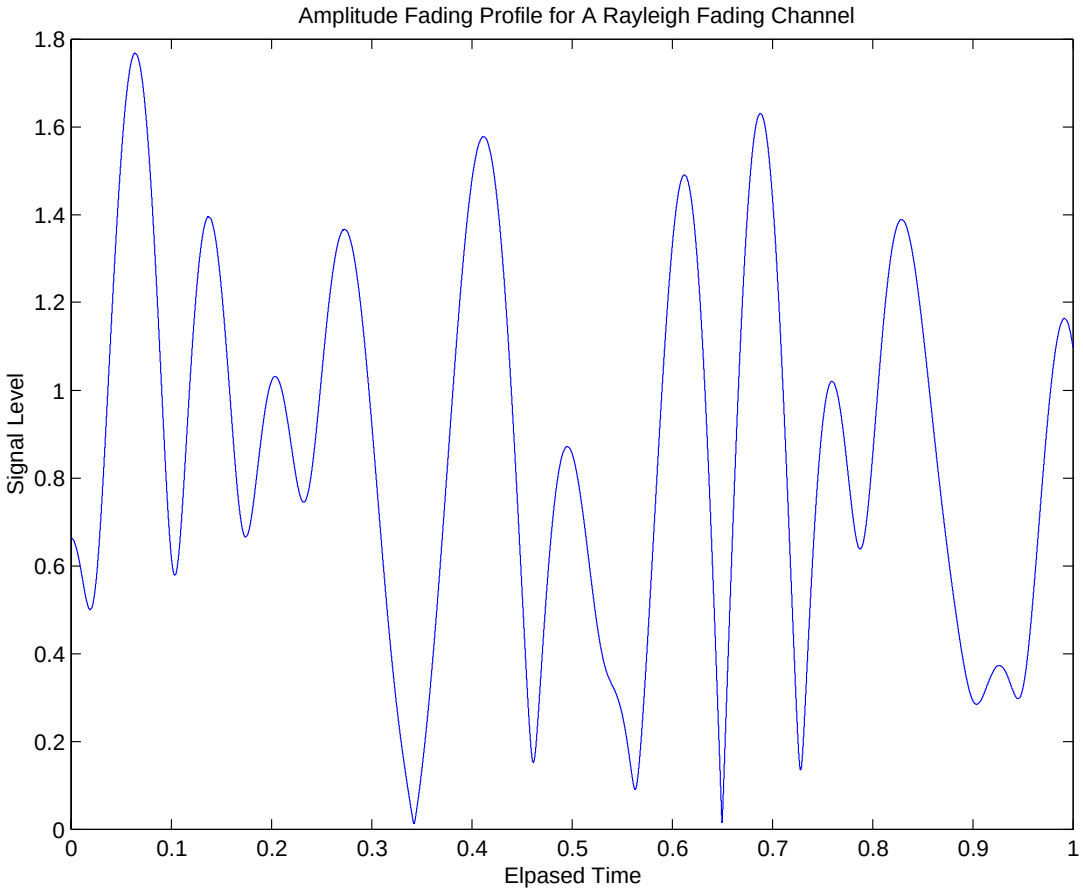
T $\equiv$ BPSK symbol duration, which equals T_s

T_s $\equiv$ Sampling period at the receiver end

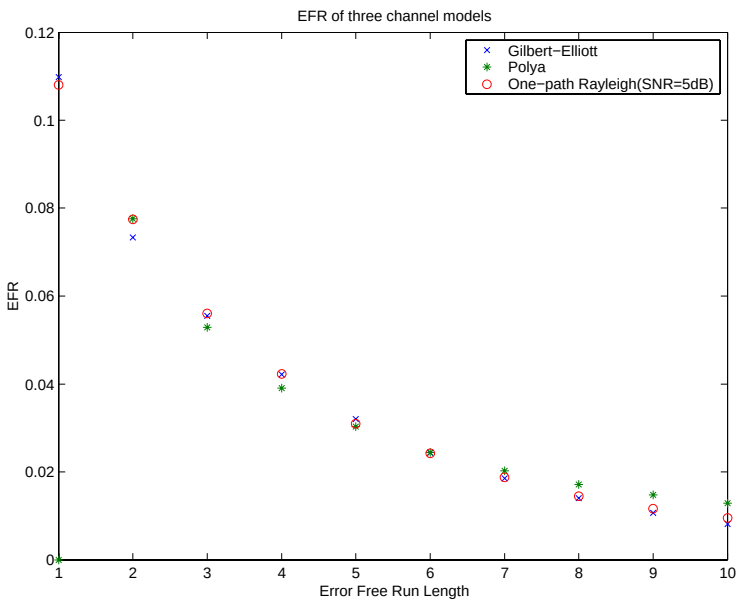
Doppler shift $=$ 8 Hz (Walking speed $= 1m/sec$)

Carrier frequency $=$ 2.4 GHz.

Simulation Results

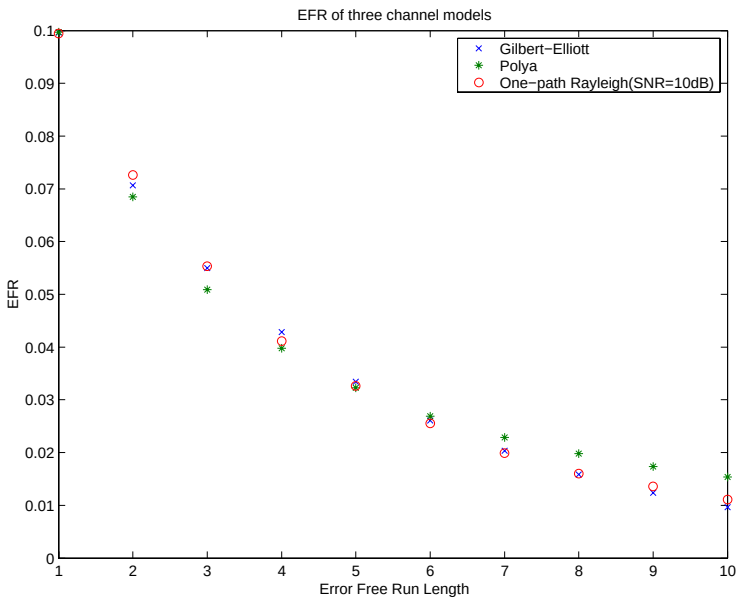


Amplitude fading profile obtained from Jake's model for the simulated Rayleigh fading channel.



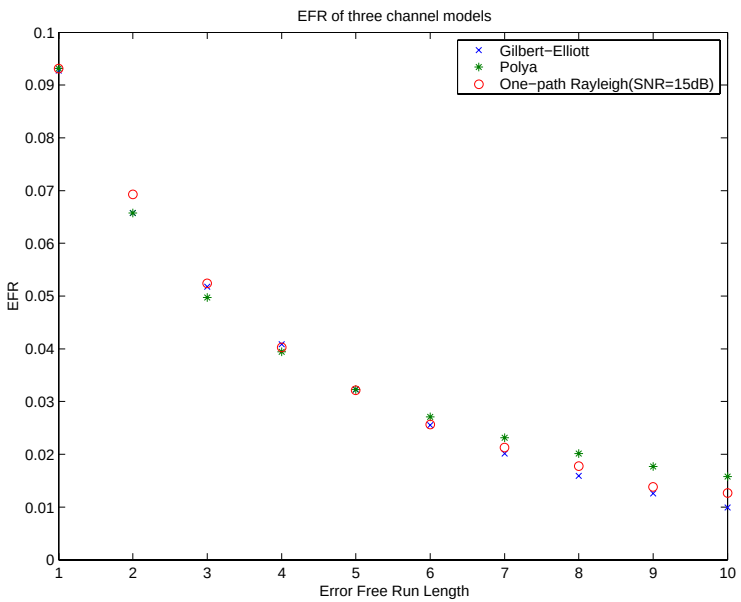
The Total Difference of EFR Between Models; condition 1

Gilbert vs. Rayleigh	0.010790
Polya vs. Rayleigh	0.125852



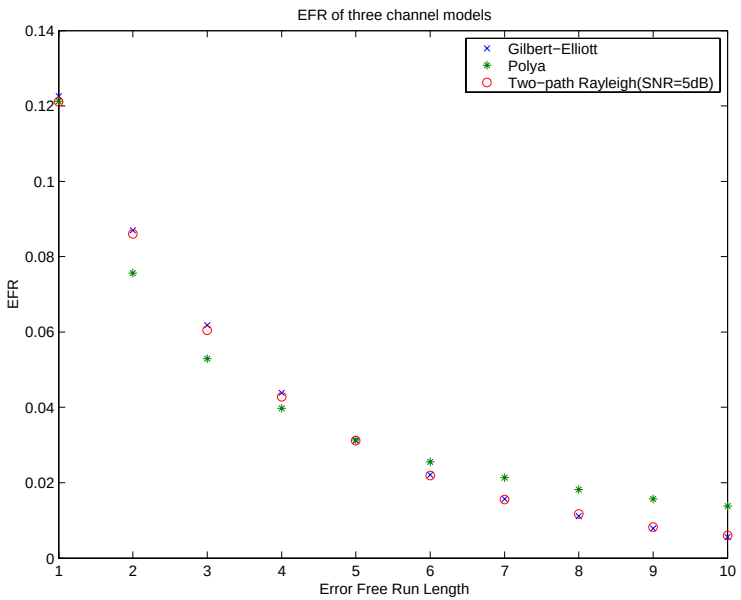
The Total Difference of EFR Between Models; condition 2

Gilbert vs.Rayleigh	0.008870
Polya vs. Rayleigh	0.026915



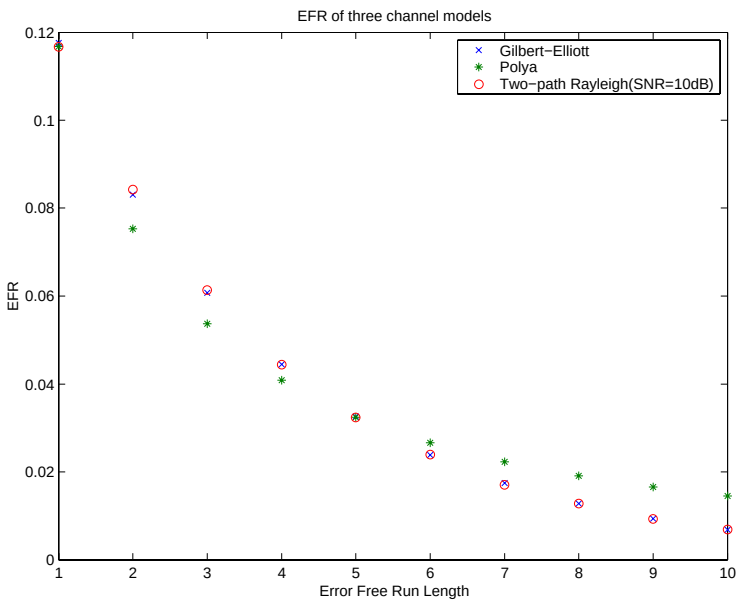
The Total Difference of EFR Between Models; condition 3

Gilbert vs.Rayleigh	0.012391
Polya vs. Rayleigh	0.019862



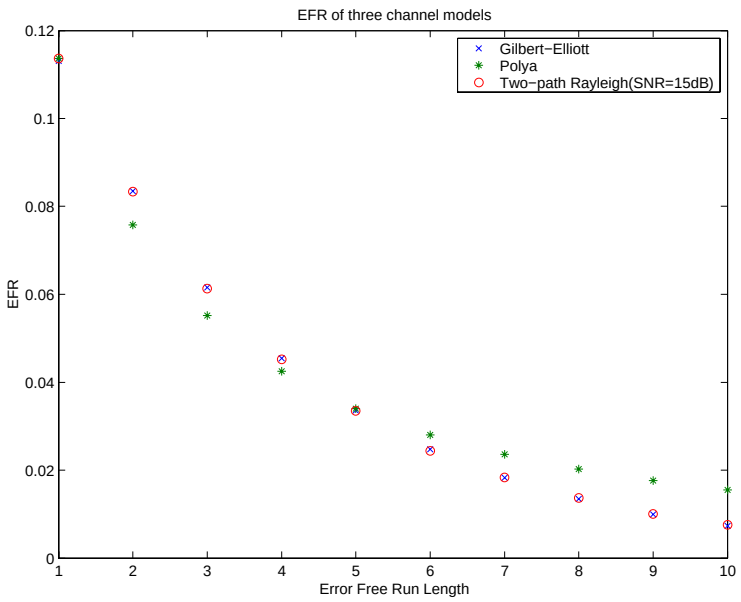
The Total Difference of EFR Between Models; condition 4

Gilbert vs. Rayleigh	0.006493
Polya vs. Rayleigh	0.052088



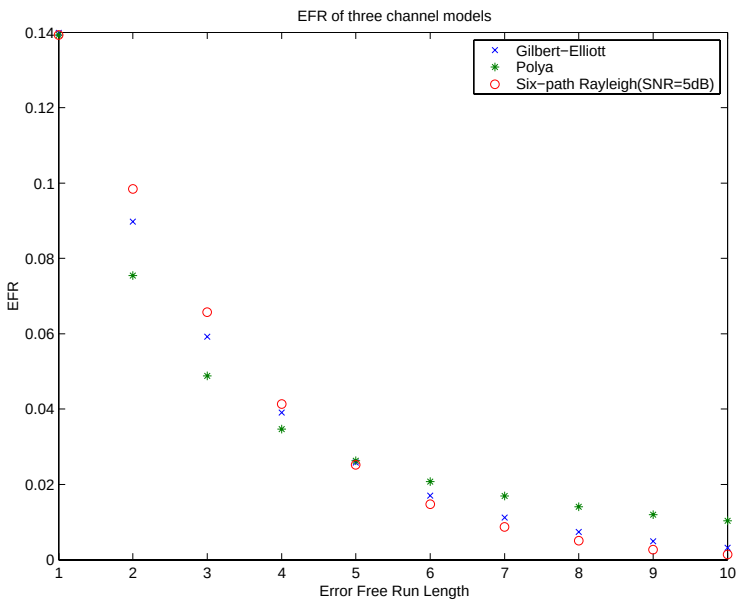
The Total Difference of EFR Between Models; condition 5

Gilbert vs.Rayleigh	0.003538
Polya vs. Rayleigh	0.049427



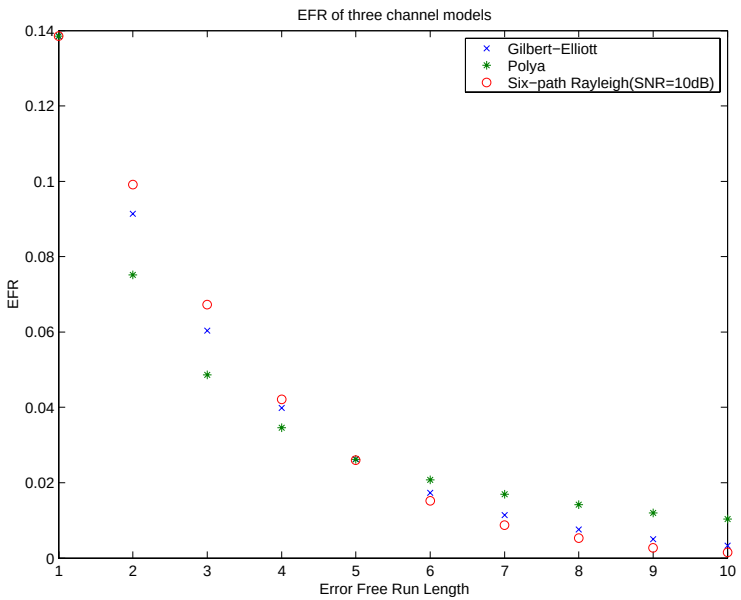
The Total Difference of EFR Between Models; condition 6

Gilbert vs.Rayleigh	0.002028
Polya vs. Rayleigh	0.04776



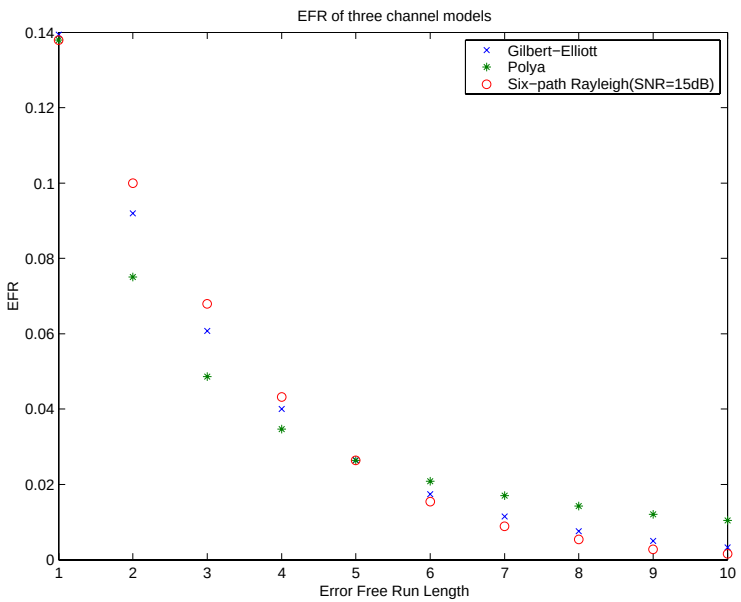
The Total Difference of EFR Between Models; condition 7

Gilbert vs. Rayleigh	0.029542
Polya vs. Rayleigh	0.088948



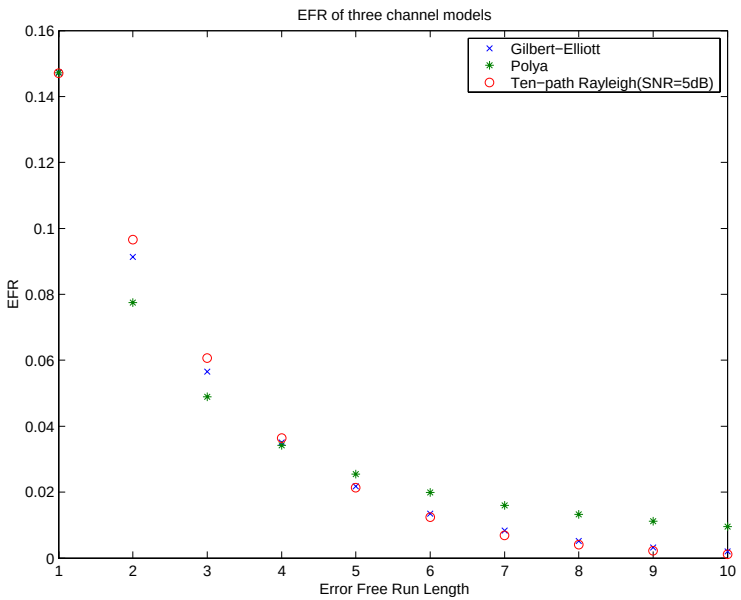
The Total Difference of EFR Between Models; condition 8

Gilbert vs. Rayleigh	0.028455
Polya vs. Rayleigh	0.091200



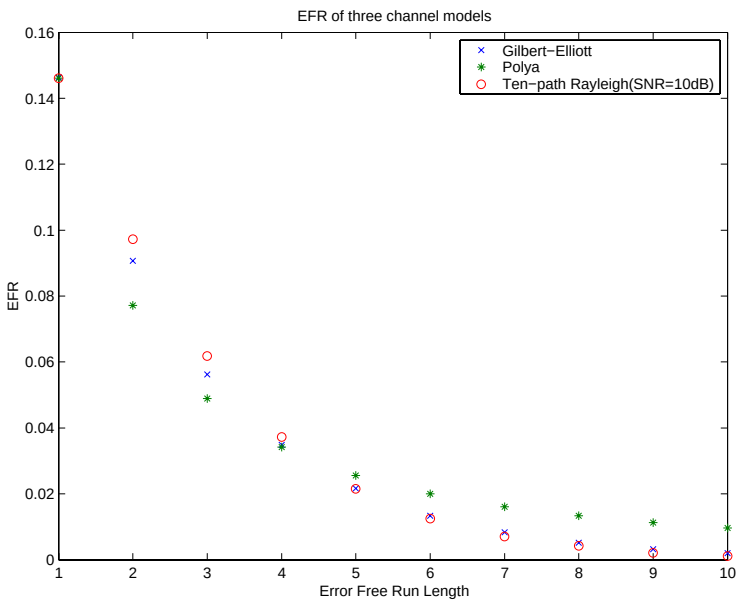
The Total Difference of EFR Between Models; condition 9

Gilbert vs. Rayleigh	0.030591
Polya vs. Rayleigh	0.093170



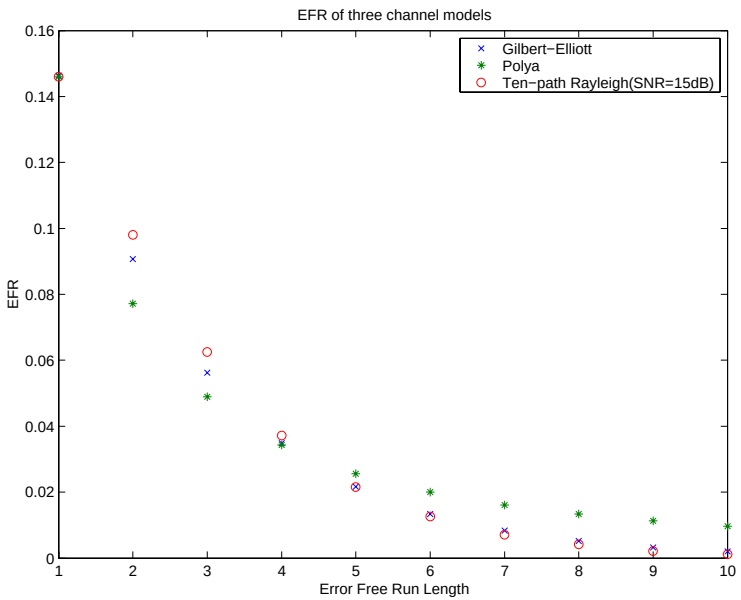
The Total Difference of EFR Between Models; condition 10

Gilbert vs. Rayleigh	0.016749
Polya vs. Rayleigh	0.080321



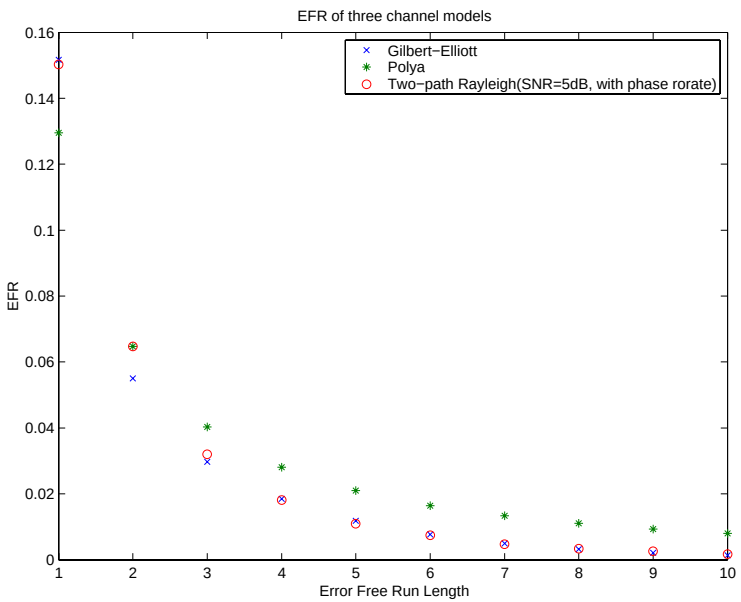
The Total Difference of EFR Between Models; condition 11

Gilbert vs. Rayleigh	0.019919
Polya vs. Rayleigh	0.083483



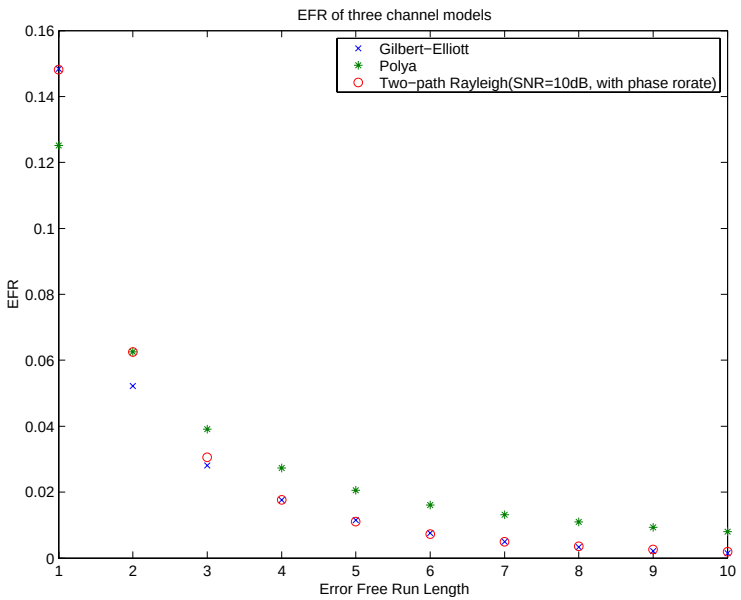
The Total Difference of EFR Between Models; condition 12

Gilbert vs. Rayleigh	0.021096
Polya vs. Rayleigh	0.084643



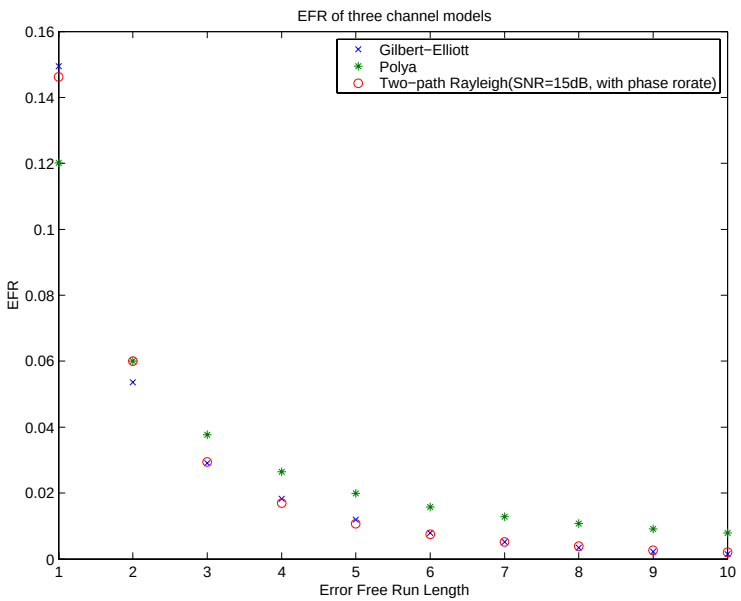
The Total Difference of EFR Between Models; condition 13

Gilbert vs. Rayleigh	0.015924
Polya vs. Rayleigh	0.087308



The Total Difference of EFR Between Models; condition 14

Gilbert vs. Rayleigh	0.015076
Polya vs. Rayleigh	0.088045



The Total Difference of EFR Between Models; condition 15

Gilbert vs. Rayleigh	0.014545
Polya vs. Rayleigh	0.088376

Model parameters for the Gilbert-Elliot and Polya channels

`Condition`	`Gilbert` (g, b, p_G, p_B)	`Polya` (R, S, Δ)
1	(0.1, 0.09, 0.24, 0.97)	(820, 2000, 1650)
2	(0.1, 0.09, 0.22, 0.98)	(700, 3500, 1650)
3	(0.11, 0.11, 0.21, 0.98)	(690, 3970, 1650)
4	(0.1, 0.068, 0.29, 1)	(790, 2420, 1700)
5	(0.1, 0.073, 0.268, 0.99)	(690, 2450, 1470)
6	(0.1, 0.071, 0.262, 1)	(662, 2670, 1335)

Condition	Gilbert (g, b, p_G, p_B)	Polya (R, S, Δ)
7	(0.1, 0.065, 0.34, 0.99)	(1005, 1520, 1817)
8	(0.1, 0.062, 0.34, 1)	(991, 1510, 1817)
9	(0.1, 0.061, 0.34, 1)	(979, 1520, 1817)
10	(0.1, 0.06, 0.38, 1)	(1184, 1517, 1817)
11	(0.1, 0.061, 0.38, 1)	(1159, 1511, 1817)
12	(0.1, 0.061, 0.38, 1)	(1156, 1511, 1817)
13	(0.14, 0.34, 0.35, 0.86)	(999, 1000, 1858)
14	(0.14, 0.38, 0.34, 0.86)	(999, 1000, 1994)
15	(0.14, 0.36, 0.34, 0.86)	(999, 1000, 2161)

Discussion on Simulation Results

- According to simulation results shown previously, the EFR of the Gilbert-Elliott model is always closer to that of the binary-quantized Rayleigh fading model than the Polya model.
- Therefore, from the aspect of EFRs, the Polya model is a **weaker** representative for the Rayleigh fading channel than the Gilbert-Elliott model.
- Another interesting observation is that the total difference of EFRs between the Gilbert-Elliott model and two-path Rayleigh fading channel model without phase rotate is particularly small, which hints that the Gilbert-Elliott model is very much suitable for representing a **two-path binary-quantized Rayleigh fading channel without phase rotate**.

Conclusions and Future Works

- In this thesis, we tried to explore the concept of channel similarity.
- Our quest proceeded by two different methodologies, an analytical methodology based on divergence, and an empirical methodology based on error free run matching.
- In the analytical methodology, we showed that divergence can be served as a guide to channel similarity; however, when this quantitative index was used to the similarity of the Gilbert-Elliott and Polya channels, only partial result was obtained.
- We then examined which of the two channel models, Gilbert-Elliott and Polya, is more suitable to approximate the binary-quantized simplification of a continuous multipath Rayleigh fading channel in terms of the EFR quantitative index.

- The simulation results show that the Polya model is apparently a **weaker** representative for the Rayleigh fading channel than the Gilbert-Elliott model from the aspect of EFR matching.
- Also from the simulations, the two-state Gilbert-Elliott model seems quite suitable to approximate a **two-path Rayleigh fading channel without phase rotate**.