A Theoretic Study on Three Low-Density Parity-Check and BCH Concatenated Coding Schemes

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- Technical Background
  - Three concatenated coding schemes: SBC-MLC, MBC-SLC and MBC-MLC
  - Decoding of BCH codes
- Analysis of the BER performances of Three Concatenated Coding Schemes
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Motivation and Contribution

• NAND flash memory has to use error-correction codes (ECCs) to ensure the data storage integrity.

• Bose-Chaudhuri-Hocquenghem (BCH) codes are one of the most powerful codes which are widely used in NAND flash memory in past years.

• A technique called Multi-Level Cell (MLC) is used in flash memory.

• Although BCH codes have been found many advantages specially suitable for the Multi-Level Cell (MLC) NAND flash memories, the storage reliability of NAND flash memory inevitably degrades when an aggressive use of MLC storage is pursued.

• This makes BCH codes inadequate for this purpose, and hence the industry naturally demands the search for more powerful ECCs.
Motivation and Contribution

• Low-density parity-check (LDPC) code is a class of powerful error control codes which has been found the potential in future solid-state drives (SSDs).

• It has an obvious drawback called error floor that prevents it from being used in the flash memory.

• It is known that the error floors of LDPC codes are perhaps a consequence of a certain number of bit errors circulated in the iterative decoding.

• A natural solution to lowering the floor is to concatenate the LDPC code with a high-rate outer algebraic code to clean up the residual systematic errors.
Motivation and Contribution

• In this thesis, we consider the concatenated scheme for BCH code and LDPC code, which means a BCH code is used as the outer code and a LDPC code is used as the inner code.

– Consider three concatenated coding schemes: SBC-MLC, MBC-SLC, and MBC-MLC.

– Derive the mathematical formulations such that discussion regarding whether these schemes can improve the error floor or not can be done without simulations.

– Selection of the BCH Code in the MBC-SLC scheme is then remarked based on the mathematical formulations.
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Scheme 1: SBC-MLC

Figure 1: Structure of the SBC-MLC coding scheme
Scheme 2: MBC-SLC

Figure 2: Structure of the MBC-SLC coding scheme
Scheme 3: MBC-MLC

Figure 3: Structure of the MBC-MLC coding scheme
Decoding of BCH Code

- The algorithm we used in BCH code is basically a bounded distance decoder.

- The received sequence can be decoded to a valid codeword as long as this sequence lies in a codeword sphere of radius $t$.

- If a sequence with $i$ errors is sent to a BCH decoder, and if the BCH decoder decodes it to a valid but wrong codeword, and also if the $t$ positions of reversed bits caused by the BCH decoder are different from the positions of these $i$ errors, the number of errors in this codeword can be as large as $t + i$. 
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  – Analysis of the BER performance of the SBC-MLC
  – Analysis of the BER performance of the MBC-SLC
    * Selection of the BCH Code in the MBC-SLC
  – Analysis of the BER performance of the MBC-MLC

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• Conclusion
Premise of Our Analysis

- Assume that the error number distribution \( \{ P_i, \forall 1 \leq i \leq k_{LDPC} \} \) is known.
  - \( P_i \) is the probability that \( i \) errors occur after the decoding of the LDPC code.

- Assume that the error number distribution is almost the same in the error floor region.

- Although our analysis can be applied to the waterfall region, we mainly focus on the BER improvement in the error floor region.
Analysis of the BER performance of the SBC-MLC

- The average BER performance of the information bits for $\beta$ LDPC codes can be formulated as:

$$I_{\text{BER, LDPC}}^{(\beta)} = \frac{\sum_{i=1}^{\beta \cdot k_{\text{LDPC}}} i \cdot S_{i}^{(\beta)}}{\beta \cdot k_{\text{LDPC}}},$$

(1)

where

$$S_{i}^{(\beta)} = \sum_{x_{1},x_{2},...,x_{\beta}} P_{x_{1}} \cdots P_{x_{\beta}}$$

$$x_{1} + \cdots + x_{\beta} = i,$$

$$0 \leq x_{1},x_{2},...,x_{\beta} \leq k_{\text{LDPC}}$$

is the probability that the sum of errors observed at the outputs of $\beta$ LDPC code decoders is equal to $i$, and $x_{m}$ denotes the number of errors in the $m$-th LDPC code block.

Note that

$$\beta \cdot k_{\text{LDPC}} = n_{\text{BCH}}.$$
Analysis of the BER performance of the SBC-MLC

- (1) can be separated into two parts, i.e.,

\[
I_{BER,LDPC}^{(β)} = \frac{\sum_{i=1}^{t} i \cdot S_i^{(β)}}{n_{BCH}} + \frac{\sum_{i=t+1}^{n_{BCH}} i \cdot S_i^{(β)}}{n_{BCH}},
\]

(2)

- After the decoding of the BCH code, (2a) can be removed.
- Assume that (2b) has a probability \( P_e \) to decode to a valid but wrong codeword; hence, the overall BER for the concatenated coding scheme can be upper-bounded by:

\[
I_{BER,SBC-MLC} \leq \frac{\sum_{i=t+1}^{n_{BCH}} (P_e \cdot (t + i) + (1 - P_e) \cdot i) \cdot S_i^{(β)}}{n_{BCH}}.
\]

(3)
Analysis of the BER performance of the SBC-MLC

- We then compare the BERs before and after applying the BCH codes. The difference can be written as:

\[
I_{\text{BER,LDPC}}^{(\beta)} - I_{\text{BER,SBC-MLC}}^{(\beta)} \geq \frac{\sum_{i=1}^{t} i \cdot S_i^{(\beta)}}{n_{\text{BCH}}} - \frac{\sum_{i=t+1}^{n_{\text{BCH}}} (P_e \cdot t) \cdot S_i^{(\beta)}}{n_{\text{BCH}}}. \tag{4}
\]

- From (4), we note that the selection of parameter \( t \) is important. This parameter, for example, will decide whether the second term in (4) is dominant or not.

- We can then estimate the system BER \( I_{\text{BER,SBC-MLC}} \) of the SBC-MLC coding scheme by selecting a proper \( t \) as:

\[
I_{\text{BER,SBC-MLC}} \approx I_{\text{BER,LDPC}}^{(\beta)} - \frac{\sum_{i=1}^{t} i \cdot S_i^{(\beta)}}{n_{\text{BCH}}}. \tag{5}
\]
Analysis of the BER performance of the MBC-SLC

- In this scheme, the average BER performance of the LDPC code decoding can be simplified to:

\[
I_{BER,LDPC} = \frac{\sum_{i=1}^{k_{LDPC}} P_i \cdot i}{k_{LDPC}}.
\]  (6)
Analysis of the BER performance of the MBC-SLC

- By targeting the average BER performance for this concatenated scheme, a uniform interleaver is assumed to be applied onto the LDPC code decoding outputs. We can re-write (6) as

\[
I_{BER,LDPC} = \frac{\sum_{i=1}^{k_{LDPC}} P_i \cdot i \cdot \sum_{w \in J} M(w, i)}{k_{LDPC}},
\]

where

\[
J \triangleq \left\{ (x_1, x_2, \cdots, x_\beta); \sum_{\ell=1}^{\beta} x_\ell = i, 0 \leq x_\ell \leq n_{BCH} \right\}.
\]

\(w = (x_1, x_2, \cdots, x_\beta)\) is an error combination from set \(J\), and

\[
M(w, i) = \frac{\binom{n_{BCH}}{x_1} \binom{n_{BCH}}{x_2} \cdots \binom{n_{BCH}}{x_\beta}}{\binom{k_{LDPC}}{i}},
\]

Note that \(\sum_{w \in J} M(w, i) = 1\).
Analysis of the BER performance of the MBC-SLC

We then observe that

\[ i \cdot \sum_{w \in \mathcal{J}} M(w, i) = \sum_{w \in \mathcal{J}} i \cdot M(w, i) = \sum_{w \in \mathcal{J}} \left( \sum_{n=1}^{\beta} x_n \right) M(w, i) \]

and separate the inner summation in (7) into two parts:

\[ \sum_{n=1}^{\beta} x_n = \sum_{n \in \mathcal{I}_w} x_n + \sum_{n \in \overline{\mathcal{I}}_w} x_n, \]

where

\[ \mathcal{I}_w \triangleq \{ \ell; 0 \leq x_\ell \leq t, 1 \leq \ell \leq \beta, (x_1, x_2, \ldots, x_\beta) = w \}, \]

and \( \overline{\mathcal{I}}_w \) is the complement of \( \mathcal{I}_w \).
Analysis of the BER performance of the MBC-SLC

• This renders:

\[
I_{\text{BER,LDPC}} = \frac{\sum_{i=1}^{k_{\text{LDPC}}} P_i \sum_{w \in J} \left( \sum_{n \in I_w} x_n \right) M(w, i)}{k_{\text{LDPC}}} \quad (8a)
\]

\[
+ \frac{\sum_{i=1}^{k_{\text{LDPC}}} P_i \sum_{w \in J} \left( \sum_{k \in I_w} x_k \right) M(w, i)}{k_{\text{LDPC}}} \quad (8b)
\]

• After performing the BCH decoding, (8a) will be eliminated, but the BCH coding blocks in (8b) will have probability \( P_e \) to be decoded to a wrong BCH codeword and probability \( 1 - P_e \) of unsuccessfully decoding.
Analysis of the BER performance of the MBC-SLC

- Along this thinking, and denoting by $|\mathcal{I}_w|$ the cardinality of $\mathcal{I}_w$, we can upper-bound the expected value of $\sum_{k \in \mathcal{I}_w} x_k$ in (8b) by

$$E_{U,\mathcal{I}_w} = \sum_{m=0}^{\mathcal{I}_w} \left( \sum_{k \in \mathcal{I}_w} x_k + t \cdot m \right) \cdot P_{m | \mathcal{I}_w}|$$  \hspace{1cm} (9)

where

$$P_{m | \mathcal{I}_w}| = \binom{|\mathcal{I}_w|}{m} P_e^m (1 - P_e)^{|\mathcal{I}_w| - m}$$

is the probability that $m$ BCH code blocks in set $\mathcal{I}_w$ decode to valid but wrong codewords.

- The upper bound of overall average coded BER is therefore

$$I_{BER, MBC-SLC}^U = \frac{\sum_{i=1}^{k_{LDPC}} P_i \cdot \sum_{w \in \mathcal{J}} E_{U,\mathcal{I}_w} \cdot M(w, i)}{k_{LDPC}}. \hspace{1cm} (10)$$
Analysis of the BER performance of the MBC-SLC

- To examine whether this scheme can improve the error floor in average or not, we have:

\[
I_{\text{BER,LDPC}} - I_{\text{BER,MBC-SLC}} \\
\geq I_{\text{BER,LDPC}} - I_{\text{BER,MBC-SLC}}^U \\
= \sum_{i=1}^{k_{\text{LDPC}}} P_i \sum_{w \in J} \left( \sum_{n \in \mathcal{I}_w} x_n - \sum_{m=0}^{\left| \mathcal{I}_w \right|} (t \cdot m) \cdot P_m \left| \mathcal{I}_w \right| \right) M(w, i)
\]

(11)
Analysis of the BER performance of the MBC-SLC

- Hence, if we can choose $t$, of which the code rate loss can be maintained in an acceptable region, and by which the excessive overestimated second term \( \sum_{m=0}^{\bar{I}_w} (t \cdot m) \cdot P_m^{\bar{I}_w} \) can be neglected, the estimation of the overall system BER can be well approximated by:

\[
I_{\text{BER,MBC-SLC}} \approx I_{\text{BER,LDPC}} - \sum_{i=1}^{k_{\text{LDPC}}} P_i \sum_{w \in J} \left( \sum_{n \in I_w} x_n \right) M(w, i) \frac{1}{k_{\text{LDPC}}}.
\]

(12)

An improvement on the error floor is thus rendered.
Selection of the BCH Code in the MBC-SLC

• We will assume the well approximation of (12) and test whether a larger BCH code will improve the average BER performance of the MBC-SLC scheme.

• By (12), the improvement is decided by:

\[
\sum_{\mathbf{w} \in \mathcal{J}} (\sum_{n \in \mathcal{I}_w} x_n) M(\mathbf{w}, i) \frac{M(\mathbf{w}, i)}{k_{\text{LDPC}}}.
\]  

(13)

• Let us first test how the parameters in coding scheme affect the improvement without considering whether the codes corresponding to these parameters exist or not.
Selection of the BCH Code in the MBC-SLC

Table 1: The values of (13) corresponding to the MBC-SLC scheme with parameters: $k_{\text{LDPC}} = 20$, $\beta = 2$, $n_{\text{BCH}} = 10$ and $t = 4$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4.83746</td>
<td>5.28483</td>
<td>5.1904</td>
</tr>
<tr>
<td>$i$</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>(13)</td>
<td>4.56013</td>
<td>3.55477</td>
<td>2.42211</td>
<td>1.40402</td>
<td>0.658728</td>
<td>0.226006</td>
<td>0.0433437</td>
</tr>
<tr>
<td>$i$</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(13)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: The values of (13) corresponding to the MBC-SLC scheme with parameters: $k_{\text{LDPC}} = 20$, $\beta = 4$, $n_{\text{BCH}} = 5$ and $t = 2$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4.20021</td>
<td>3.83127</td>
<td>3.32817</td>
</tr>
<tr>
<td>$i$</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>(13)</td>
<td>4.20021</td>
<td>3.83127</td>
<td>3.32817</td>
<td>2.74149</td>
<td>2.12384</td>
<td>1.52606</td>
<td>0.993292</td>
</tr>
<tr>
<td>$i$</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(13)</td>
<td>0.561146</td>
<td>0.251806</td>
<td>0.0701754</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Selection of the BCH Code in the MBC-SLC

• Tables 1 and 2 hint that
  – when $i$ (i.e., the number of errors) is small, using a larger BCH code can improve the error floors more;
  – when the error number distribution places more mass for larger $i$ (such as $i \geq 9$ in Tables 1 and 2), a smaller BCH code seems to be favored.
Analysis of the BER performance of the MBC-MLC

• We can similarly get the BER formulation of the MBC-MLC coding scheme from that of the MBC-SLC coding scheme.

• We still assume an uniform interleaver is placed between the BCH coders and the LDPC coders.

• The only difference is that the number of LDPC code decoders is changed from one to $\gamma$. 
As similar to (1), we derive the average BER performance of the information bits of these $\gamma$ LDPC codes as:

$$I^{(\gamma)}_{\text{BER,LDPC}} = \frac{\sum_{i=1}^{\gamma \cdot k_{\text{LDPC}}} i \cdot S_i^{(\gamma)}}{\gamma \cdot k_{\text{LDPC}}}, \quad (14)$$

where

$$S_i^{(\gamma)} = \sum_{x_1, x_2, \ldots, x_\gamma \atop x_1 + \cdots + x_\gamma = i \atop 0 \leq x_1, x_2, \ldots, x_\gamma \leq k_{\text{LDPC}}} P_{x_1} \cdots P_{x_\gamma}$$

is the probability that the number of bits errors at the outputs of $\gamma$ LDPC code decoders is equal to $i$. 
Analysis of the BER performance of the MBC-MLC

- Be reminded that the upper bound of overall coded BER of the MBC-SLC coding scheme is given by:

\[ I_{BER,MBC-SLC}^U = \sum_{i=1}^{k_{LDPC}} P_i \cdot \sum_{w \in \mathcal{J}} E_{U,I} \cdot M(w, i). \]  

(15)

- Note that in the above formula, \( P_i \) is the error number distribution of one LDPC code.

- We then replace it by \( S_i^{(\gamma)} \) for \( \gamma \) LDPC codes. As a result, the upper bound of overall coded BER of the MBC-MLC coding scheme is equal to:

\[ I_{BER,MBC-MLC}^{U,\gamma} = \sum_{i=1}^{\gamma k_{LDPC}} S_i^{(\gamma)} \cdot \sum_{w \in \mathcal{J}} E_{U,I} \cdot M(w, i). \]  

(16)
Analysis of the BER performance of the MBC-MLC

• Hence,
\[
I_{\text{BER,LDPC}}^{(\gamma)} - I_{\text{BER,MBC-MLC}}^{(\gamma)} \\
\geq I_{\text{BER,LDPC}}^{(\gamma)} - I_{\text{BER,MBC-MLC}}^{U,\gamma} \\
= \sum_{i=1}^{\gamma^{k_{\text{LDPC}}}} S_i^{(\gamma)} \sum_{w \in J} \left( \sum_{n \in I_w} x_n - \sum_{m=0}^{\left|I_w\right|} (t \cdot m) \cdot P_m^{\left|I_w\right|} \right) M(w, i) \\
\gamma \cdot k_{\text{LDPC}}
\]
(17)

• From (17), the discussion regarding the selection of \( t \) is the same as that for the MBC-SLC scheme; hence, we omit them.

• We conclude this section by pointing out that for a proper choice of \( t \),
\[
I_{\text{BER,MBC-MLC}} \approx I_{\text{BER,LDPC}}^{(\gamma)} - \sum_{i=1}^{\gamma^{k_{\text{LDPC}}}} S_i^{(\gamma)} \sum_{w \in J} \left( \sum_{n \in I_w} x_n \right) M(w, i) \\
\gamma \cdot k_{\text{LDPC}}
\]
(18)
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Simulation Results

Table 3: Error number distribution \( \{P_i, \forall 1 \leq i \leq k_{LDPC}\} \) for the QC(4590,3835) LDPC code at SNR = 3.5 dB. Only the non-zero \( P_i \) values are listed in this table.

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_i )</td>
<td>0.962107</td>
<td>2.16901\times 10^{-5}</td>
<td>0.0344656</td>
<td>0.000715773</td>
<td>0.000520562</td>
<td>0.00104112</td>
</tr>
<tr>
<td>( i )</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>( P_i )</td>
<td>0.00010845</td>
<td>0.000173521</td>
<td>4.33802\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
</tr>
<tr>
<td>( i )</td>
<td>16</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td>26</td>
<td>33</td>
</tr>
<tr>
<td>( P_i )</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
</tr>
<tr>
<td>( i )</td>
<td>36</td>
<td>39</td>
<td>40</td>
<td>41</td>
<td>45</td>
<td>46</td>
</tr>
<tr>
<td>( P_i )</td>
<td>4.33802\times 10^{-5}</td>
<td>8.67604\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
</tr>
<tr>
<td>( i )</td>
<td>47</td>
<td>48</td>
<td>49</td>
<td>51</td>
<td>52</td>
<td>53</td>
</tr>
<tr>
<td>( P_i )</td>
<td>2.16901\times 10^{-5}</td>
<td>4.33802\times 10^{-5}</td>
<td>4.33802\times 10^{-5}</td>
<td>6.50703\times 10^{-5}</td>
<td>4.33802\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
</tr>
<tr>
<td>( i )</td>
<td>56</td>
<td>57</td>
<td>58</td>
<td>65</td>
<td>67</td>
<td>75</td>
</tr>
<tr>
<td>( P_i )</td>
<td>4.33802\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
<td>2.16901\times 10^{-5}</td>
</tr>
</tbody>
</table>
Simulation Results

Table 4: Numerical calculation of the difference term in (5) according to the error number distribution in Table 3. In this table, $n_{BCH} = 8191$ and $\beta = 2$. (The SBC-MLC scheme consisting of one (8191, 8139, 4) BCH code and two QC(4590,3835) LDPC codes.)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\sum_{i=1}^{t} i \cdot S_{i}^{(3)} / n_{BCH}$</th>
<th>$I_{BER,LDPC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.55456×10^{-9}</td>
<td>3.76136×10^{-5}</td>
</tr>
<tr>
<td>4</td>
<td>2.61558×10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>
Simulation Results

Figure 4: Simulated and calculated BER performances of the SBC-MLC scheme consisting of one (8191, 8139, 4) BCH code and two QC(4590,3835) LDPC codes. Also shown is the performance of the LDPC code.
Table 5: Numerical calculation of the difference term in (12) according to the error number distribution in Table 3. In this table, $k_{\text{LDPC}} = 3835$ and $\beta = 7$. (The MBC-SLC scheme consisting of seven (511, 493, 2) BCH codes and one QC(4590, 3835) LDPC code.)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\sum_{i=1}^{k_{\text{LDPC}}} P_i \sum_{w \in \mathcal{J}} \left( \sum_{n \in \mathcal{I}_w} x_n \right) M(w, i)$</th>
<th>$I_{\text{BER,LDPC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.37034 \times 10^{-5}$</td>
<td>$3.76136 \times 10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>$3.25249 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
Simulation Results

Figure 5: Simulated and calculated BER performances of the MBC-SLC scheme consisting of seven (511,493, 2) BCH codes and one QC(4590,3835) LDPC code. Also shown is the performance of the LDPC code.
Simulation Results

Table 6: Error number distribution \{P_i, \forall 1 \leq i \leq k_{\text{LDPC}}\} for the QC(2286,1914) LDPC code at SNR= 4 dB. Only the non-zero \(P_i\) values are listed in this table.

<table>
<thead>
<tr>
<th>(i)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_i)</td>
<td>0.992169</td>
<td>7.83073\times10^{-6}</td>
<td>6.26458\times10^{-5}</td>
<td>0.00670311</td>
<td>0.000195768</td>
<td>0.000681274</td>
</tr>
<tr>
<td>(i)</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>(P_i)</td>
<td>2.34922\times10^{-5}</td>
<td>0.000125292</td>
<td>7.83073\times10^{-6}</td>
<td>1.56615\times10^{-5}</td>
<td>7.83073\times10^{-6}</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Numerical calculation of the difference term in (18) according to the error number distribution in Table 6. In this table, \(k_{\text{LDPC}} = 1914\) and \(\gamma = 2\). (The MBC-SLC scheme consisting of eight (511,483, 3) BCH codes and two QC(2286,1914) LDPC code. Also shown is the performance of the LDPC code.)

<table>
<thead>
<tr>
<th>(t)</th>
<th>(\frac{\gamma k_{\text{LDPC}}}{\sum_{i=1}^{\gamma k_{\text{LDPC}}} S_i^{(\gamma)} \sum_{w \in J} \left( \sum_{n \in I_w} x_n \right) M(w, i)} \cdot k_{\text{LDPC}})</th>
<th>(I_{\text{BER,LDPC}}^{(\gamma)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.11843\times10^{-5}</td>
<td>1.75598\times10^{-5}</td>
</tr>
<tr>
<td>2</td>
<td>1.65464\times10^{-5}</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.7589\times10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>
Simulation Results

Figure 6: Simulated and calculated BER performances of the MBC-SLC scheme consisting of eight (511,483, 3) BCH codes and two QC(2286,1914) LDPC code. Also shown is the performance of the LDPC code.
Simulation Results

Figure 7: Calculated BER performances of the MBC-SLC scheme for the QC(2286,1914) LDPC code concatenated with different BCH codes. Also shown is the performance of the LDPC code.
Simulation Results

Figure 8: Simulated BER performances of the MBC-SLC scheme for the QC(2286,1914) LDPC code concatenated with different BCH codes. Also shown is the performance of the LDPC code.
Outline

- Motivation and Contribution
- Technical Background
- Analysis of the BER performance of Three Concatenated Coding Schemes
- Simulation Results
- Conclusion
Conclusion

• Our analysis and simulations show that the concatenated scheme can improve the error floor of the LDPC-only scheme by choosing a proper $t$.

• By investigating the MBC-SLC scheme for different BCH codes subject to a fixed system code rate, we found that a larger $t$ may be a better choice.

• When the SBC-MLC scheme is composed of a BCH code with a stronger error correcting capability, its performance is often better than that of the MBC-SLC scheme.
Conclusion

EB/N0 (dB)

QC(2286,1914) LDPC code
MBC−SLC,(255,239,2)BCH code
MBC−SLC,(511,484,3)BCH code
MBC−SLC,(1023,973,5)BCH code
SBC−MLC,(4095,3879,18)BCH code
Thank You for Your Attention