Fixed-Length Joint Source-Channel Coding System for Generally Non-uniform Sources

Po-Han Lin

Institute of Communications Engineering

National Chiao Tung University

Hsin Chu, Taiwan 30050, R.O.C.

Outline

- Introduction
- Preliminaries
- Fixed-Length Joint Source-Channel Block Code
- Modified Turbo Code
- Simulation Results
- Conclusion

Introduction

- The communication system nowadays usually treats the source coding and channel coding separately for their design convenience.

- Alternatively, one may combine the source coding and channel coding into one to hopefully reduce the system complexity.

- Such a joint source-channel code design is often **variable-length** in nature.

- Engineering challeges of joint source-channel **variable-length** error-correcting (VLEC) code.
  - Varying decoding delay.
  - Error propagation when codewords are sequentially concatenated in transmission.

- For this reason, we will focus on the joint source-channel **fixed-length** error-correcting code (FLEC) design in this thesis.

In this thesis, two different approaches to design FLECs are proposed.

- Approach 1 : Derive the union bound for error rates of FLECs; then generate the code that has an acceptably good union bound value.

- Approach 2 : Modify the turbo decoder to suit the transceiving of n-ary non-uniform source statistics.

Preliminaries

Definitions and Notations

- Denote by $\boldsymbol{r} = (r_1, r_2, \ldots, r_\ell)$ the received vector.

- Define the hard-decision sequence $\boldsymbol{y} = (y_1, y_2, \ldots, y_\ell)$, where $y_i = \begin{cases} 0, & \text{if } r_i > 0 \\ 1, & \text{otherwise} \end{cases}$.

- $\boldsymbol{c}_i = (c_{i1}, c_{i2}, \ldots, c_{i\ell})$ is the codeword corresponding the ith symbol.

- $\Pr(\boldsymbol{c}_i) = p_i$ is the transmission probability of ith symbol.

- $h_i = d_{\mathrm{H}}(\boldsymbol{c}_i, \boldsymbol{y})$ is the Hamming distance between $\boldsymbol{c}_i$ and $\boldsymbol{y}$.

- The log-likelihood ratio of the ith received scalar is given by $\phi_i = \frac{4}{N_0} r_i$, where N_0 is the one-sided power spectrum density of the AWGN noise.

- $b = Q\left(\sqrt{\frac{2E_b R}{N_0}}\right)$ denotes the crossover probability of the hard-decision AWGN channel, where $Q(u) = \int_t^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ is the Q-function, E_b is the bit energy and R is the code rate.

MAP Decoding Criterion

- For hard-decision decoding, the MAP decision $\boldsymbol{c}_m$ upon $\boldsymbol{y}$ satisfies

$$h_m \left[\ln(b) - \ln(1-b)\right] + \ln p_m \geq h_i \left[\ln(b) - \ln(1-b)\right] + \ln p_i \quad \forall\ i.$$

- For soft-decision decoding, the MAP decision $\boldsymbol{c}_m$ upon $\boldsymbol{r}$ satisfies

$$\sum_{j=1}^{\ell} (-1)^{c_{m,j}} \phi_j + 2 \ln p_m \geq \sum_{j=1}^{\ell} (-1)^{c_{i,j}} \phi_j + 2 \ln p_i \quad \forall\ i.$$

Joint Source-Channel Block Code

Union Bound

- Let $K_h = \begin{cases} \displaystyle\sum_{e=(h+1)/2}^{h} \binom{h}{e} b^e (1-b)^{h-e} & \text{for odd } h \\ \displaystyle\frac{1}{2}\binom{h}{h/2} b^{h/2} (1-b)^{h/2} + \sum_{e=h/2+1}^{h} \binom{h}{e} b^e (1-b)^{h-e} & \text{for even } h \end{cases}$

- Define $A_h = \displaystyle\sum_{i=1}^{n} \sum_{j:d_{\mathrm{H}}(\boldsymbol{c}_i,\boldsymbol{c}_j)=h} p_i$

- Then, given a source distribution and the codeword length ℓ, the symbol error probability (SEP) can be bounded as following:

$$P_e \leq \sum_{h=1}^{\ell} A_h \times K_h$$

- In this thesis, we fix the code rate $R = 1/4$ and also the signal-to-noise ratio per information bit $E_{\mathrm{b}}/N_0 = 10$ dB when determining the code based on the union bound.

Construction of Joint Source-Channel Block Code

Code size $n = 4$ and codeword length $\ell = 7$.

$$\begin{aligned} \boldsymbol{c}_1 &= ********, & g_1 &= 0 \\ \boldsymbol{c}_2 &= ********, & g_2 &= 0 \\ \boldsymbol{c}_3 &= ********, & g_3 &= 0 \\ \boldsymbol{c}_4 &= ********, & g_4 &= 0 \\ & & g_{\text{total}} &= g_1 + g_2 + g_3 + g_4 = 0 \end{aligned}$$

$$\begin{cases} y = 0 \\ g_i = \sum_{j=1}^{n} K_{a_{i,j,y}} \times (p_i + p_j) \\ a_{i,j,y} = d_{\text{H}}\left(c_{i,1}c_{i,2}\ldots c_{i,y-1}, c_{j,1}c_{j,2}\ldots c_{j,y-1}\right). \end{cases}$$

Construction of Joint Source-Channel Block Code

$$\boldsymbol{c}_1 = 0{*}{*}{*}{*}{*}{*}{*}, \qquad g_1 = 0$$

$$\boldsymbol{c}_2 = {*}{*}{*}{*}{*}{*}{*}{*}, \qquad g_2 = 0$$

$$\boldsymbol{c}_3 = {*}{*}{*}{*}{*}{*}{*}{*}, \qquad g_3 = 0$$

$$\boldsymbol{c}_4 = {*}{*}{*}{*}{*}{*}{*}{*}, \qquad g_4 = 0$$

$$g_{\text{total}} = 0$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 &= 0{*}{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\
\boldsymbol{c}_2 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\
\boldsymbol{c}_3 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\
\boldsymbol{c}_4 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\
& & g_{\text{total}} &= 0
\end{aligned}
$$

$$
\begin{aligned}
r_0 &= K_0 \times (p_2 + p_1) = 0.45 \\
r_1 &= K_1 \times (p_2 + p_1) = 0.0114
\end{aligned}
$$

If $r_0 > r_1$, this bit is 1; others, 0.

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 &= 0{*}{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\
\boldsymbol{c}_2 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\
\boldsymbol{c}_3 &= \color{red}{1}\color{black}{*}{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\
\boldsymbol{c}_4 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\
& & g_{\text{total}} &= 0
\end{aligned}
$$

$$r_0 = K_0 \times (p_3 + p_1) + K_1 \times (p_3 + p_2) = 0.4523$$
$$r_1 = K_1 \times (p_3 + p_1) + K_0 \times (p_3 + p_2) = 0.1014$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 &= 0{*}{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\
\boldsymbol{c}_2 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\
\boldsymbol{c}_3 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\
\boldsymbol{c}_4 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\
& & g_{\text{total}} &= 0
\end{aligned}
$$

$$
r_0 = K_0 \times (p_4 + p_1) + K_1 \times (p_4 + p_2) + K_1 \times (p_4 + p_3) = 0.4125
$$
$$
r_1 = K_1 \times (p_4 + p_1) + K_0 \times (p_4 + p_2) + K_0 \times (p_4 + p_3) = 0.1104
$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_2 &= 10{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0911 \\
\boldsymbol{c}_3 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_3 &= 0.0911 \\
\boldsymbol{c}_1 &= 0{*}{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\
\boldsymbol{c}_4 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0101 \\
& & g_{\text{total}} &= 0.2231
\end{aligned}
$$

$$
\begin{cases}
y = 1 \\
g_i = \sum_{j=1}^{n} K_{a_{i,j,y}} \times (p_i + p_j) \\
a_{i,j,y} = d_{\mathrm{H}} \left(c_{i,1} c_{i,2} \ldots c_{i,y-1}, c_{j,1} c_{j,2} \ldots c_{j,y-1} \right).
\end{cases}
$$

Construction of Joint Source-Channel Block Code

$$\begin{aligned} \boldsymbol{c}_2 &= 10{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0911 \\ \boldsymbol{c}_3 &= 11{*}{*}{*}{*}{*}{*}, & g_3 &= 0.0911 \\ \boldsymbol{c}_1 &= 0{*}{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\ \boldsymbol{c}_4 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0101 \\ & & g_{\text{total}} &= 0.2231 \end{aligned}$$

$$r_0 = K_0 \times (p_3 + p_2) = 0.09$$

$$r_1 = K_1 \times (p_3 + p_2) = 0.0023$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_2 &= 10{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0911 \\
\boldsymbol{c}_3 &= 11{*}{*}{*}{*}{*}{*}, & g_3 &= 0.0911 \\
\boldsymbol{c}_1 &= 0{\color{red}0}{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\
\boldsymbol{c}_4 &= 1{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0101 \\
& & g_{\text{total}} &= 0.2231
\end{aligned}
$$

$$
r_0 = K_1 \times (p_1 + p_2) + K_2 \times (p_1 + p_3) = 0.0228
$$

$$
r_1 = K_2 \times (p_1 + p_2) + K_1 \times (p_1 + p_3) = 0.0228
$$

$$
\begin{aligned}
\boldsymbol{c}_2 &= 10{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0911 \\
\boldsymbol{c}_3 &= 11{*}{*}{*}{*}{*}{*}, & g_3 &= 0.0911 \\
\boldsymbol{c}_1 &= 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\
\boldsymbol{c}_4 &= 11{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0101 \\
& & g_{\text{total}} &= 0.2231
\end{aligned}
$$

$$r_0 = K_0 \times (p_4 + p_2) + K_1 \times (p_4 + p_3) + K_1 \times (p_4 + p_1) = 0.0617$$
$$r_1 = K_1 \times (p_4 + p_2) + K_0 \times (p_4 + p_3) + K_2 \times (p_4 + p_1) = 0.0617$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 =& 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\
\boldsymbol{c}_2 =& 11{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0473 \\
\boldsymbol{c}_3 =& 10{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0034 \\
\boldsymbol{c}_4 =& 11{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0053 \\
& & g_{\text{total}} &= 0.0868
\end{aligned}
$$

Construction of Joint Source-Channel Block Code

$$\begin{aligned} \boldsymbol{c}'_1 &= \color{red}{00}\color{black}{******}, & g_1 &= 0 \\ \boldsymbol{c}'_2 &= ********, & g_2 &= 0 \\ \boldsymbol{c}'_3 &= ********, & g_3 &= 0 \\ \boldsymbol{c}'_4 &= ********, & g_4 &= 0 \\ & & g_{\text{total}} &= 0 \end{aligned}$$

Construction of Joint Source-Channel Block Code

$$\begin{aligned} \boldsymbol{c}'_1 &= 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\ \boldsymbol{c}'_2 &= 01{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\ \boldsymbol{c}'_3 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\ \boldsymbol{c}'_4 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\ & & g_{\text{total}} &= 0 \end{aligned}$$

$$\begin{aligned} r_{00} &= K_0 \times (p_2 + p_1) = 0.45 \\ r_{01} &= K_1 \times (p_2 + p_1) = 0.0114 \\ r_{10} &= K_1 \times (p_2 + p_1) = 0.0114 \\ r_{11} &= K_2 \times (p_2 + p_1) = 0.0114 \end{aligned}$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}'_1 &= 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\
\boldsymbol{c}'_2 &= 01{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\
\boldsymbol{c}'_3 &= {\color{red}10}{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\
\boldsymbol{c}'_4 &= {*}{*}{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\
& & g_{\text{total}} &= 0
\end{aligned}
$$

$$
\begin{aligned}
r_{00} &= K_0 \times (p_3 + p_1) + K_1 \times (p_3 + p_2) = 0.4523 \\
r_{01} &= K_1 \times (p_3 + p_1) + K_0 \times (p_3 + p_2) = 0.1014 \\
r_{10} &= K_1 \times (p_3 + p_1) + K_2 \times (p_3 + p_2) = 0.0137 \\
r_{11} &= K_2 \times (p_3 + p_1) + K_1 \times (p_3 + p_2) = 0.0137
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{c}_1' &= 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0 \\
\boldsymbol{c}_2' &= 01{*}{*}{*}{*}{*}{*}, & g_2 &= 0 \\
\boldsymbol{c}_3' &= 10{*}{*}{*}{*}{*}{*}, & g_3 &= 0 \\
\boldsymbol{c}_4' &= 11{*}{*}{*}{*}{*}{*}, & g_4 &= 0 \\
& & g_{\text{total}} &= 0
\end{aligned}
$$

$$
\begin{aligned}
r_{00} &= K_0 \times (p_4 + p_1) + K_1 \times (p_4 + p_2) + K_1 \times (p_4 + p_3) = 0.4125 \\
r_{01} &= K_1 \times (p_4 + p_1) + K_0 \times (p_4 + p_2) + K_2 \times (p_4 + p_3) = 0.0617 \\
r_{10} &= K_1 \times (p_4 + p_1) + K_2 \times (p_4 + p_2) + K_0 \times (p_4 + p_3) = 0.0617 \\
r_{11} &= K_2 \times (p_4 + p_1) + K_1 \times (p_4 + p_2) + K_1 \times (p_4 + p_3) = 0.0129
\end{aligned}
$$

$$\begin{aligned} \boldsymbol{c}_1' &= 00{*}{*}{*}{*}{*}{*}, & g_1 &= 0.0308 \\ \boldsymbol{c}_2' &= 01{*}{*}{*}{*}{*}{*}, & g_2 &= 0.0034 \\ \boldsymbol{c}_3' &= 10{*}{*}{*}{*}{*}{*}, & g_3 &= 0.0034 \\ \boldsymbol{c}_4' &= 11{*}{*}{*}{*}{*}{*}, & g_4 &= 0.0004 \\ & & g_{\text{total}} &= 0.0380 \end{aligned}$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 &= 00{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}_2 &= 01{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}_3 &= 10{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}_4 &= 11{*}{*}{*}{*}{*}{*}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{c}'_1 &= 0{*}{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}'_2 &= 1{*}{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}'_3 &= 1{*}{*}{*}{*}{*}{*}{*} \\
\boldsymbol{c}'_4 &= 1{*}{*}{*}{*}{*}{*}{*}
\end{aligned}
$$

Construction of Joint Source-Channel Block Code

$$\boldsymbol{c}_1 = 000*****$$
$$\boldsymbol{c}_2 = 011*****$$
$$\boldsymbol{c}_3 = 100*****$$
$$\boldsymbol{c}_4 = 111*****$$

$$\boldsymbol{c}'_1 = 010*****$$
$$\boldsymbol{c}'_2 = 100*****$$
$$\boldsymbol{c}'_3 = 101*****$$
$$\boldsymbol{c}'_4 = 110*****$$

$$\boldsymbol{c}_1 = 0000{*}{*}{*}{*}$$
$$\boldsymbol{c}_2 = 0111{*}{*}{*}{*}$$
$$\boldsymbol{c}_3 = 1000{*}{*}{*}{*}$$
$$\boldsymbol{c}_4 = 1111{*}{*}{*}{*}$$

$$\boldsymbol{c}'_1 = 0000{*}{*}{*}{*}$$
$$\boldsymbol{c}'_2 = 0111{*}{*}{*}{*}$$
$$\boldsymbol{c}'_3 = 1011{*}{*}{*}{*}$$
$$\boldsymbol{c}'_4 = 1101{*}{*}{*}{*}$$

$$\boldsymbol{c}_1 = 00000{*}{*}{*}$$
$$\boldsymbol{c}_2 = 01110{*}{*}{*}$$
$$\boldsymbol{c}_3 = 10111{*}{*}{*}$$
$$\boldsymbol{c}_4 = 11011{*}{*}{*}$$

$$\boldsymbol{c}'_1 = 00000{*}{*}{*}$$
$$\boldsymbol{c}'_2 = 01111{*}{*}{*}$$
$$\boldsymbol{c}'_3 = 10011{*}{*}{*}$$
$$\boldsymbol{c}'_4 = 11101{*}{*}{*}$$

Construction of Joint Source-Channel Block Code

$$\boldsymbol{c}_1 = 000001{*}{*}$$
$$\boldsymbol{c}_2 = 011100{*}{*}$$
$$\boldsymbol{c}_3 = 101110{*}{*}$$
$$\boldsymbol{c}_4 = 110111{*}{*}$$

$$\boldsymbol{c}'_1 = 000010{*}{*}$$
$$\boldsymbol{c}'_2 = 011100{*}{*}$$
$$\boldsymbol{c}'_3 = 101101{*}{*}$$
$$\boldsymbol{c}'_4 = 110110{*}{*}$$

Construction of Joint Source-Channel Block Code

$$
\begin{aligned}
\boldsymbol{c}_1 &= 0000010* \\
\boldsymbol{c}_2 &= 0111001* \\
\boldsymbol{c}_3 &= 1011100* \\
\boldsymbol{c}_4 &= 1101111*
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{c}'_1 &= 0000001* \\
\boldsymbol{c}'_2 &= 0111010* \\
\boldsymbol{c}'_3 &= 1011100* \\
\boldsymbol{c}'_4 &= 1101110*
\end{aligned}
$$

$$\boldsymbol{c}_1 = 00000100$$
$$\boldsymbol{c}_2 = 01110010$$
$$\boldsymbol{c}_3 = 10111001$$
$$\boldsymbol{c}_4 = 11011111$$

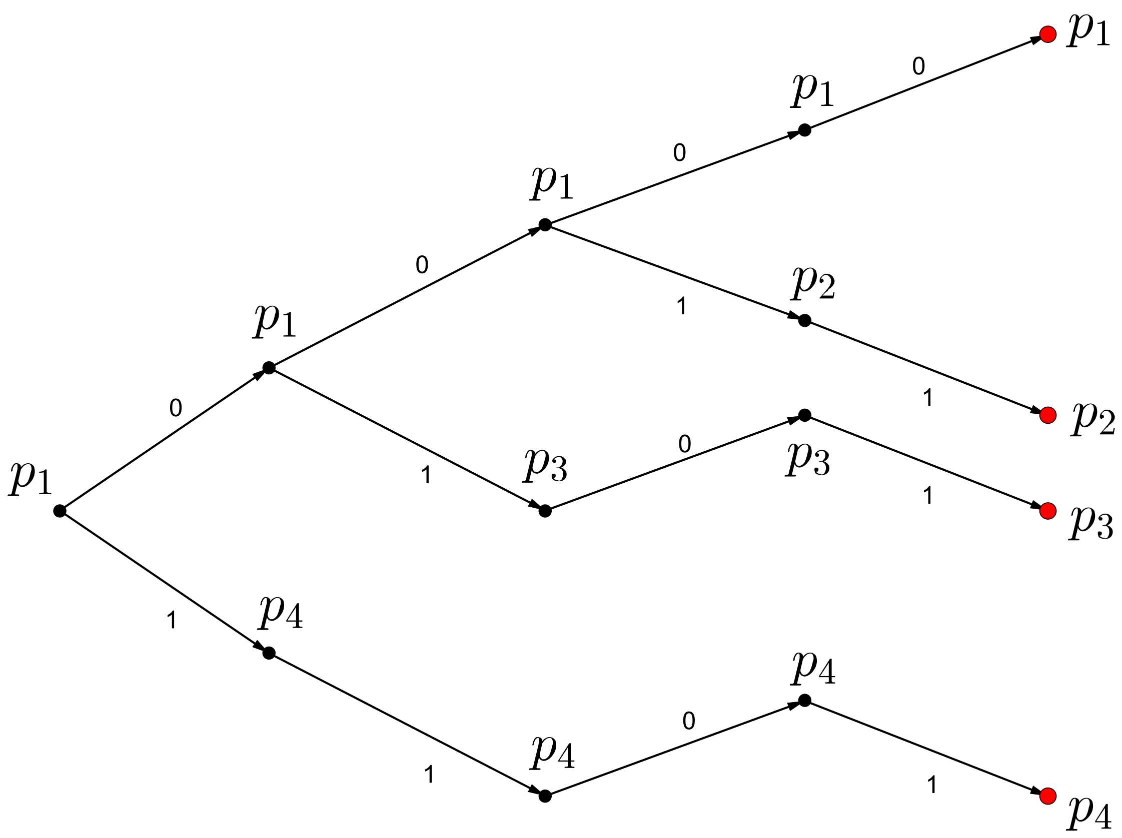
$$\boldsymbol{c}'_1 = 00000100$$
$$\boldsymbol{c}'_2 = 01110001$$
$$\boldsymbol{c}'_3 = 10111000$$
$$\boldsymbol{c}'_4 = 11011101$$

Step 0. Construct the binary code tree corresponding to the given codebook. Each leaf node is associated with a probability corresponding to the probability of occurrence for the source letter that is encoded to this codeword. The probability associated with a non-leaf node is the largest one among all probabilities associated with those leaf nodes that are offsprings of this non-leaf node.

Step 1. Calculate the hard-decision sequence $\boldsymbol{y}$*. Initialize the metric of the root node as 0, and push the root node into the stack.*

Decoding of Joint Source-Channel Block Code

A sample of the code tree that is generated based on $n = 4$, $\ell = 4$ and source distribution $p_1 > p_2 > p_3 > p_4$.



$\boldsymbol{c}_1 = 0000$

$\boldsymbol{c}_2 = 0011$

$\boldsymbol{c}_3 = 0101$

$\boldsymbol{c}_4 = 1101$

Decoding of Joint Source-Channel Block Code

Step 2. Extract the node (at level j) with the maximal metric value e from the stack.

Discard this extracted node if

$$2(\ell - j_M) + \ln q_M \geq (j_M - j) + \ln q,$$

where

$$\begin{cases} q \text{ is the probability associated with this node,} \\ q_M \text{ is the largest probability associated with those nodes} \\ \qquad \text{that have been visited thus far, and} \\ j_M \text{ is the level at which the node that decides } q_M \text{ is located.} \end{cases}$$

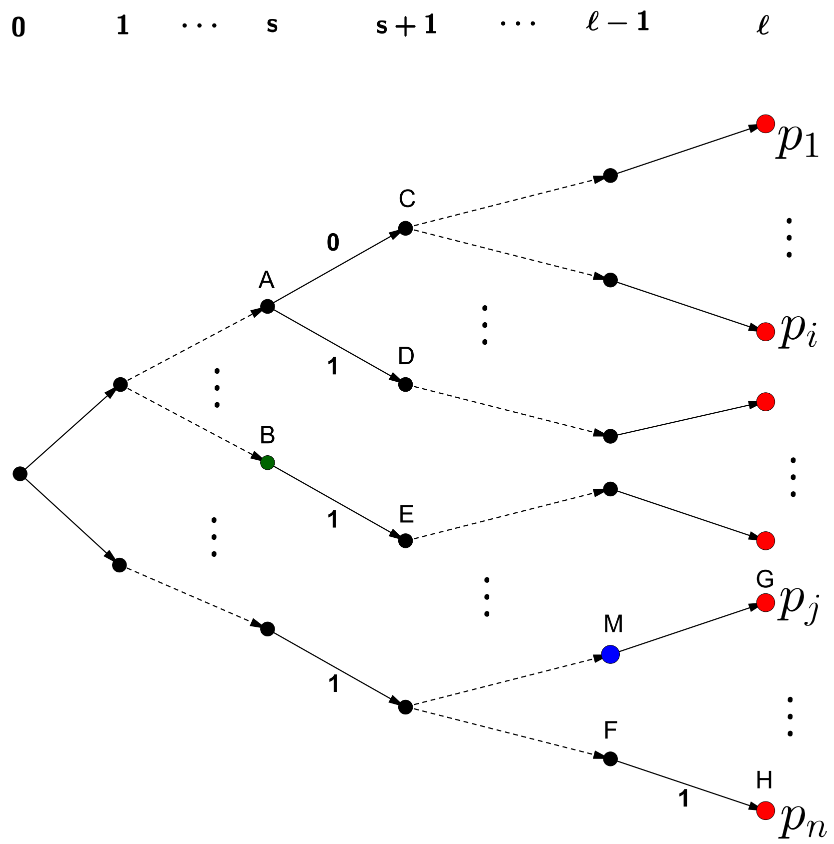
else if the extracted node is a leaf node, output the codeword corresponding to the leaf node and stop the algorithm.

else, perform the following procedure for all child nodes.

- *If the child node is a non-leaf node, set the metric of this child node to*
$$\begin{cases} e + (c_{i,j} \oplus y_j) \ln\left(\frac{b}{1-b}\right), & \textit{hard-decision decoding} \\ e + (-1)^{c_{i,j}} \phi_i, & \textit{soft-decision decoding} \end{cases}$$
where $c_{i,j}$ is the code bit corresponding to the tree branch connecting the extracted node and the child node.
- *Else, set the metric of this child node to*
$$\begin{cases} e + (c_{i,j} \oplus y_j) \ln\left(\frac{b}{1-b}\right) + \ln(p_i), & \textit{hard-decision decoding} \\ e + (-1)^{c_{i,j}} \phi_i + 2\ln(p_i), & \textit{soft-decision decoding} \end{cases}$$

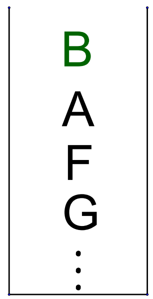
Push the child node into the stack.

Decoding of Joint Source-Channel Block Code

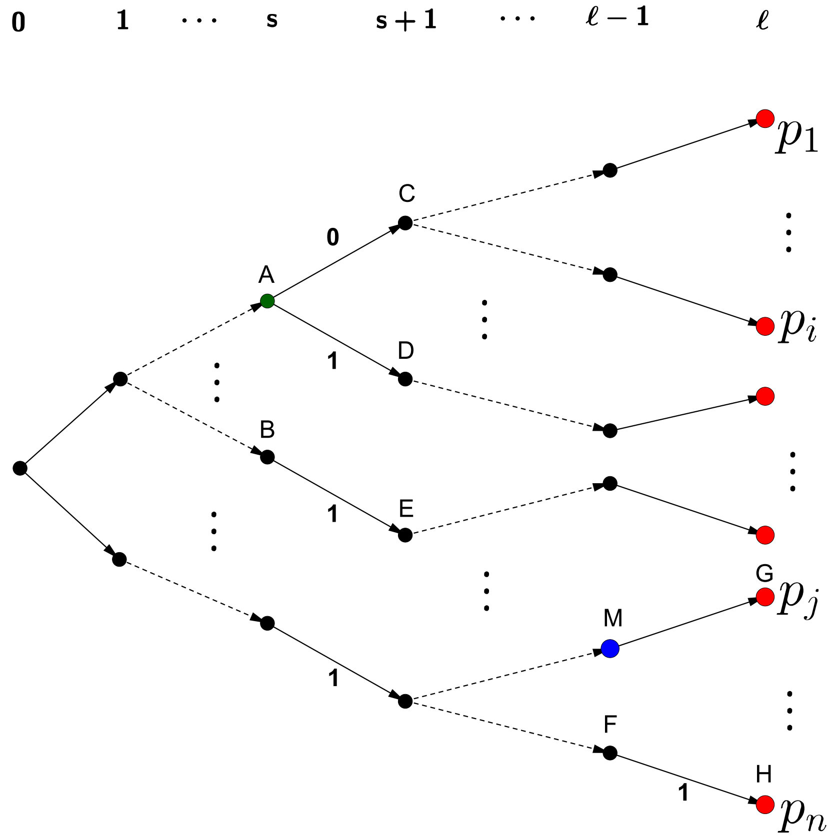


$$2\left[\ell - (\ell - 1)\right] - \ln q_M \leq \left[(\ell - 1) - s\right] - \ln q_B$$

$\Rightarrow$ Node B is Discarded



Decoding of Joint Source-Channel Block Code

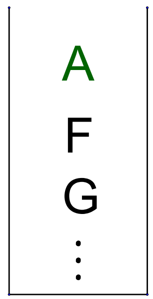


$$2\left[\ell-(\ell-1)\right]-\ln q_M \geq \left[(\ell-1)-s\right]-\ln q_A$$

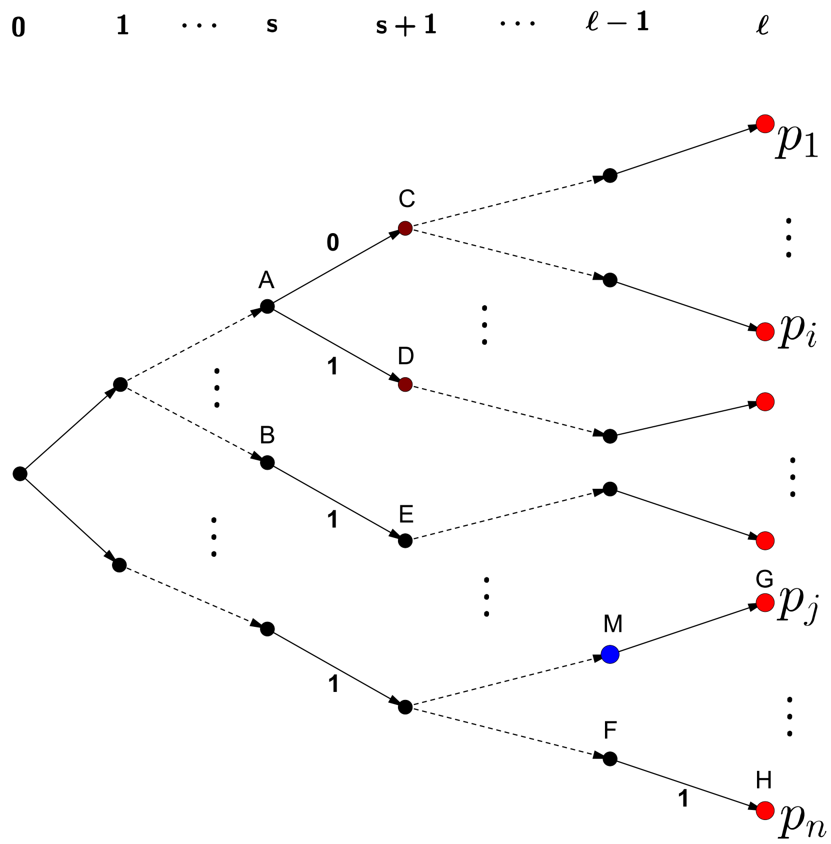
$\Rightarrow$ Node A is not Discarded

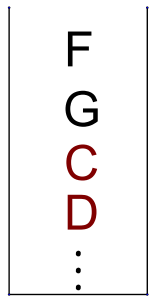
$$e_C = e_A + (0 \oplus y_{s+1}) \ln\left(\frac{b}{1-b}\right)$$

$$e_D = e_A + (1 \oplus y_{s+1}) \ln\left(\frac{b}{1-b}\right)$$

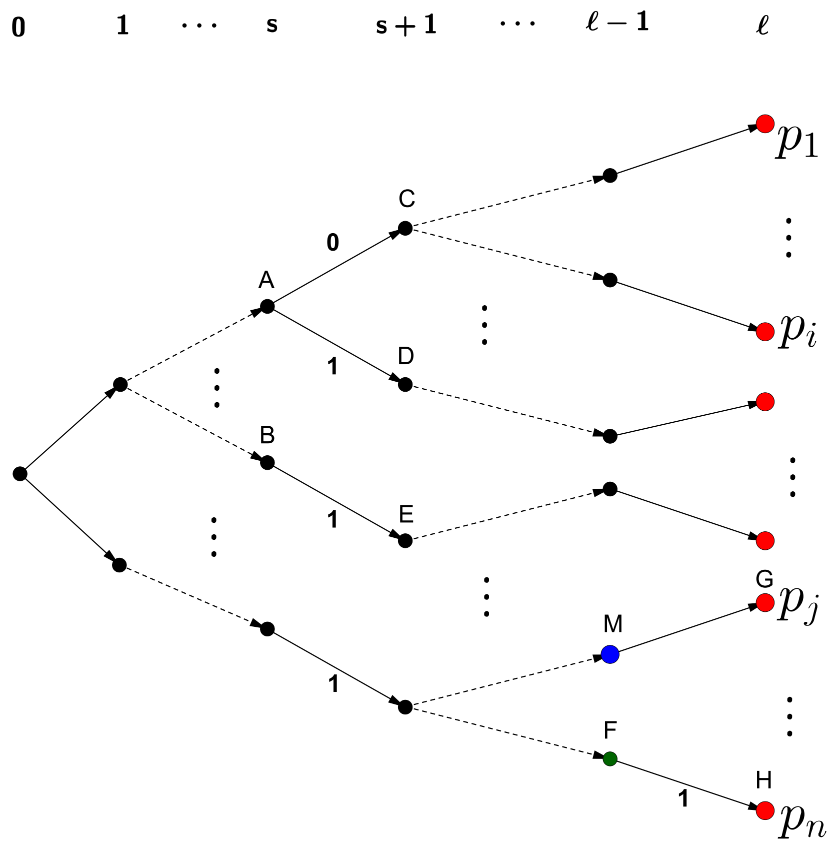


Decoding of Joint Source-Channel Block Code





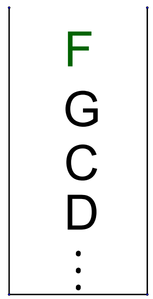
Decoding of Joint Source-Channel Block Code



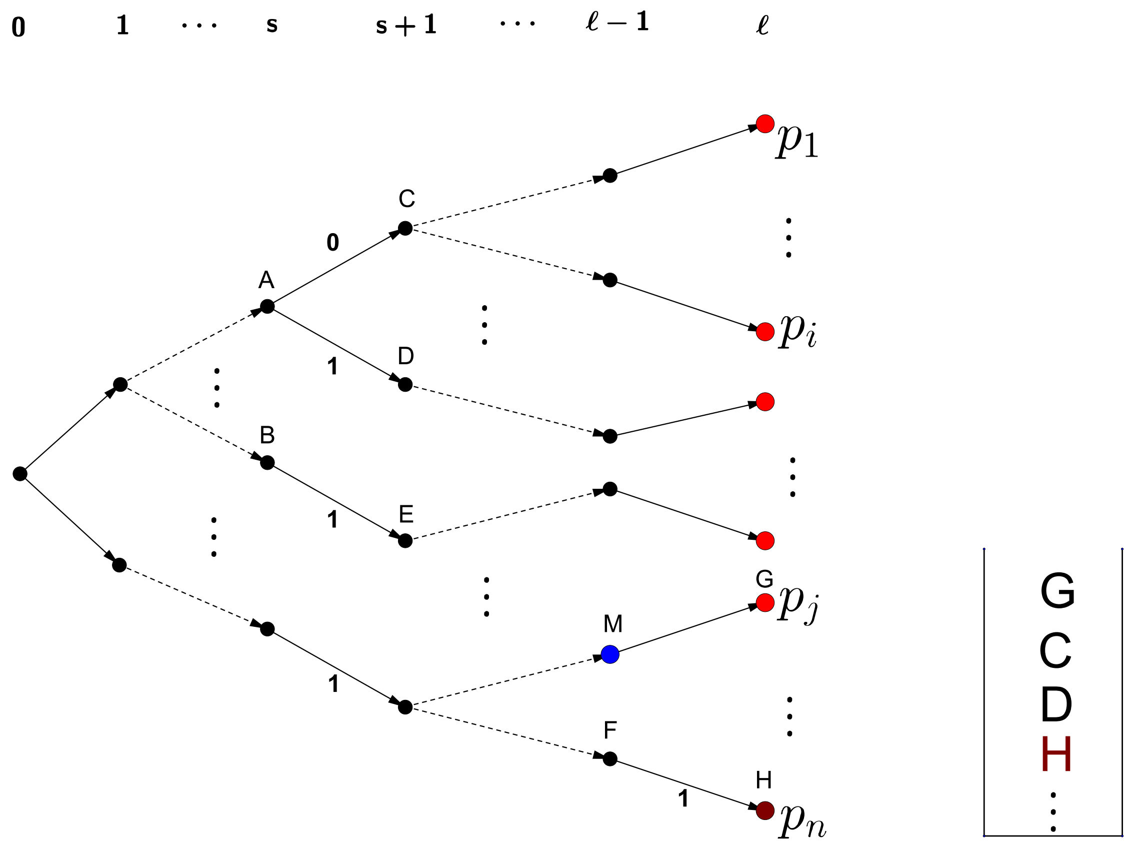
$$2\left[\ell-(\ell-1)\right]-\ln q_M \geq 0-\ln q_F$$

$\Rightarrow$ Node F is not Discarded

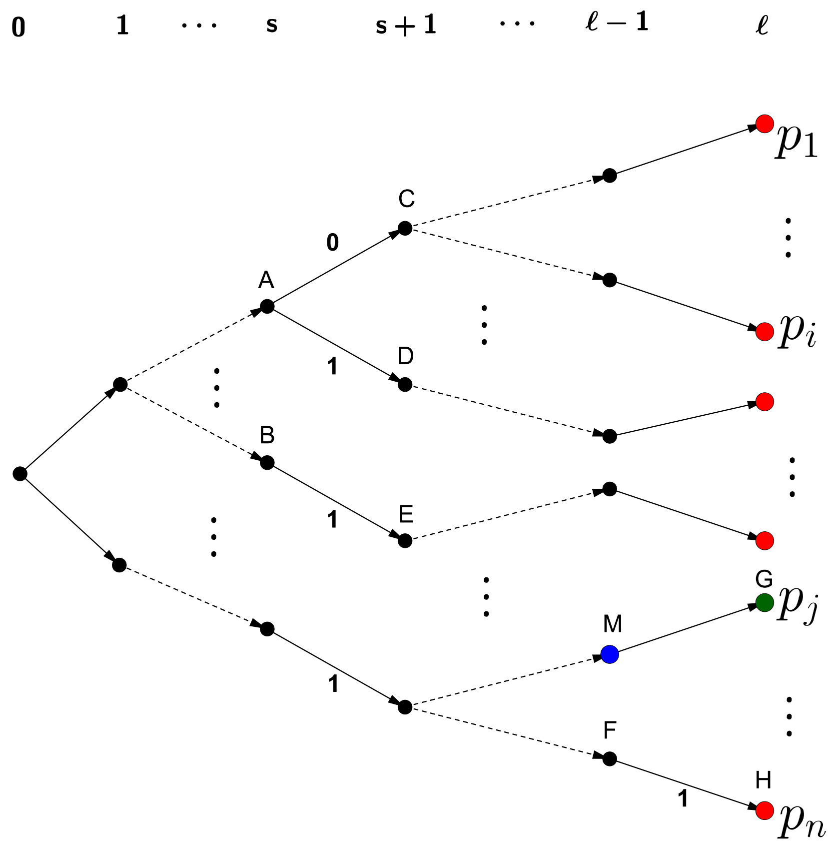
$$e_H = e_F + (1 \oplus y_{s+1}) \ln\left(\frac{b}{1-b}\right) + \ln p_n$$



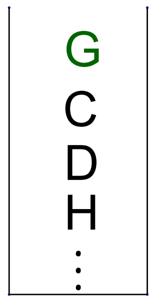
Decoding of Joint Source-Channel Block Code



Decoding of Joint Source-Channel Block Code



Node G is a leaf node
Decoding end



Modified Turbo Code

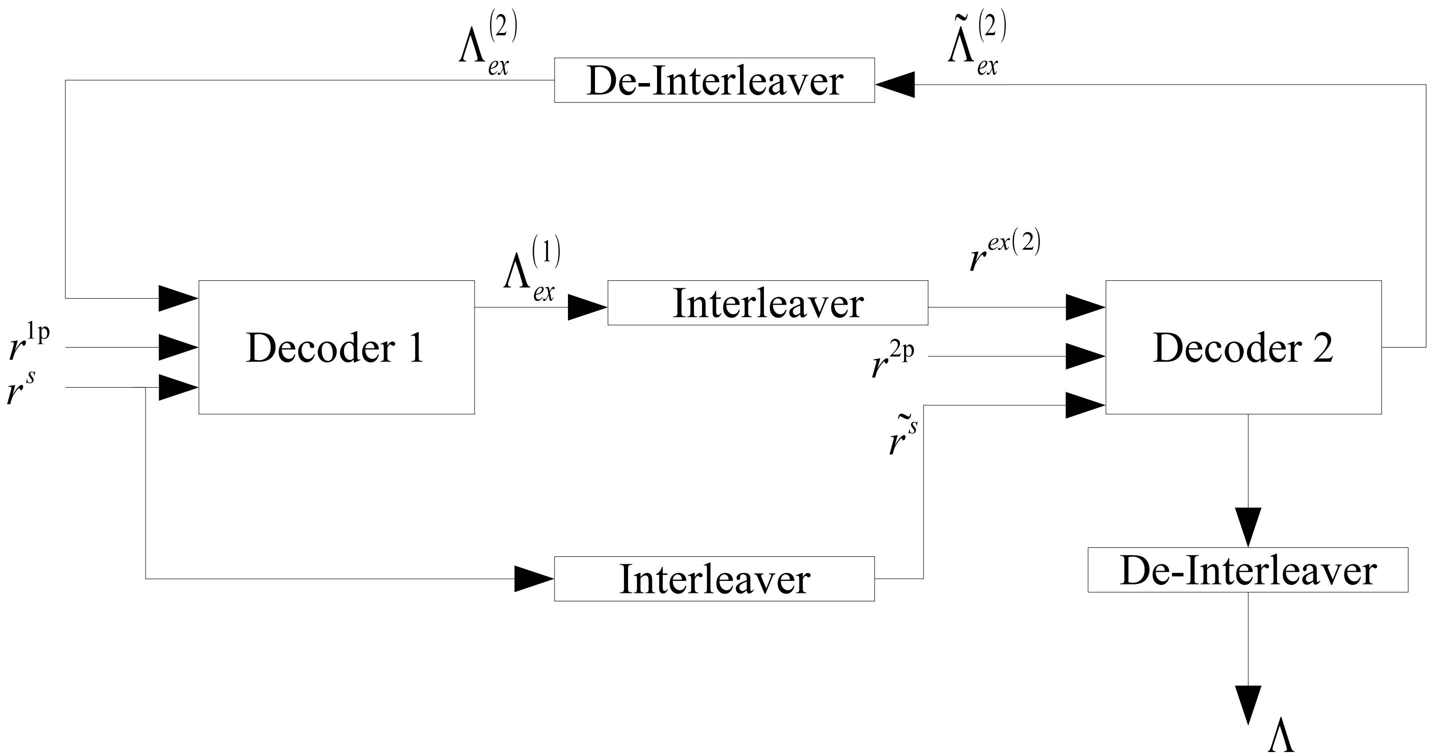
Definitions and Notations

- Let $g = \lceil \log_2(n) \rceil$.
- Binary-index the symbol "alphabetically" using g bits as follows:

$$\begin{aligned} s_1 &= (00\ldots00) \\ s_2 &= (00\ldots01) \\ s_3 &= (00\ldots10) \\ &\vdots \end{aligned}$$

- Assume the ith bit u_i depends on the previous t bits $u_{i-1}u_{i-2}\ldots u_{i-t}$.
- Denote $\boldsymbol{U}_i = u_{i-1}u_{i-2}\ldots u_{i-t} = u_{i-1}\boldsymbol{U}_{i-1}$
- S_w^i is the state w at level i.
- T_c is the set of state pair $\left(S_{\bar{w}}^{i-1}, S_w^i\right)$ such that $S_{\bar{w}}^{i-1} \xrightarrow{u_i=c} S_w^i$
- $\boldsymbol{x}_i^j = \boldsymbol{x}_i\boldsymbol{x}_{i+1}\ldots\boldsymbol{x}_{j-1}\boldsymbol{x}_j$, where $\boldsymbol{x}_i = \left(x_i^s, x_i^{1p}, x_i^{2p}\right)$.
- $\boldsymbol{x}_i \xrightarrow{\text{channel}} \boldsymbol{r}_i$, where $\boldsymbol{r} = \boldsymbol{r}_1^\ell = \boldsymbol{r}_1\boldsymbol{r}_2\ldots\boldsymbol{r}_\ell$.

Block Diagram of Turbo Decoder



Given $\{u_i\}_{i=1}^{\ell}$ is a binary uniform i.i.d process, the decoding metric for BCJR algorithm is (originally) derived as follows:

$$\begin{aligned}
\Pr\left(S_{\bar{w}}^{i-1}, S_w^i, \boldsymbol{r}\right) &= \Pr\left(S_{\bar{w}}^{i-1}, S_w^i, \boldsymbol{r}_1^{i-1}, \boldsymbol{r}_i, \boldsymbol{r}_{i+1}^{\ell}\right) \\
&= \Pr\left(\boldsymbol{r}_{i+1}^{\ell} \mid S_{\bar{w}}^{i-1}, S_w^i, \boldsymbol{r}_1^{i-1}, \boldsymbol{r}_i\right) \Pr\left(S_{\bar{w}}^{i-1}, S_w^i, \boldsymbol{r}_1^{i-1}, \boldsymbol{r}_i\right) \quad (1) \\
&= \Pr\left(\boldsymbol{r}_{i+1}^{\ell} \mid S_w^i\right) \Pr\left(S_{\bar{w}}^{i-1}, S_w^i, \boldsymbol{r}_1^{i-1}, \boldsymbol{r}_i\right) \quad (2) \\
&\vdots
\end{aligned}$$

However, if $\{u\}_1^{\ell}$ is not an i.i.d process, the equalities of (1) and (2) are no longer valid.

The Alternative Decomposition

Define

$$\alpha\left(\boldsymbol{V}_t, S_{\bar{w}}^{i-1}\right) \triangleq \Pr\left(S_{\bar{w}}^{i-1}, \boldsymbol{U}_i = \boldsymbol{V}_t \mid \boldsymbol{r}_1^{i-1}\right)$$

$$\beta\left(c, \boldsymbol{V}_t, S_w^i\right) \triangleq \Pr\left(\boldsymbol{r}_{i+1}^{\ell} \mid u_i = c, \boldsymbol{U}_i = \boldsymbol{V}_t, S_w^i\right)$$

$$\gamma\left(c, \boldsymbol{V}_t, S_{\bar{w}}^{i-1}, S_w^i\right) \triangleq \Pr\left(u_i = c, S_w^i, \boldsymbol{r}_i \mid S_{\bar{w}}^{i-1}, \boldsymbol{U}_i = \boldsymbol{V}_t\right)$$

- $\Lambda^{(1)}(i) = \ln \dfrac{\sum\limits_{\left(S_{\bar{w}}^{i-1}, S_w^i\right) \in T_1} \sum\limits_{\boldsymbol{V}_t \in \{0,1\}^t} \alpha\left(\boldsymbol{V}_t, S_{\bar{w}}^{i-1}\right)\gamma\left(1, \boldsymbol{V}_t, S_{\bar{w}}^{i-1}, S_w^i\right)\beta\left(1, \boldsymbol{V}_t, S_w^i\right)}{\sum\limits_{\left(S_{\bar{w}}^{i-1}, S_w^i\right) \in T_0} \sum\limits_{\boldsymbol{V}_t \in \{0,1\}^t} \alpha\left(\boldsymbol{V}_t, S_{\bar{w}}^{i-1}\right)\gamma\left(0, \boldsymbol{V}_t, S_{\bar{w}}^{i-1}, S_w^i\right)\beta\left(0, \boldsymbol{V}_t, S_w^i\right)}$

Calculation of γ

- $\bar{M}$ is the estimated mean of $r_i^{\mathrm{ex}(2)}$, i.e., $\bar{M} = \frac{1}{\ell}\sum_{i=1}^{\ell}\left|r_i^{\mathrm{ex}(2)}\right|$.
- $\bar{\sigma}^2$ is the estimated variance of $r_i^{\mathrm{ex}(2)}$, i.e., $\bar{\sigma}^2 = \frac{1}{\ell-1}\sum_{i=1}^{\ell}\left(\mid r_i^{\mathrm{ex}(2)} \mid - \bar{M}\right)^2$
- $$\Rightarrow \Pr\left(r_i^{\mathrm{ex}(2)}\middle| u_i = c\right) = \begin{cases} 1, & \text{if } \bar{\sigma}^2 = 0 \\ \frac{1}{\sqrt{2\pi\bar{\sigma}^2}} \exp\left\{ -\frac{\left[r_i^{\mathrm{ex}(2)} - (2u_i - 1)\,\bar{M}\right]^2}{2\bar{\sigma}^2} \right\}, & \text{otherwise} \end{cases}$$
- The function γ is then given as:

$$\begin{aligned} &\gamma\left(c, \boldsymbol{V}_t, S_{\bar{w}}^{i-1}, S_w^i\right) \\ &= \Pr\left(u_i = c \mid \boldsymbol{U}_i = \boldsymbol{V}_t\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{\left(r_i^s - x_i^s\right)^2 + \left(r_i^{(1p)} - x_i^{(1p)}\right)^2}{2\sigma^2} \right\} \\ &\quad \times \Pr\left(r_i^{\mathrm{ex}(2)} \mid u_i = c\right). \end{aligned}$$

Recursively Computation of α

$$\alpha\left(\boldsymbol{V}_t, S_{\bar{w}}^{i-1}\right) \triangleq \Pr\left(S_{\bar{w}}^{i-1}, \boldsymbol{U}_i = \boldsymbol{V}_t \mid \boldsymbol{r}_1^{i-1}\right)$$

Case 1. $i \neq mg + 1$, where $g = \lceil \log_2(n) \rceil$.

$$\alpha\left(\boldsymbol{V}_t, S_{\bar{w}}^{i-1}\right) = \frac{\sum\limits_{w': \left(S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right) \in T_{v_1}} \alpha\left(\boldsymbol{V}_{t-1}, S_{w'}^{i-2}\right) \gamma\left(v_1, \boldsymbol{V}_{t-1}, S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right)}{\sum\limits_{\bar{w}=0}^{15} \sum\limits_{\boldsymbol{V}_t \in \{0,1\}^t} \sum\limits_{w': \left(S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right) \in T_{v_1}} \alpha\left(\boldsymbol{V}_{t-1}, S_{w'}^{i-2}\right) \gamma\left(v_1, \boldsymbol{V}_{t-1}, S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right)}.$$

Recursively Computation of α

Case 2. $i = mg + 1$

$$\alpha\left(\boldsymbol{V}_0, S_{\bar{w}}^{i-1}\right) \frac{\sum_{c'=0}^{1} \sum_{w':\left(S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right) \in T_{c'}} \Pr\left(S_{w'}^{i-2} \mid \boldsymbol{r}_1^{i-2}\right) \Pr\left(S_{\bar{w}}^{i-1}, \boldsymbol{r}_{i-1} \mid S_{w'}^{i-2}\right)}{\sum_{\bar{w}=0}^{15} \sum_{c'=0}^{1} \sum_{w':\left(S_{w'}^{i-2}, S_{\bar{w}}^{i-1}\right) \in T_{c'}} \Pr\left(S_{w'}^{i-2} \mid \boldsymbol{r}_1^{i-2}\right) \Pr\left(S_{\bar{w}}^{i-1}, \boldsymbol{r}_{i-1} \mid S_{w'}^{i-2}\right)},$$

where

$$\Pr\left(S_{w'}^{i-2} \mid \boldsymbol{r}_1^{i-2}\right) = \sum_{\boldsymbol{V}_g \in \{0,1\}^g} \alpha\left(\boldsymbol{V}_g S_{w'}^{i-2}\right),$$

and

$$\Pr\left(S_{\bar{w}}^{i-1}, \boldsymbol{r}_{i-1} \mid S_{w'}^{i-2}\right)$$

$$= \Pr\left(u_{i-1} = c'\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{\left(r_i^s - x_i^s\right)^2 + \left(r_i^{(1p)} - x_i^{(1p)}\right)^2}{2\sigma^2}\} \Pr\left(r_i^{\mathrm{ex}(2)} \mid u_i = c\right).$$

Recursively Computation of β

$$\beta\left(c, \boldsymbol{V}_t, S_w^i\right) \triangleq \Pr\left(\boldsymbol{r}_{i+1}^{\ell} \mid u_i = c, \boldsymbol{U}_i = \boldsymbol{V}_t, S_w^i\right)$$

Case 1. $i \neq mg$.

$$\beta\left(c, \boldsymbol{V}_t, S_w^i\right) = \sum_{c'=0}^{1} \beta\left(c', \boldsymbol{V}_{t+1}, S_{\bar{w}}^{i+1}\right) \gamma\left(c', \boldsymbol{V}_{t+1}, S_w^i, S_{\bar{w}}^{i+1}\right).$$

Case 2. $i = mg$.

$$\beta\left(c, \boldsymbol{V}_t, S_w^i\right) = \sum_{c'=0}^{1} \beta\left(c', \boldsymbol{V}_0, S_{\bar{w}}^{i+1}\right) \gamma\left(c', \boldsymbol{V}_0, S_w^i, S_{\bar{w}}^{i+1}\right).$$

Initial Values of α and β

(Suppose there are 16 states.)

$$\alpha\left(\boldsymbol{V}_0, S_0^0\right) = 1 \;,\; \alpha\left(\boldsymbol{V}_0, S_w^0\right) = 0 \text{ for every } \; 0 < w \leq 16$$

$$\beta\left(0, \boldsymbol{V}_k, S_w^\ell\right) = \beta\left(1, \boldsymbol{V}_k, S_w^\ell\right) = \frac{1}{2} \text{ for every } 0 \leq w \leq 16$$

where $k \equiv (\ell - 1) \bmod g$.

Extrinsic Information

$$\Lambda^{(1)}(i) = \Lambda^{(1)}_{\mathrm{ch}}(i) + \Lambda^{(1)}_{\mathrm{ap}}(i) + \Lambda^{(1)}_{\mathrm{ex}}(i)$$

$$\begin{aligned}
\Longleftrightarrow \Lambda^{(1)}_{\mathrm{ex}}(i) &= \Lambda^{(1)}(i) - \Lambda^{(1)}_{\mathrm{ch}}(i) - \Lambda^{(1)}_{\mathrm{ap}}(i) \\
&= \Lambda^{(1)}(i) - \ln \frac{\Pr\left(r_i^s \mid u_i = 1\right)}{\Pr\left(r_i^s \mid u_i = 0\right)} - \ln \frac{\Pr\left(r_i^{(ex)} \mid u_i = 1\right)}{\Pr\left(r_i^{(ex)} \mid u_i = 0\right)} \\
&= \begin{cases} \Lambda^{(1)}(i) - \frac{2}{\sigma^2} r_i^s & \text{if } \bar{\sigma}^2 = 0 \\ \Lambda^{(1)}(i) - \frac{2}{\sigma^2} r_i^s - \frac{2\bar{M}}{\bar{\sigma}^2} r_i^{(ex)} & \text{otherwise} \end{cases}
\end{aligned}$$

Adjust the Extrinsic Information for Decoder 2

- Implicitly treat the information bit sequence after interleaving as an i.i.d uniform process.
- $\{\Lambda_{\text{ex}}^{(2)}\}_1^\ell$ is the de-interleaved sequence for $\{\tilde{\Lambda}_{\text{ex}}^{(2)}\}_1^\ell$.

$$
\begin{aligned}
&\Lambda_{\text{ex}}^{(2)}(i) \\
&= v\Lambda_{\text{ex}}^{(2)}(i) + (1-v)\ln\left[\frac{\displaystyle\sum_{\boldsymbol{V}_t\in\{0,1\}^t} \Pr(u_i = 1 \mid \boldsymbol{U}_i = \boldsymbol{V}_t)\Pr(\hat{u}_{i-t} = v_t)\ldots\Pr(\hat{u}_{i-1} = v_1)}{\displaystyle\sum_{\boldsymbol{V}_t\in\{0,1\}^t} \Pr(u_i = 0 \mid \boldsymbol{U}_i = \boldsymbol{V}_t)\Pr(\hat{u}_{i-t} = v_t)\ldots\Pr(\hat{u}_{i-1} = v_1)}\right]
\end{aligned}
$$

where

$$
\Pr(\hat{u}_i = 0) = \frac{1}{1 + e^{\Lambda_{\text{ex}}^{(2)}(i)}}, \quad \Pr(\hat{u}_i = 1) = \frac{e^{\Lambda_{\text{ex}}^{(2)}(i)}}{1 + e^{\Lambda_{\text{ex}}^{(2)}(i)}},
$$

and

$$
v = \begin{cases} 1, & \text{if } (i-1) \bmod g = 0 \\ 0.85, & \text{otherwise} \end{cases}
$$

Simulation Results

System Setting

For the joint source-channel block code, we use three different first-order Markov sources to generate the source distribution.

Case 1.

$$\Pr(u_i = 0 \mid u_{i-1} = 0) = \Pr(u_i = 1 \mid u_{i-1} = 1) = 0.9$$
$$\Pr(u_1 = 0) = 0.9$$

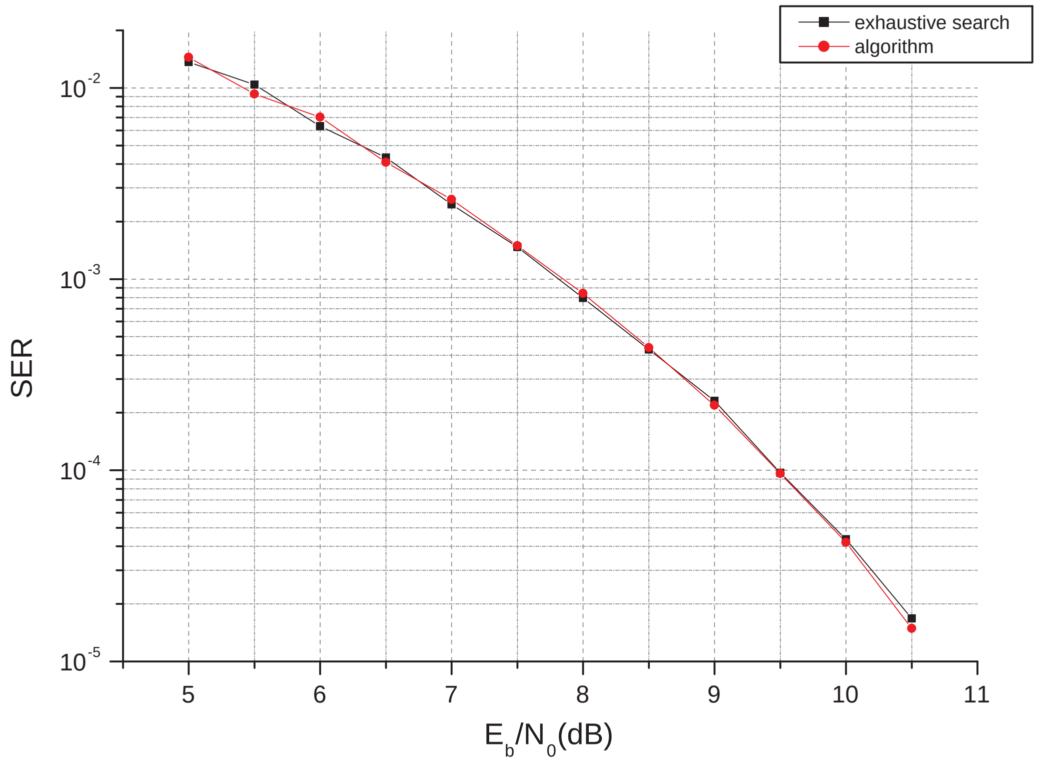
Case 2.

$$\Pr(u_i = 0 \mid u_{i-1} = 0) = \Pr(u_i = 1 \mid u_{i-1} = 1) = 0.95$$
$$\Pr(u_1 = 0) = 0.5$$

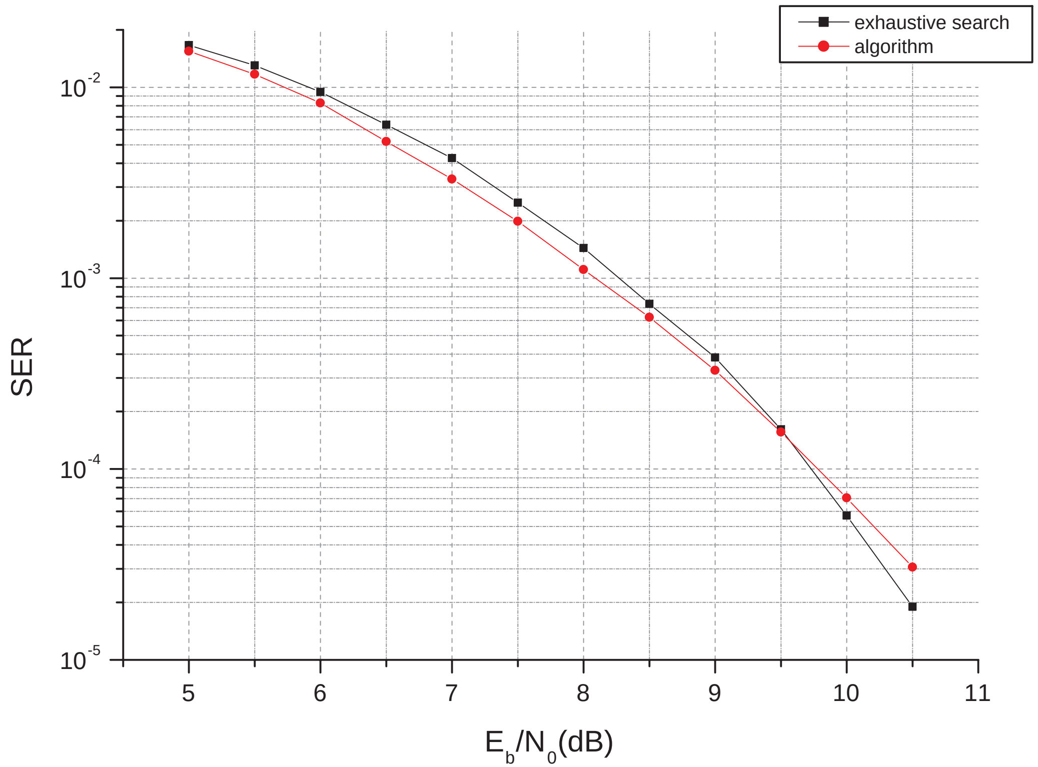
Case 3.

$$\Pr(u_i = 0 \mid u_{i-1} = 0) = \Pr(u_i = 1 \mid u_{i-1} = 1) = 0.55$$
$$\Pr(u_1 = 0) = 0.55$$

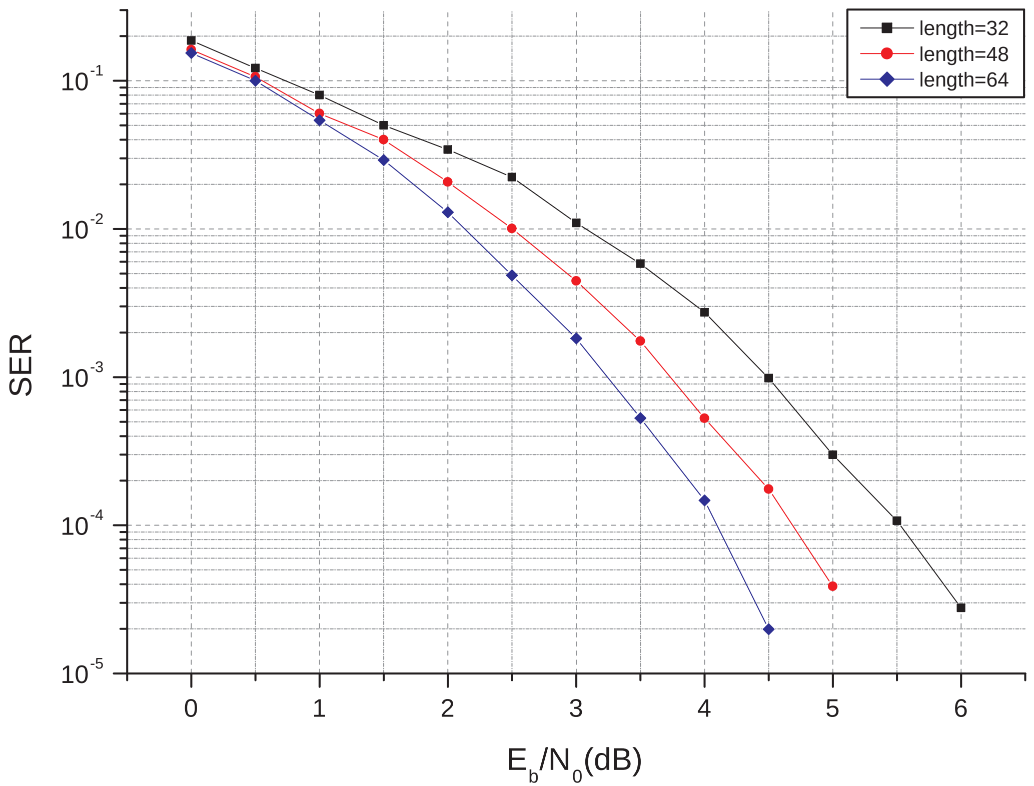
Simulation Results of JSC Block Code



Performance comparison between the JSC block code constructed by exhaustive search and that by our proposed algorithm with codeword length $\ell = 8$. The source follows Case 1 and hard-decision decoding is employed.

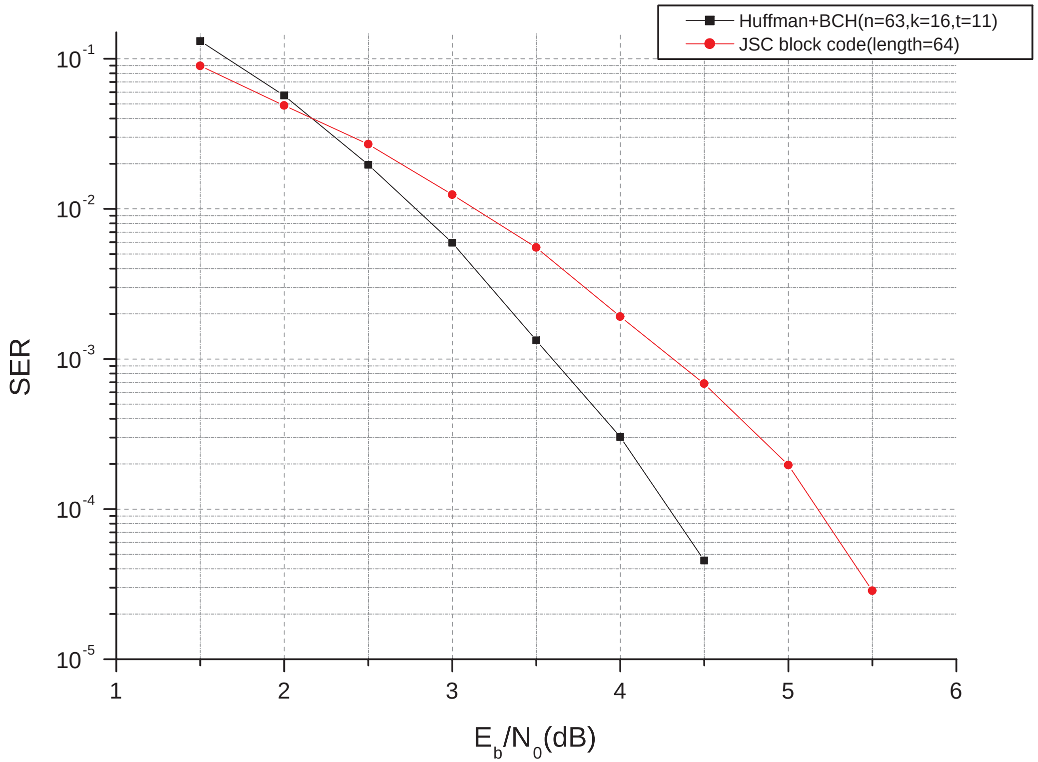


Performance comparison between the JSC block code constructed by exhaustive search and that by our proposed algorithm with codeword length $\ell = 8$. The source follows Case 2 and hard-decision decoding is employed.



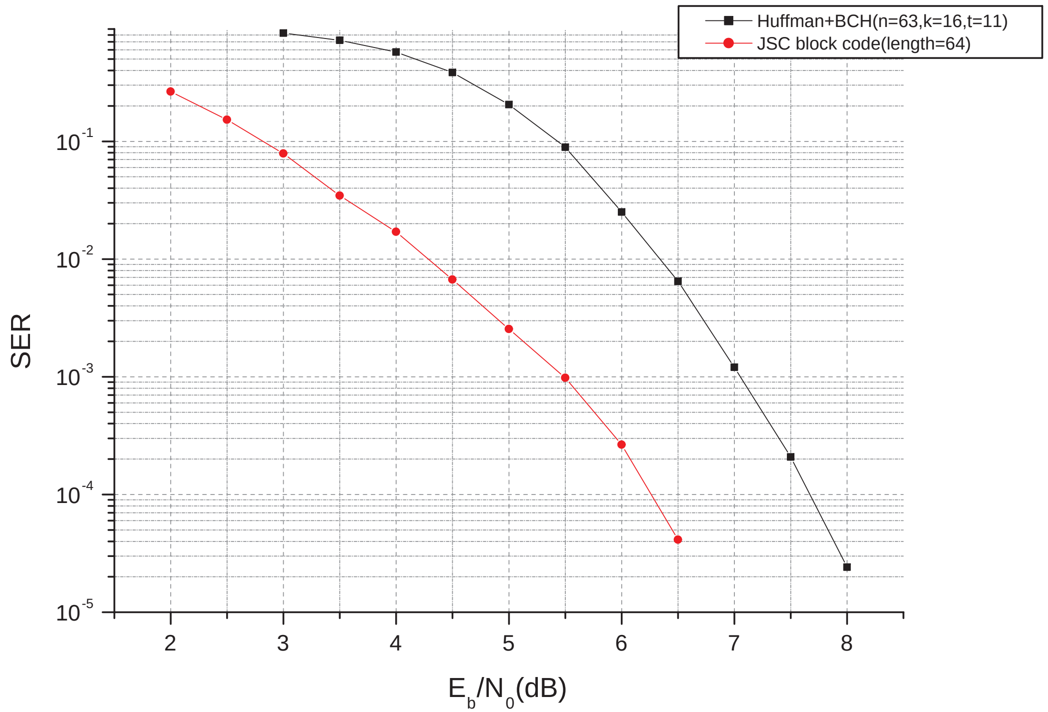
Performance comparison of the proposed joint source-channel block codes for different codeword length. The source follows Case 1 and soft-decision decoding is employed.

Simulation Results of JSC Block Code

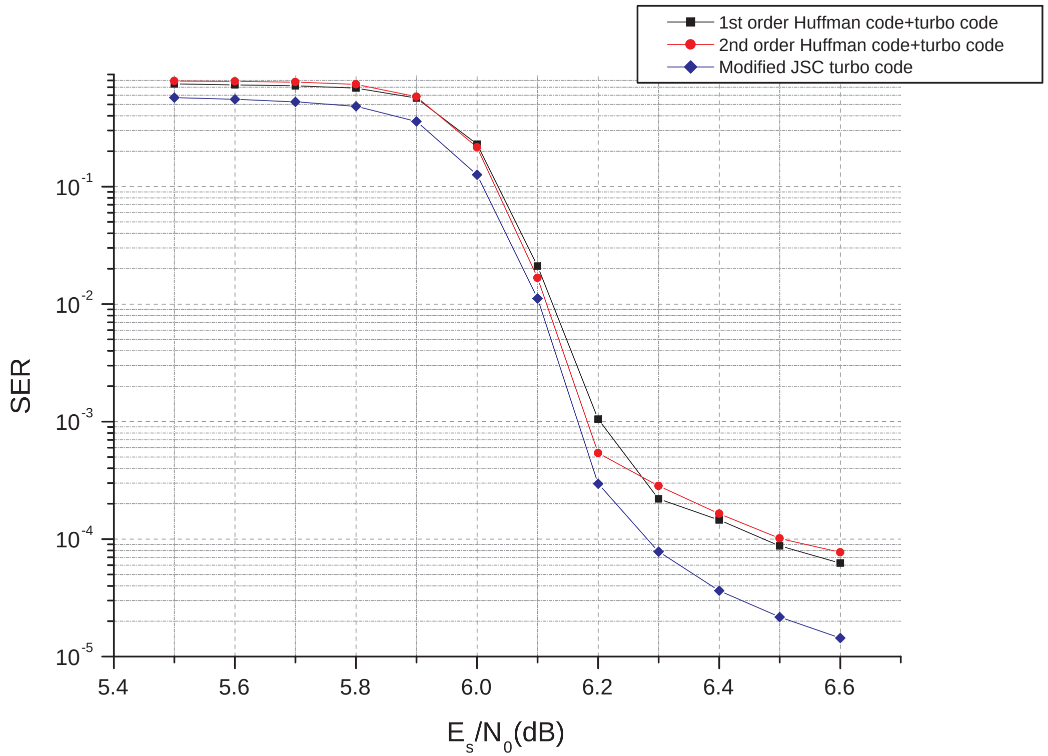


Performance comparison between the joint source-channel block code with codeword length 64 and the tandem scheme (i..e, the Huffman code + $(63, 16, 11)$ BCH code). The source follows Case 1 and hard-decision decoding is employed.

Simulation Results of JSC Block Code

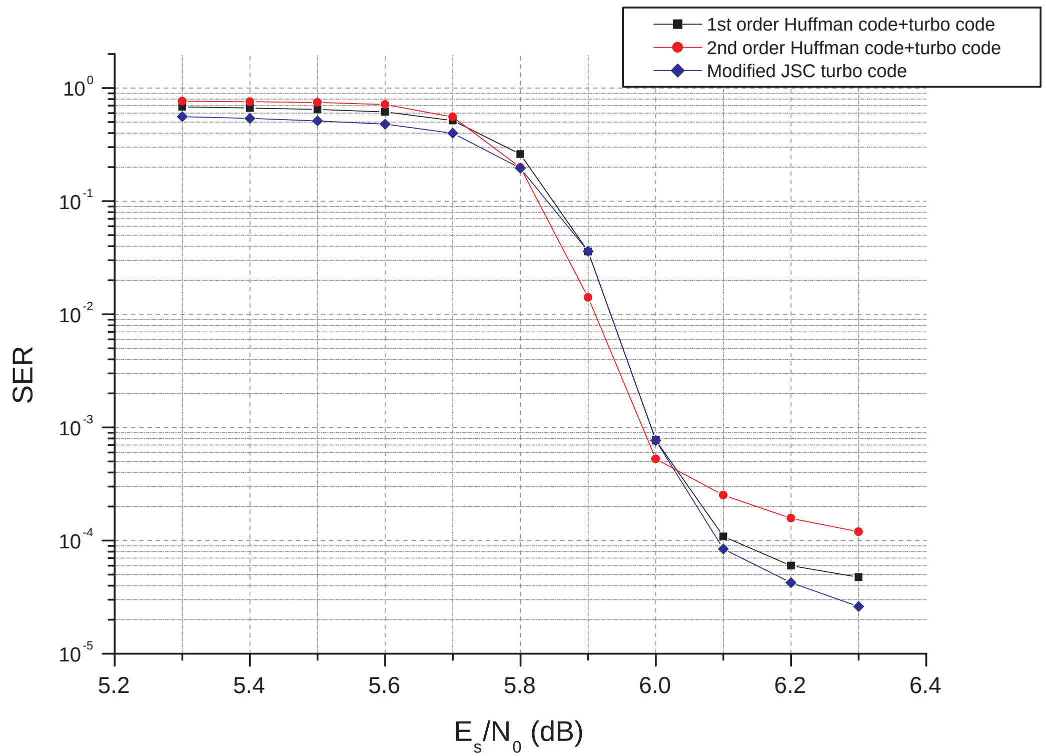


Performance comparison between the joint source-channel block code with codeword length 64 and the tandem scheme (i..e, the Huffman code + $(63, 16, 11)$ BCH code). The source follows Case 3 and hard-decision decoding is employed.



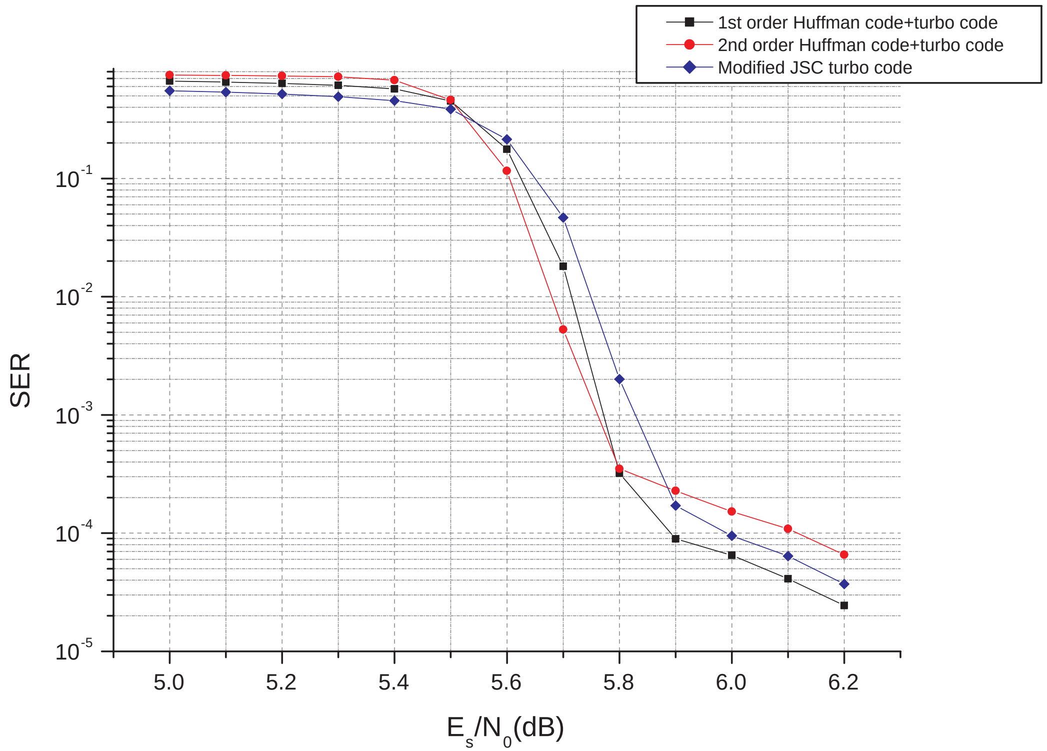
Performance comparison between the joint source-channel turbo code and the tandem scheme (i.e., the Huffman code + a traditional turbo code) under the source with per-letter source entropy 3.9711 bits. The interleaver size is $\ell = 128 \times 128$.

Simulation Results of Modified Turbo Code



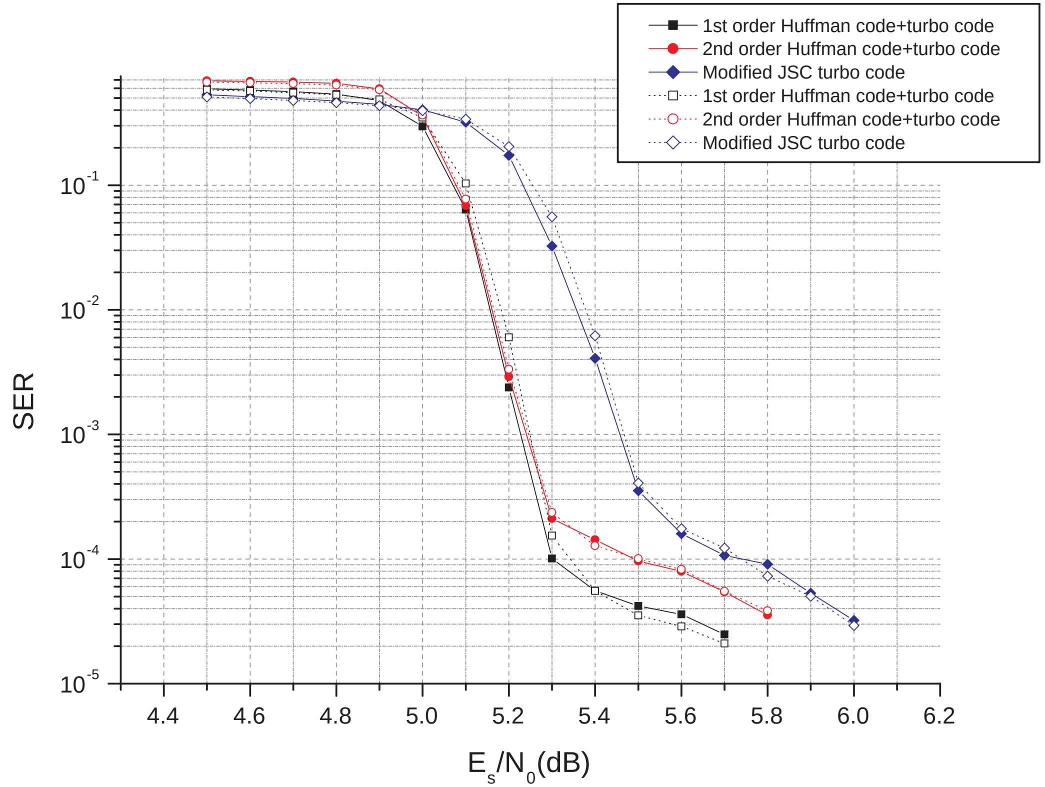
Performance comparison between the joint source-channel turbo code and the tandem scheme (i.e., the Huffman code + a traditional turbo code) under the source with per-letter source entropy 3.79021 bits. The interleaver size is $\ell = 128 \times 128$.

Simulation Results of Modified Turbo Code

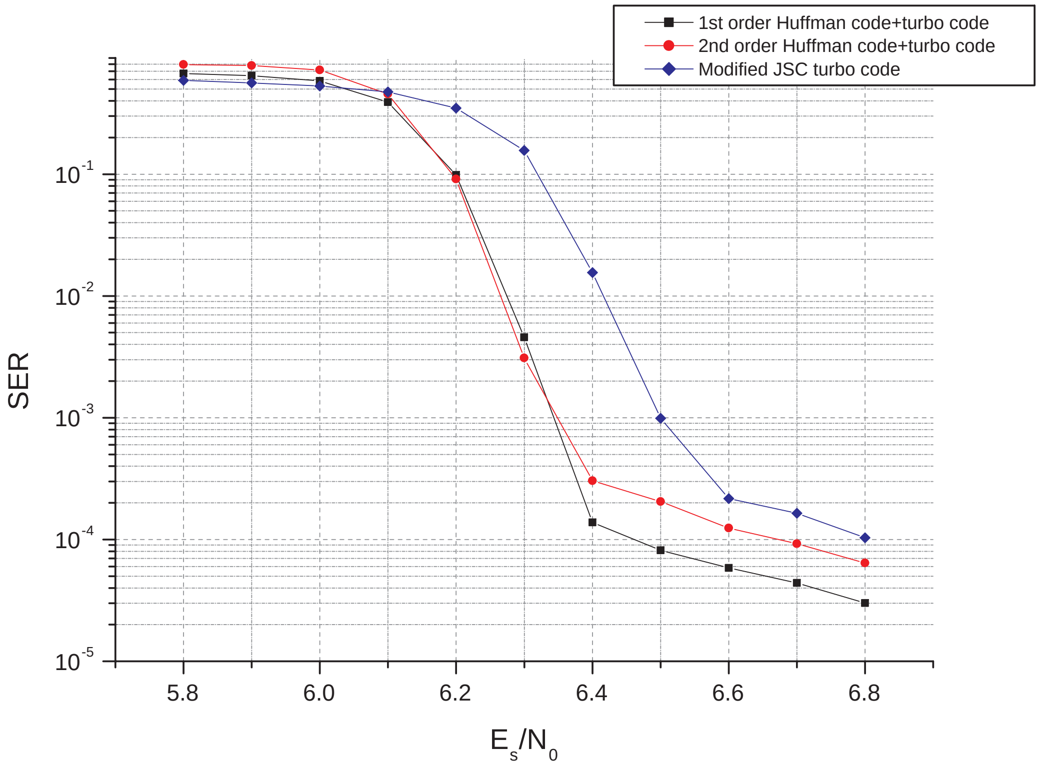


Performance comparison between the joint source-channel turbo code and the tandem scheme (i.e., the Huffman code + a traditional turbo code) under the source with per-letter source entropy 3.59095 bits. The interleaver size is $\ell = 128 \times 128$.

Simulation Results of Modified Turbo Code

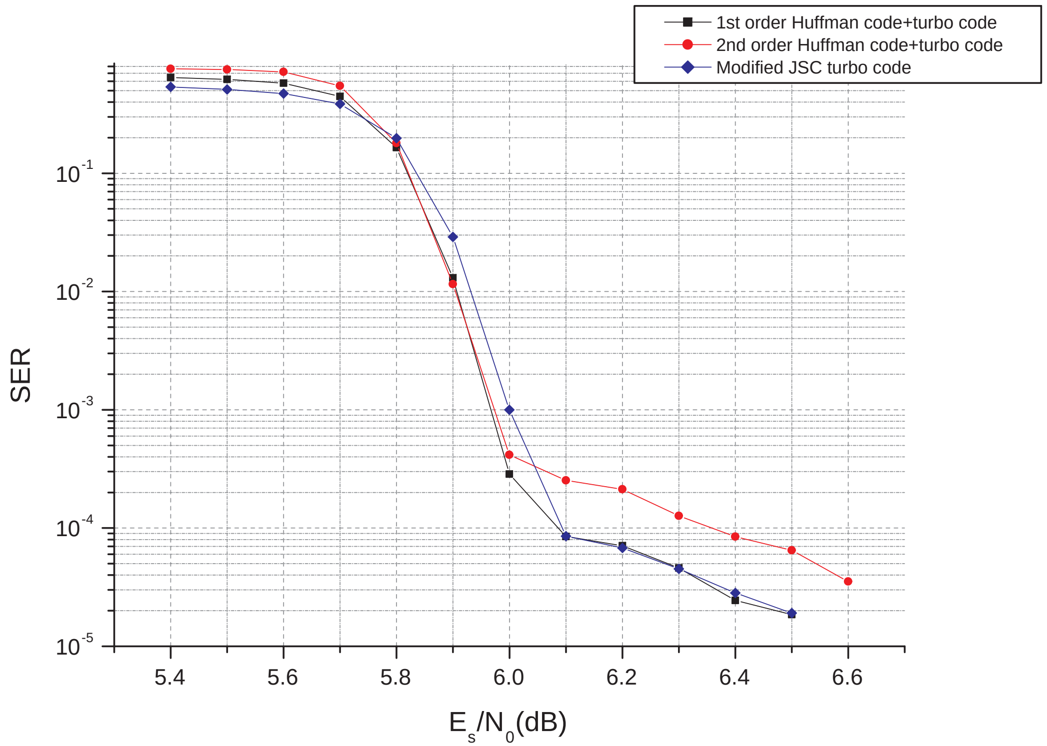


Performance comparison under two different source distributions with the same per-letter source entropy 3.20 bits. The interleaver size is $\ell = 128 \times 128$.

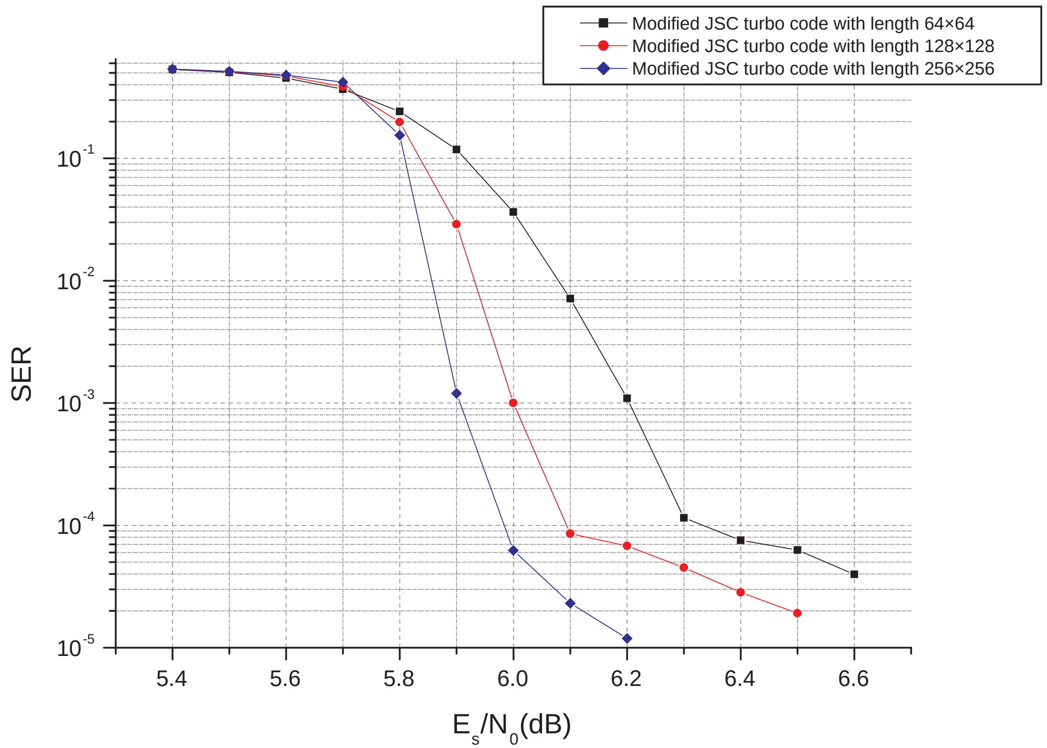


Performance comparison between the joint source-channel turbo code and the tandem scheme (i.e., the Huffman code + a traditional turbo code) under the 26-English-letter text source. The interleaver size is $\ell = 128 \times 128$.

Simulation Results of Modified Turbo Code



Performance comparison between the joint source-channel turbo code and the tandem scheme (i.e., the Huffman code + a traditional turbo code) under the normalized 16-English-letter text source. The interleaver size is $\ell = 128 \times 128$.



Performance comparison of the modified JSC turbo code by testing different interleaver sizes under the normalized 16-English-letter text source.

Conclusion

- For a given source distribution and codeword length, we suggest an algorithm to construct the JSC block code. A corresponding low-complexity sub-optimal decoding algorithm is also proposed.

- We additionally propose to modify the turbo decoder specifically for non-uniform sources and long block length.

- For both methods, the proposed joint source-channel codes outperform the respective tandem scheme when the Huffman compression efficiency of the target source is low.

Thank You for Your Attention.

Appendix

Source distribution of 26 English alphabet symbols. The source entropy is 4.12091 bits.

English alphabet	probability	English alphabet	probability
E	0.14878610	T	0.09354149
A	0.08833733	O	0.07245769
R	0.06872164	N	0.06498532
H	0.05831331	I	0.05644515
S	0.05537763	D	0.04376834
L	0.04123298	U	0.02762209
P	0.02575393	F	0.02455297
M	0.02361889	C	0.02081665
W	0.01868161	G	0.01521216
Y	0.01521216	B	0.01267680
V	0.01160928	K	0.00867360
X	0.00146784	J	0.00080064
Q	0.00080064	Z	0.00053376

Appendix

Normalized source distribution of the 16 most probable English letters. The source entropy is 3.78611 bits.

English alphabet	probability	English alphabet	probability
E	0.1627270	T	0.1023060
A	0.0966141	O	0.0792466
R	0.0751605	N	0.0710741
H	0.0637770	I	0.0617338
S	0.0605662	D	0.0478692
L	0.0450963	U	0.0302101
P	0.0281670	F	0.0268535
M	0.0258319	C	0.0227671