Low-Complexity Soft-Output Sphere Decoding with Modified Repeated Tree Search Strategy

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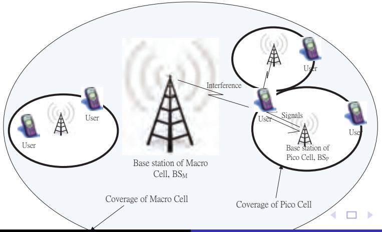
April 30, 2013

Introduction

- Various detectors for Multiple-In-Multiple-Out (MIMO) technologies have been proposed, yet to achieve best complexity and performance tradeoff still remain a challenge.
- We proposed a low complexity soft-output sphere decoding called Modified RTS that can achieve good complexity-performance tradeoff and is hardware implementable.
- We further apply modified RTS for interference cancellation with unknown interference modulation.

Heterogeneous Network

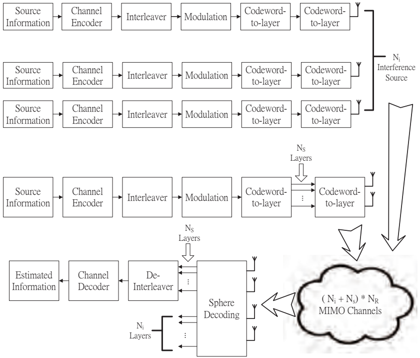
- In a heterogeneous network, power of Interferences (from other users) may be equal to or even larger than power of signals; hence, traditional interference cancellation techniques may fail under such circumstance.
- BS_P and BS_M may transmit at the same time, while the user in the Pico cell may not know the modulation of signals from BS_M.



System Block Diagram

- System Model $\mathbf{y} = \mathbf{H}_s\mathbf{P}_s\mathbf{x}_s + \mathbf{H}_i\mathbf{P}_i\mathbf{x}_i + \sum_k \mathbf{H}_k\mathbf{P}_k\mathbf{x}_k + \mathbf{n}$

- $\mathbf{H}$ denots the channel matrix and $\mathbf{P}$ denotes the precoder matrix.
- Length of $\mathbf{x}_s$ is N_s and length of $\mathbf{x}_i$ is N_i.



Linear Receivers

- MMSE Type I

$$w_{MMSE,1} = \widetilde{\mathbf{H}}_s^H \left(\widetilde{\mathbf{H}_s} \widetilde{\mathbf{H}_s}^H + diag(\sigma_{IN}^2) \right)^{-1}$$

- MMSE Type II

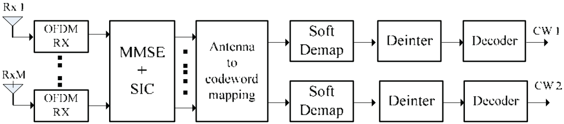
$$w_{MMSE,2} = \widetilde{\mathbf{H}_s}^H \left(\widetilde{\mathbf{H}_s} \widetilde{\mathbf{H}_s}^H + \widetilde{\mathbf{H}_i} \widetilde{\mathbf{H}_i}^H + diag(\sigma_{IN}^2) \right)^{-1}$$

- Interference Rejection Combining (IRC)

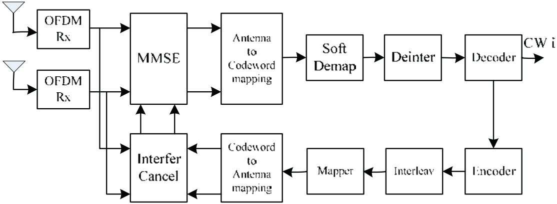
$$w_{IRC} = \widetilde{\mathbf{H}_s}^H \left(\widetilde{\mathbf{H}_s} \widetilde{\mathbf{H}_s}^H + \widetilde{\mathbf{H}_i} \widetilde{\mathbf{H}_i}^H + \sum_k \widetilde{\mathbf{H}}_k \widetilde{\mathbf{H}}_k^H + \sigma_N^2 \cdot I \right)^{-1}$$

Sequential Interference Cancellation (SIC)

- Symbol-SIC



- Codeword-SIC



Sphere Decoding

- Simplified system model

$$
\begin{aligned}
\mathbf{y} &= \mathbf{H}_s\mathbf{P}_s\mathbf{x}_s + \mathbf{H}_i\mathbf{P}_i\mathbf{x}_i + \sum_k \mathbf{H}_k\mathbf{P}_k\mathbf{x}_k + \mathbf{n} \\
&= [\mathbf{H}_s\mathbf{P}_s \quad \mathbf{H}_i\mathbf{P}_i][\mathbf{x}_s \quad \mathbf{x}_i]^T + \sum_k \mathbf{H}_k\mathbf{P}_k\mathbf{x}_k + \mathbf{n} \\
&= \mathbf{H}\mathbf{x} + \mathbf{n}'
\end{aligned}
$$

where $\mathbf{H} \triangleq [\mathbf{H}_s\mathbf{P}_s \quad \mathbf{H}_i\mathbf{P}_i]$, $\mathbf{x} \triangleq [\mathbf{x}_s \quad \mathbf{x}_i]^T$, and $\mathbf{n}' \triangleq \sum_k \mathbf{H}_k\mathbf{P}_k\mathbf{x}_k + \mathbf{n}$ is the equivalent noise.

Pre-processing Step

- First, QR-decompose channel matrix $\mathbf{H}$:

$$\mathbf{H} = \mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0}_{(N_\mathrm{R}-N_\mathrm{T})\times N_\mathrm{T}} \end{bmatrix}$$

- Then, we obtain $\tilde{\mathbf{y}} = \mathbf{Q}^H\mathbf{y} = \mathbf{Q}^H\mathbf{H}\mathbf{x} + \mathbf{Q}^H\mathbf{n} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$, which in matrix form is:

$$\begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_{N_\mathrm{T}} \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_\mathrm{T}} \\ 0 & r_{2,2} & \cdots & r_{2,N_\mathrm{T}} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & r_{N_\mathrm{T},N_\mathrm{T}} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_\mathrm{T}} \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \vdots \\ \tilde{n}_{N_\mathrm{T}} \end{bmatrix}.$$

Tree Search Step

- After pre-processing, the maximum-likelihood (ML) decision can be found by solving:

$$\begin{aligned}\hat{\mathbf{x}}_{\mathrm{ML}} &= \arg\min_{\mathbf{x}\in\mathcal{O}^{N_\mathrm{T}}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 \\ &= \arg\min_{\mathbf{x}\in\mathcal{O}^{N_\mathrm{T}}} \sum_{i=1}^{N_\mathrm{T}} \left| \tilde{y}_i - \sum_{j=i}^{N_\mathrm{T}} r_{i,j} x_j \right|^2 .\end{aligned}$$

LLR computation for each bit

- Max-log approximation of LLR for bit $x_{j,b}$

$$LLR(x_{j,b}) = \min_{\mathbf{x}\in\mathcal{X}_{j,b}^{(0)}} \| \mathbf{y} - \mathbf{H}\mathbf{x} \|^2 - \min_{\mathbf{x}\in\mathcal{X}_{j,b}^{(1)}} \| \mathbf{y} - \mathbf{H}\mathbf{x} \|^2 .$$

- For the QR-decomposition-refined system: $\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$

$$LLR(x_{j,b}) = \min_{\mathbf{x}\in\mathcal{X}_{j,b}^{(0)}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 - \min_{\mathbf{x}\in\mathcal{X}_{j,b}^{(1)}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 .$$

Tree Traversal Strategies

- Repeated Tree Search (RTS)
- Single Tree Search (STS) [1]
- Smart Ordering Candidate Adding (SOCA) [2]

Some other complexity reduction algorithms such as LLR clipping, sorting and regularization can be combined with these tree search strategies.

[1] C. Studer, A. Burg, and H. Bölcskei, "Soft-Output Sphere Decoding: Algorithms and VLSI Implementation," *IEEE J. Select. Areas Commun.*, vol. 26, no. 2, pp. 290-300, Feb. 2008.

[2] D. L. Milliner, E. Zimmermann, J. R. Barry and G. Fettweis, "A fixed-complexity smart candidate adding algorithm for soft-output MIMO detection," *IEEE Trans. Signal Process.*, vol. 3, no. 6, pp. 1016-1025, Dec. 2009.

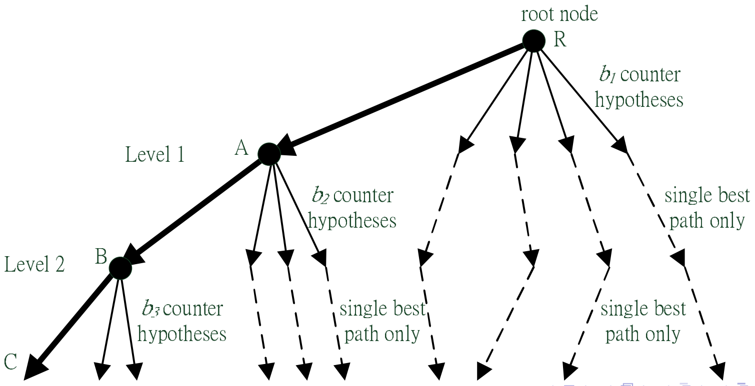
Modified RTS (1)

- A two-stage tree traversal strategy similar to RTS
- A depth-first tree search algorithm called Schnorr-Euchner sphere decoder (SESD) is used in the first stage.
- A complexity upper limit T_1 is set for the first stage, and hence a near ML path is found instead of ML path.
- The second stage finds those counter hypothesis paths based on the near-ML path found in the first stage.
- Vector $\mathbf{b} = [b_1, \cdots, b_{N_\mathrm{T}}]$ is pre-defined to specify the number of counter hypothesis paths to be extended at each level.
- The complexity upper limit for the second stage is

$$T_2 = \sum_{i=1}^{N_\mathrm{T}} b_i \left(N_\mathrm{T} + 1 - i \right).$$

Modified RTS (2)

- Due to LLR clipping, only counter hypothesis paths whose distance to the near-ML path is less than $L_{\max}$ are kept.



Simulation Setting

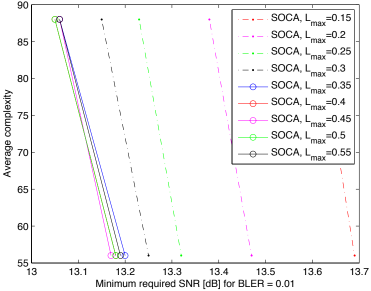
- 4×4 MIMO system with 16QAM constellation is considered.
- Fast fading and slow fading are both examined.
- Coding schemes
  - Turbo code of rate 1/2 and codeword length 2000 bits with 40×50 block interleaver is used at transmitter, and 8-iteration Max-Log-MAP decoder is adopted at receiver.
  - (2,1,8) convolutional code of rate 1/2 and codeword length 720 bits with 15×48 block interleaver is used at transmitter, and Viterbi decoder is adopted at receiver.
- The performance metric: SNR required to achieve a block error rate (BLER) of 10^{-2} after channel decoding.

Complexity Limit for Modified RTS and SOCA

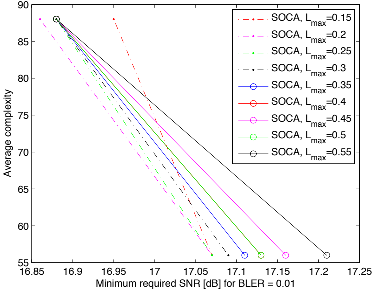
- Modified RTS:
  - Upper complexity limit T_1 for the first stage of the modified RTS is set to 30.
  - $\mathbf{b} = [4444], [4442], [4422], [4222]$ and $[2222]$, respectively, result in $T_2 = 40, 38, 34, 28, 20$.
- SOCA:
  b_1 is set to 16, 14, 12, 10, 8, 6 and 4, respectively resulting complexities of 88, 80, 72, 64, 56, 48, 40.

Effect of L_{max} on SOCA

Fast Fading



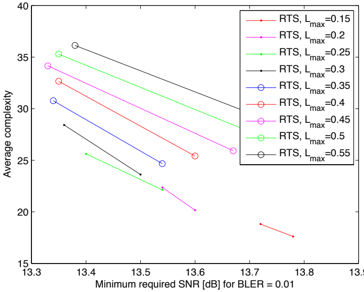
Slow Fading



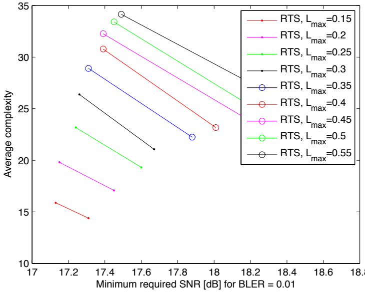
$b_1 = 16$ and 8 have been examined.

Effect of L_{max} on Modified RTS

Fast Fading

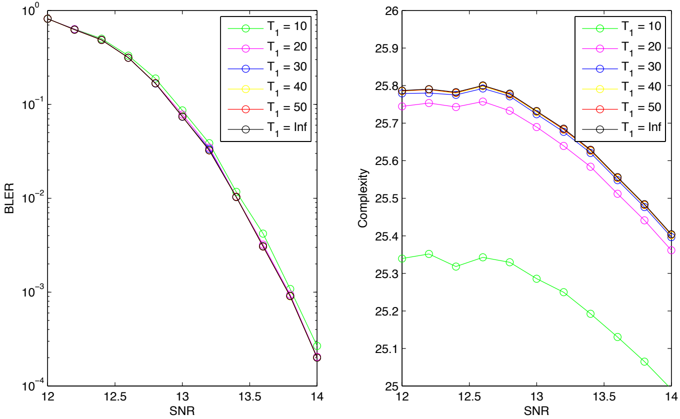


Slow Fading

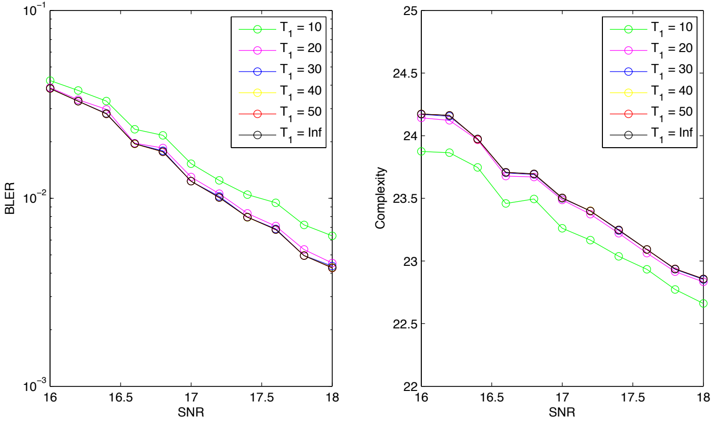


$\mathbf{b} = [4444]$ and $[2222]$ have been examined.

Effect of T_1 on Modified RTS in Fast Fading Channels

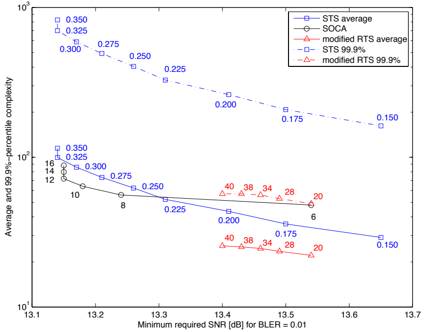


Effect of T_1 on Modified RTS in Slow Fading Channels

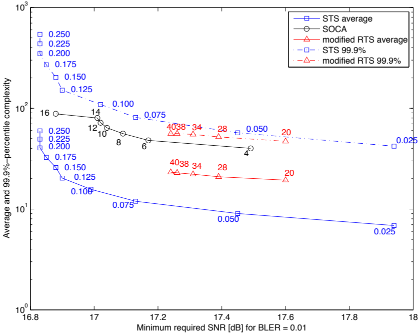


Performance versus Complexity

Fast Fading, Turbo Code



Slow Fading, Turbo Code



Classification for Unknown Interference Modulation

- The SD algorithm is capable of executing ML detection with "augmented" modulation scheme, which takes "all" possible modulation schemes into consideration.
- The modulation scheme of the interference source is kept the same for a number of detection, so receiver can decide the modulation scheme after N resource elements (REs) is received.

General Likelihood Ratio Test (GLRT)

- Original GLRT

$$\min_{\mathbf{i}\in \mathrm{I}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 \gtrless_{I}^{II} \min_{\mathbf{i}\in \mathrm{II}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2$$

- Modified GLRT

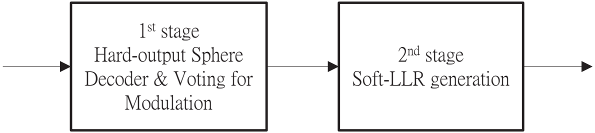
$$\min_{\mathbf{i}\in \mathrm{I}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 + 2N_0 m_i \log | \chi_I | \gtrless_{I}^{II}$$

$$\min_{\mathbf{i}\in \mathrm{II}} \| \tilde{\mathbf{y}} - \mathbf{R}\mathbf{x} \|^2 + 2N_0 m_i \log | \chi_{II} |$$ [3]

[3] B. Shim, and I. Kang, "Joint Modulation Classification and Detection Using Sphere Decoding," *Signal Processing Letters, IEEE*, vol. 16, no. 9, pp. 778-781, Sep. 2009.

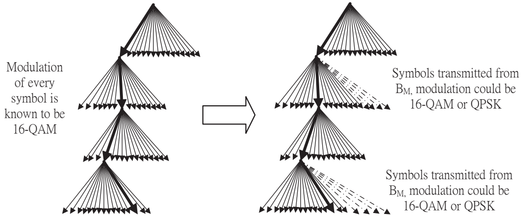
New Modified RTS for Unknown Interference Modulation

- The first stage runs hard-decision SD for N REs with augmented modulation scheme to generate N votes.
- N votes jointly decide the modulation of interference.
- The second stage is executed according the modulation determined in the first stage.



Modification on First Stage(1)

- SESD is still adopted for the first stage, except number of candidates is changed.



Modification on First Stage(2)

- After running the modified first stage for N REs, the modulation of each interference source can be decided by voting.
- Due to the unbalanced numbers of constellation points, we propose an “unfair voting” method by introduce a bias V_b such that the decision rule is

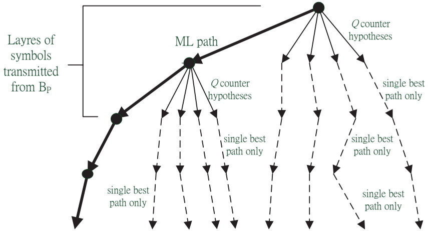
$$V_I + V_b \gtrless_{II}^{I} V_{II}.$$

Modification on First Stage(3)

- Some REs may have ML path corresponding to different modulation scheme from the one obtained from Unfair Voting.
- These REs need to execute the SESD again according to the modulation scheme decided previously.

Modification on Second Stage

- Generate counter hypothesis paths according to previously determined modulation.
- Only soft outputs of desired signal parts from BS_P are of interest.



Simulation Setting

- $N_s = N_i = 2, N_R = 4$, which corresponds to the system model

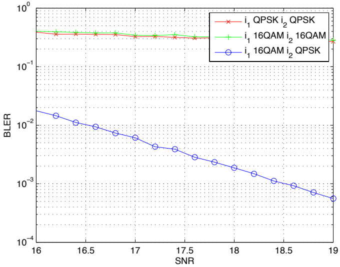
$$\mathbf{y} = \mathbf{H}_s\mathbf{P}_s\mathbf{x}_s + \mathbf{H}_i\mathbf{P}_i\mathbf{x}_i + \sum_k \mathbf{H}_k\mathbf{P}_k\mathbf{x}_k + \mathbf{n}$$

$$= [\mathbf{H}_s\mathbf{P}_s \quad \mathbf{H}_i\mathbf{P}_i] \begin{bmatrix} x_1 \\ x_2 \\ i_1 \\ i_2 \end{bmatrix} + \mathbf{n}'.$$

- Modulation of $\mathbf{x}$ is 16QAM, and modulation of $\mathbf{i}$ may be QPSK or 16QAM.
- A 3GPP-specified punctured turbo code of rate 1/2 is adopted and codeword length is 1920 bits.
- Slow fading is considered.
- L_{max} is set to be 0.2.

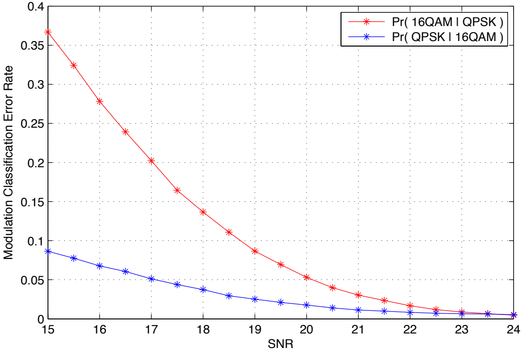
BLER Performance with Correct/Wrong Modulation of Interference

The modulation scheme of the interference source i_1 and i_2 are 16QAM and QPSK, respectively.



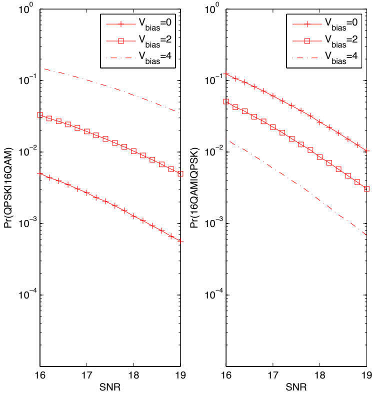
Modulation Classification Error Rate with Single RE

The modulations of i_1 and i_2 are randomly chosen.

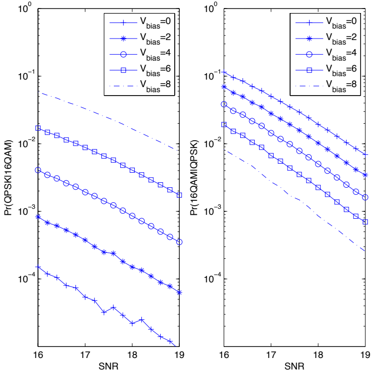


Optimization of v_b When $N = 8$ and $N = 16$

N=8

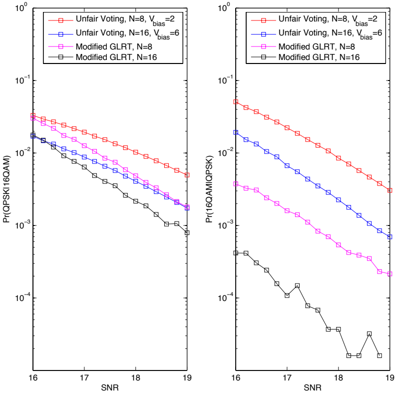


N=16



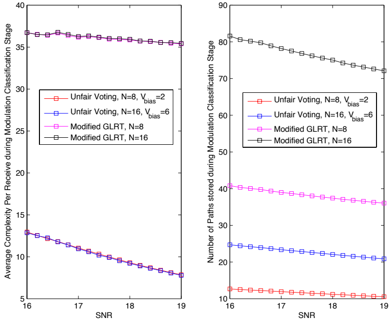
Optimization of v_b When N=8 and N=16

- Modified GLRT has better performance at a price that more paths need to be searched and stored.
- $\Pr(QPSK|16QAM)$ and $\Pr(16QAM|QPSK)$ could be further balanced to improve overall BLER.

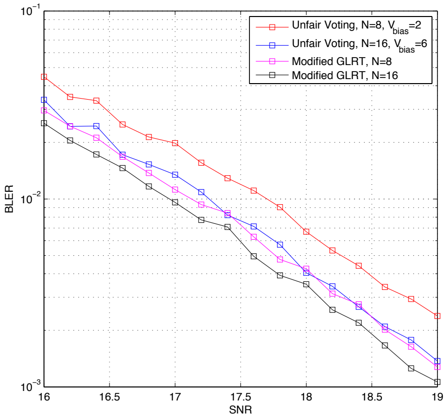


Average Complexity and Paths Stored During Modulation Classification Stage

- Average complexity during modulation classification stage is proportional to number of total hypotheses.
- Number of paths stored is proportional to $N \times$ number of total hypotheses.



BLER Performance Comparison



Conclusion and Future Work

- We propose a low-complexity Sphere Decoder for MIMO detection that is hardware implementable.
- We propose a simple method to determine the unknown modulation scheme of interference.
- This is only a preliminary research for the joint modulation classification and detection. It would be interesting to find the analytical evidence as well as more efficient and reliable algorithms for this challenge.

Thanks for Your Attention!