Combined Channel Estimation and Sensor Fault Protection in Wireless Sensor Networks

Prepared by Si-Yao Huang

Advisory by Prof. Po-Ning Chen

Institute of Communications Engineering

National Chiao Tung University

Hsinchu, Taiwan 300, R.O.C

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Outline

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Introduction

- Wireless sensor networks operate in a harsh environment, training sequence has been used to detect the faulty sensor under limited energy support.
- Skoglund et al proposed to use computer-searched nonlinear codes for combined channel estimation and error protection in slow fading channel in order to release the power consumption on the training sequence.
- In this thesis, we consider to apply Skoglund's concept to the wireless sensor network. The objective is to provide good fault-tolerant capability for the wireless sensor networks.
- The simulation results are finally given under Rayleigh fading channel, spatially independent channel.

System Model

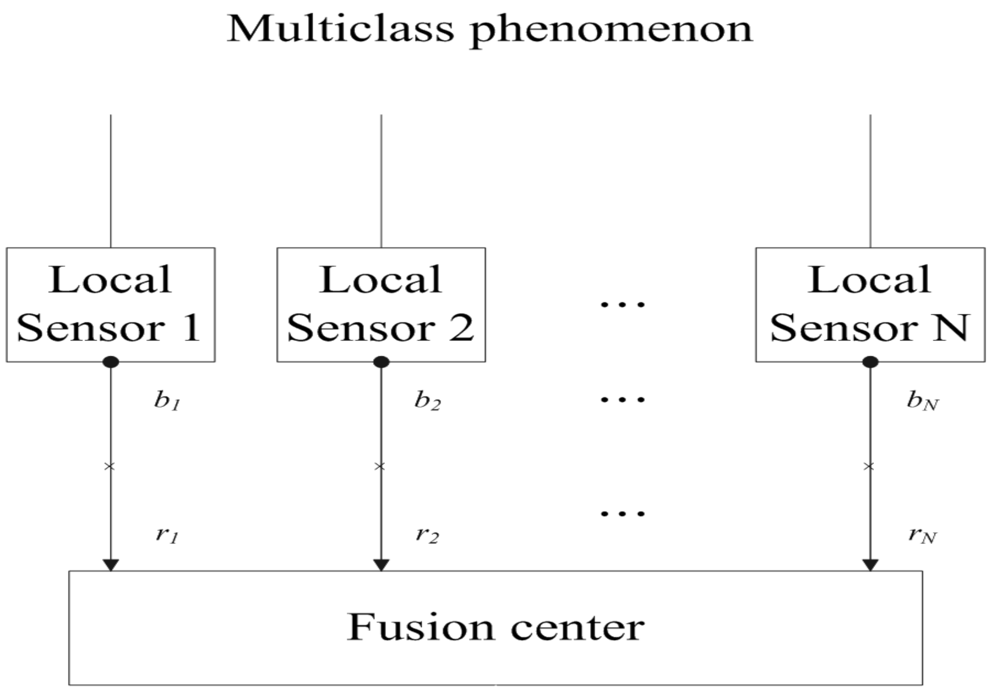

Figure 1: System model of wireless sensor networks

- The communication channel between sensors and fusion center is assumed to be Rayleigh Fading channel.
- The prior probability of each hypothesis is assumed equal.
- Given that the true hypothesis is H_ℓ, the probability of sensor i favoring hypothesis $H_{\tilde{\ell}}$ is denoted by $P^{(i)}_{\tilde{\ell}|\ell}$.
- The transmitted code matrix is $\mathbb{C} = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \cdots & \boldsymbol{b}_K \end{bmatrix}_{N \times K}$, where $\boldsymbol{b}_i = \begin{bmatrix} b_{i,1} & b_{i,2} & \cdots & b_{i,N} \end{bmatrix}^T$, and $\boldsymbol{b}_i \in \{\pm 1\}^N$

- The received complex matrix at the fusion center is

$$\mathbb{R} = \begin{bmatrix} \boldsymbol{r}_1 & \boldsymbol{r}_2 & \cdots & \boldsymbol{r}_K \end{bmatrix}_{L \times K},$$

where

$$\boldsymbol{r}_i = \begin{bmatrix} b_{i,1} & 0 & \cdots & 0 \\ b_{i,2} & b_{i,1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ b_{i,P} & b_{i,P-1} & \cdots & b_{i,1} \\ \vdots & \ddots & \ddots & \vdots \\ b_{i,N} & b_{i,N-1} & & b_{N-P+1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{i,N} \end{bmatrix}_{L \times P} \boldsymbol{h}_i + \boldsymbol{n}_i$$

$L = N + P - 1$, the channel coefficients

$$\boldsymbol{h}_i = \begin{bmatrix} h_{i,1} & \cdots & h_{i,P} \end{bmatrix}^T$$

are assumed constant over the transmission of codeword $\boldsymbol{b}_i$, and $\boldsymbol{n}_i$ is the zero-mean complex Gaussian noise over the ith wireless channel link.

- The maximum-likelihood (ML) decision for the true hypothesis is given by:

$$\begin{aligned}\hat{\ell} &= \arg\min_{1\leq\ell\leq M}\sum_{k=1}^{K}\left\|\boldsymbol{r}_k - \mathbb{B}_k^{(\ell)}\boldsymbol{h}_k\right\|^2 \\ &= \arg\min_{1\leq\ell\leq M}\sum_{k=1}^{K}\left\|\boldsymbol{r}_k - \mathbb{P}_{B_k}^{(\ell)}\boldsymbol{r}_k\right\|^2\end{aligned}$$

where $\mathbb{P}_{B_k}^{(\ell)} = \mathbb{B}_k^{(\ell)}[(\mathbb{B}_k^{(\ell)})^T\mathbb{B}_k^{(\ell)}]^{-1}(\mathbb{B}_k^{(\ell)})^T$, and the non-zero column components of matrix $\mathbb{B}_k^{(\ell)}$ is the codeword $\boldsymbol{b}_k^{(\ell)}$ for sensor k with respect to hypothesis H_ℓ.

Pairwise Error Probability Union Bound Under Rayleigh Fading

- The detection error P_e at the fusion center can be given by:

$$P_e = \sum_{m=1}^{M} \Pr(H_m) \Pr(\hat{\ell} \neq m | H_m).$$

- Then, by denoting the local decision at sensor i by s_i, we obtain:

$$\begin{aligned} &\Pr(\hat{\ell} \neq m | H_m) \\ &= \sum_{\ell_1=1}^{M} \sum_{\ell_2=1}^{M} \cdots \sum_{\ell_K=1}^{M} \Pr(\hat{\ell} \neq m | H_m, s_1 = H_{\ell_1}, s_2 = H_{\ell_2}, \cdots, s_K = H_{\ell_K}) \\ &\qquad \times \Pr(s_1 = H_{\ell_1} | H_m) \Pr(s_2 = H_{\ell_2} | H_m) \cdots \Pr(s_K = H_{\ell_K} | H_m) \end{aligned}$$

$$
\begin{aligned}
&\Pr(\hat{\ell} \neq m | H_m, s_1 = H_{\ell_1}, s_2 = H_{\ell_2}, \cdots, s_K = H_{\ell_K}) \\
\leq & \sum_{\ell=1,\ell\neq m}^{M} \frac{1}{2\pi} \int_0^{\infty} \left(\int_{-\infty}^{\infty} \prod_{n=1}^{\bar{L}} \left(1 - 2j\bar{\lambda}_n t\right)^{-k_n/2} e^{-jtr} dt \right) dr \\
= & \sum_{\ell=1,\ell\neq m}^{M} \sum_{n=1}^{\bar{L}} \frac{1}{(k_n-1)!} \left[\frac{\partial^{(k_n-1)}}{\partial x^{(k_n-1)}} F_n(x) \right]_{x=\bar{\lambda}_n},
\end{aligned}
$$

where

$$
F_n(x) = x^{\sum_{i=1}^{\bar{L}} k_i - 1} \prod_{u=1,u\neq n}^{\bar{L}} (x - \bar{\lambda}_u)^{-k_u}.
$$

We finally conclude:

$$
\begin{aligned}
P_e \leq & \sum_{m=1}^{M} \Pr(H_m) \sum_{\ell_1=1}^{M} \sum_{\ell_2=1}^{M} \cdots \sum_{\ell_K=1}^{M} P_{\ell_1|m}^{(1)} P_{\ell_2|m}^{(2)} \cdots P_{\ell_K|m}^{(K)} \\
& \times \left(\sum_{\ell=1,\ell\neq m}^{M} \sum_{n=1}^{\bar{L}} \frac{1}{(k_n-1)!} \left[\frac{\partial^{(k_n-1)}}{\partial x^{(k_n-1)}} F_n(x) \right]_{x=\bar{\lambda}_n} \right). \qquad (1)
\end{aligned}
$$

- Perfect-sensor-observation assumption that $P^{(i)}_{\ell|m} = 1$ when $\ell = m$, equation(1) reduces to:

$$P_e \;\le\; \sum_{m=1}^{M} \Pr(H_m) \sum_{\ell=1,\ell\neq m}^{M} \sum_{n=1}^{\bar{L}} \frac{1}{(k_n-1)!} \left[\frac{\partial^{(k_n-1)}}{\partial x^{(k_n-1)}} F_n(x) \right]_{x=\bar{\lambda}_n}$$

- Faulty sensor situation

$$\begin{aligned}
&\Pr(\hat{\ell} \neq m | H_m) \\
&= \sum_{\boldsymbol{c}\in\{\pm 1\}^N} \sum_{\ell_2=1}^{M} \sum_{\ell_3=1}^{M} \cdots \sum_{\ell_K=1}^{M} \Pr(\hat{\ell} \neq m | H_m, \boldsymbol{b}_1 = \boldsymbol{c}, s_2 = H_{\ell_2}, \cdots, s_K = H_{\ell_K}) \\
&\qquad \times \Pr(\boldsymbol{b}_1 = \boldsymbol{c}) P^{(2)}_{\ell_2|m} \cdots P^{(K)}_{\ell_K|m}.
\end{aligned}$$

  – For stuck-at-one fault, $\Pr\left(\boldsymbol{b}_1 = \begin{bmatrix} -1 & -1 & \cdots & -1 \end{bmatrix}^T\right) = 1.$

  – For random fault, $\Pr(\boldsymbol{b}_1 = \boldsymbol{c}) = \frac{1}{2^N}$ for every $\boldsymbol{c} \in \{\pm 1\}^N$.

Code Search

- Simulated annealing is a popular algorithm for finding the optimum solution of certain problems.
- We employ the simulated annealing to find the optimum codebook in our wireless sensor network system. The error probability bound in (3) is regarded as the system entropy.
- In order to further simplify the code search process.

$$
\begin{aligned}
&\Pr(\hat{\ell} \neq m | H_m, s_1 = H_{\ell_1}, s_2 = H_{\ell_2}, \cdots, s_K = H_{\ell_K}) \\
&\leq \sum_{\ell=1,\ell\neq m}^{M} \Pr\left(\sum_{k=1}^{K} \boldsymbol{r}_k^H \left(\mathbb{P}_{B_k}^{(\ell)} - \mathbb{P}_{B_k}^{(m)} \right) \boldsymbol{r}_k > 0 \right. \\
&\qquad\qquad\qquad \left| s_1 = H_{\ell_1}, s_2 = H_{\ell_2}, \cdots, s_K = H_{\ell_K} \right) \qquad (2) \\
&\leq \sum_{\ell=1,\ell\neq m}^{M} \sum_{k=1}^{K} \Pr\left(\boldsymbol{r}_k^H \left(\mathbb{P}_{B_k}^{(\ell)} - \mathbb{P}_{B_k}^{(m)} \right) \boldsymbol{r}_k > 0 \middle| s_k = H_{\ell_k} \right). \qquad (3)
\end{aligned}
$$

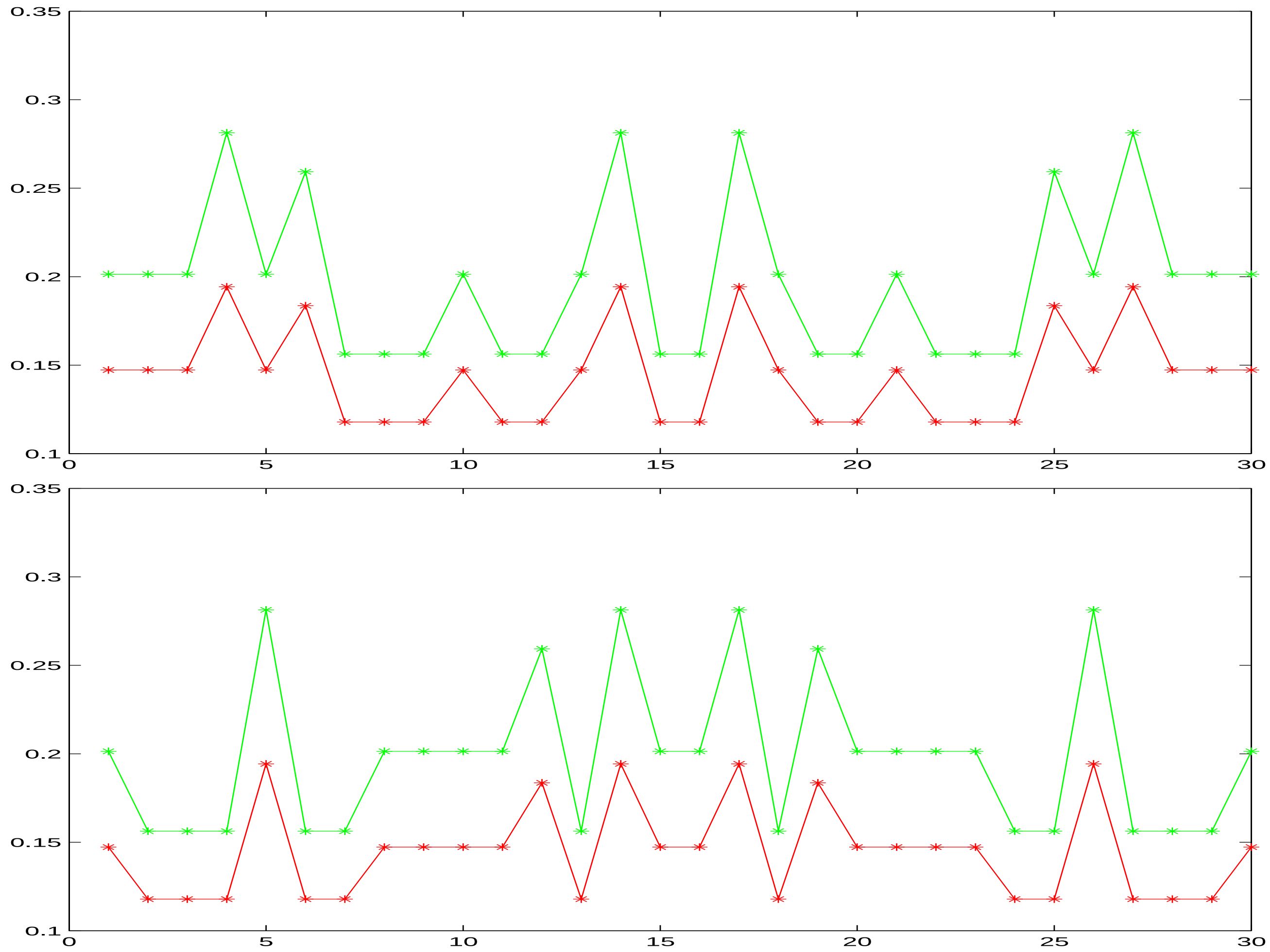

Figure 2: The bounds in (2) (red points) and (3) (green points) for sixty different codes.

Simulation Results

- Memory order $(P-1)=1$.
- $E[|h_{i,1}|^2]=E[|h_{i,2}|^2]=1/2$, and $\{\boldsymbol{h}_i\}_{i=1}^{K}$ are also independent across sensors.
- The codeword search in SNR = 10 dB.
- $\mathbb{P}_B=\mathbb{B}[\mathbb{B}^T\mathbb{B}]^{-1}\mathbb{B}^T=(-\mathbb{B})[(-\mathbb{B})^T(-\mathbb{B})]^{-1}(-\mathbb{B})^T=\mathbb{P}_{-B}$.

- LSE(LS): LS-channel-estimate scheme.
- COM: Combined-channel-estimation-and-sensor-fault-protection scheme.
- "Nx", "Sy" and "Hz" (either in the figure title or in the figure legend) will respectively denote "x bits transmitted per sensor", "y sensors" and "z hypotheses".

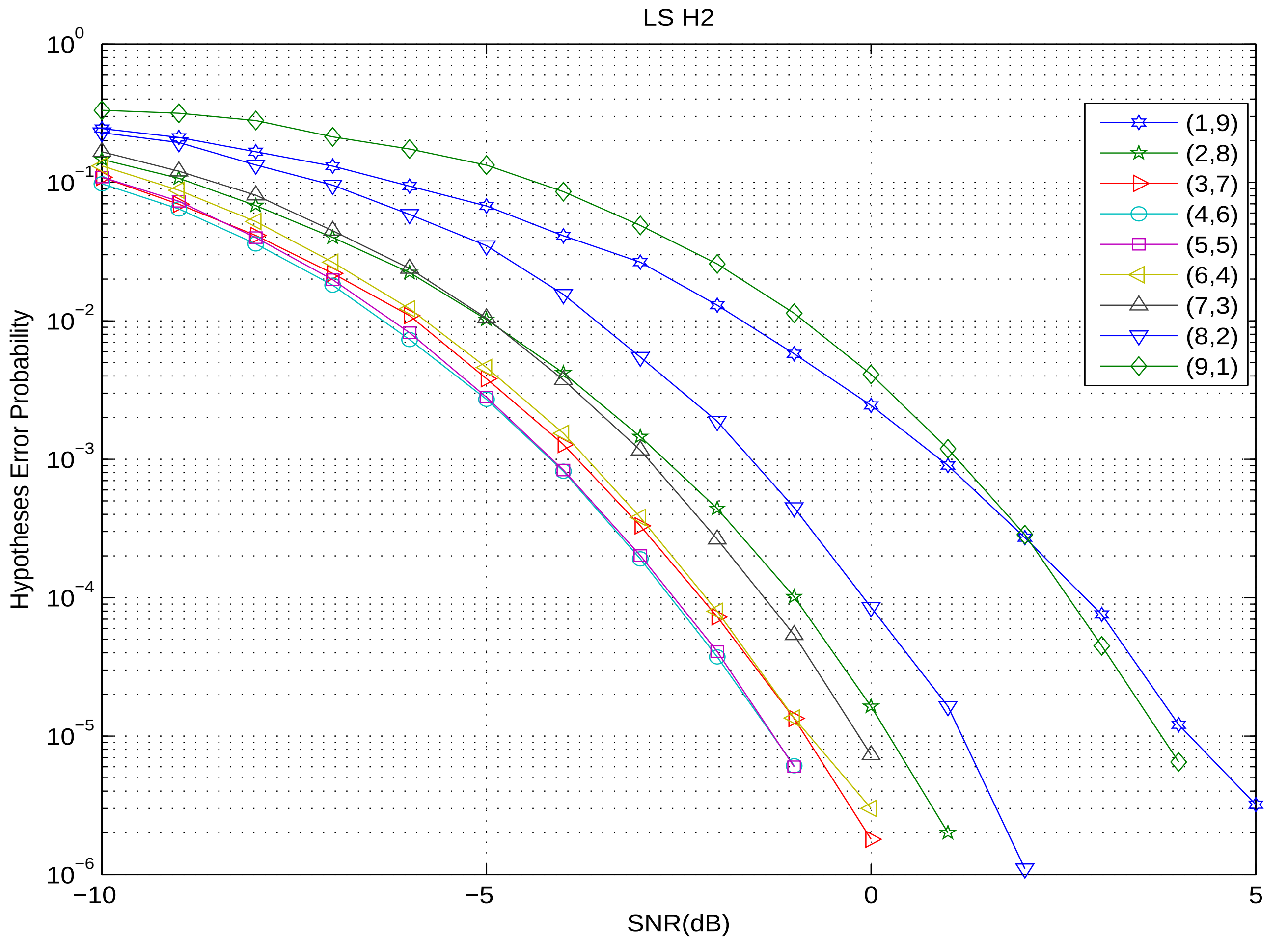

Figure 3: Performances of hypothesis detection based on LS estimate of channel coefficients in terms of training sequences. Ten sensors, each transmitting ten bits, and two hypotheses are assumed. In the legend, LS(x, y) represents that x training bits and y code bits are transmitted.

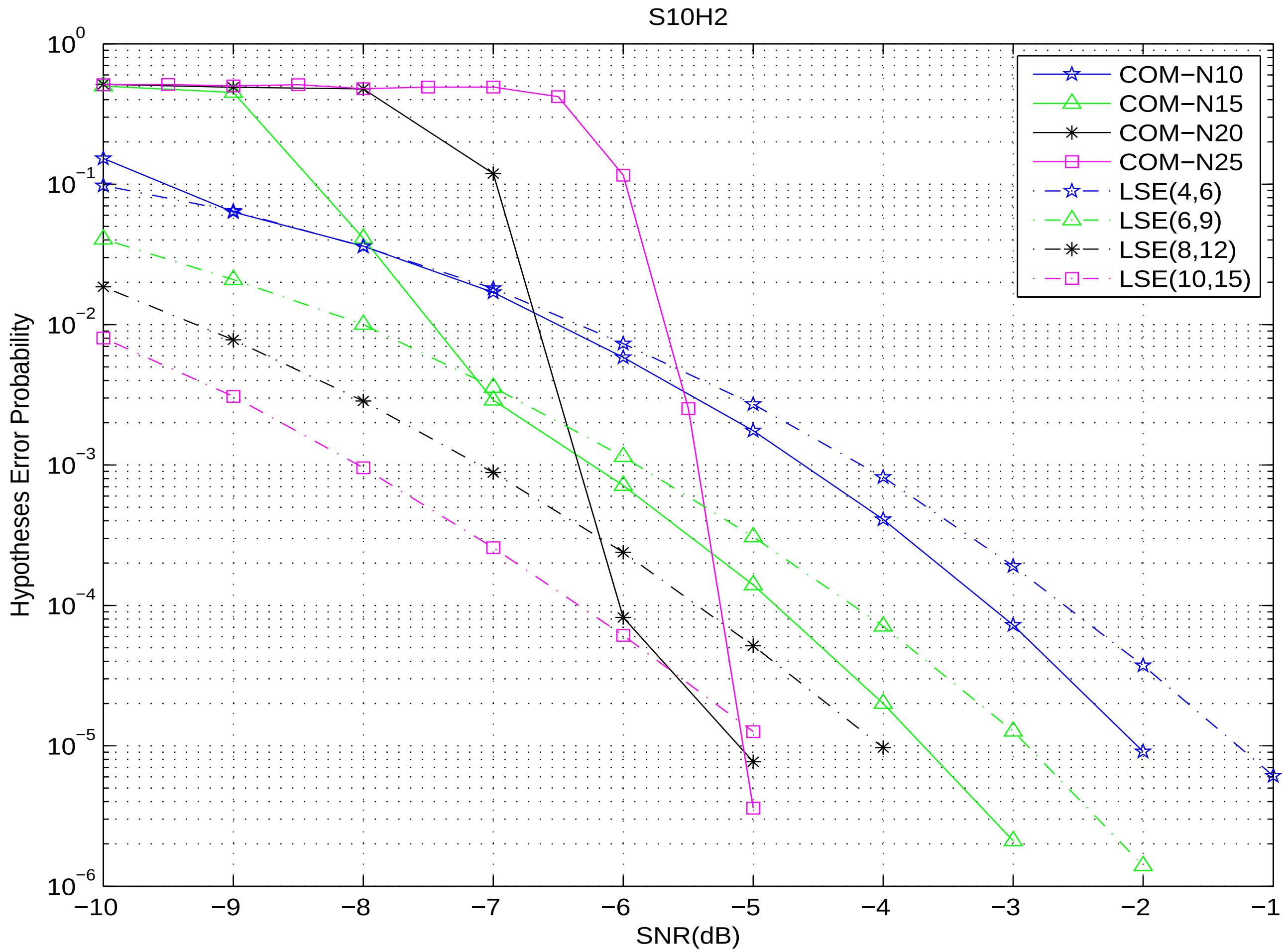

Figure 4: Performances of hypothesis detection. "S10H2" represents ten sensors and two hypotheses are assumed in this figure.

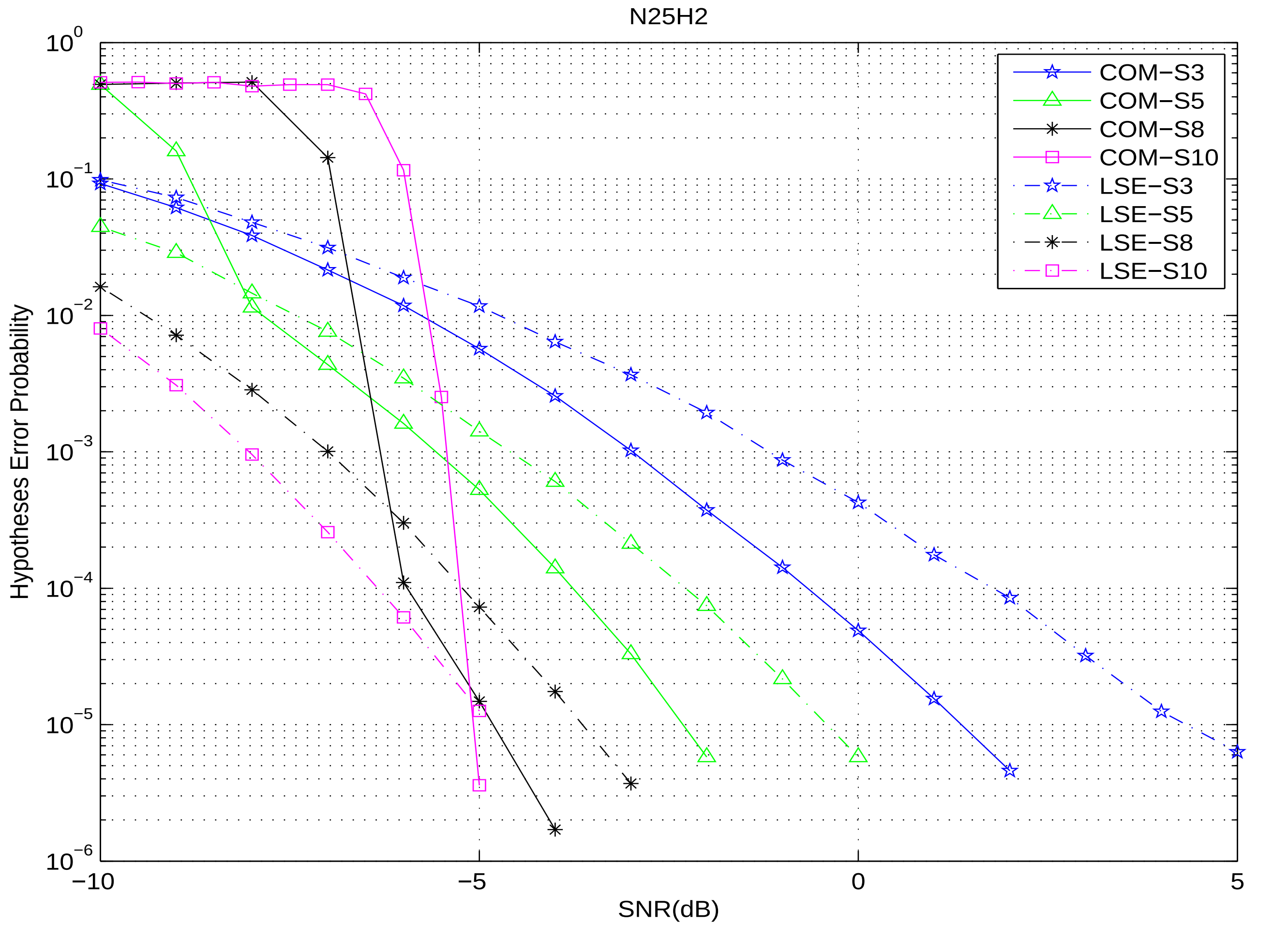

Figure 5: Performances of hypothesis detection. "N25H2" represents 25 bits per sensor and two hypotheses are assumed in this figure.

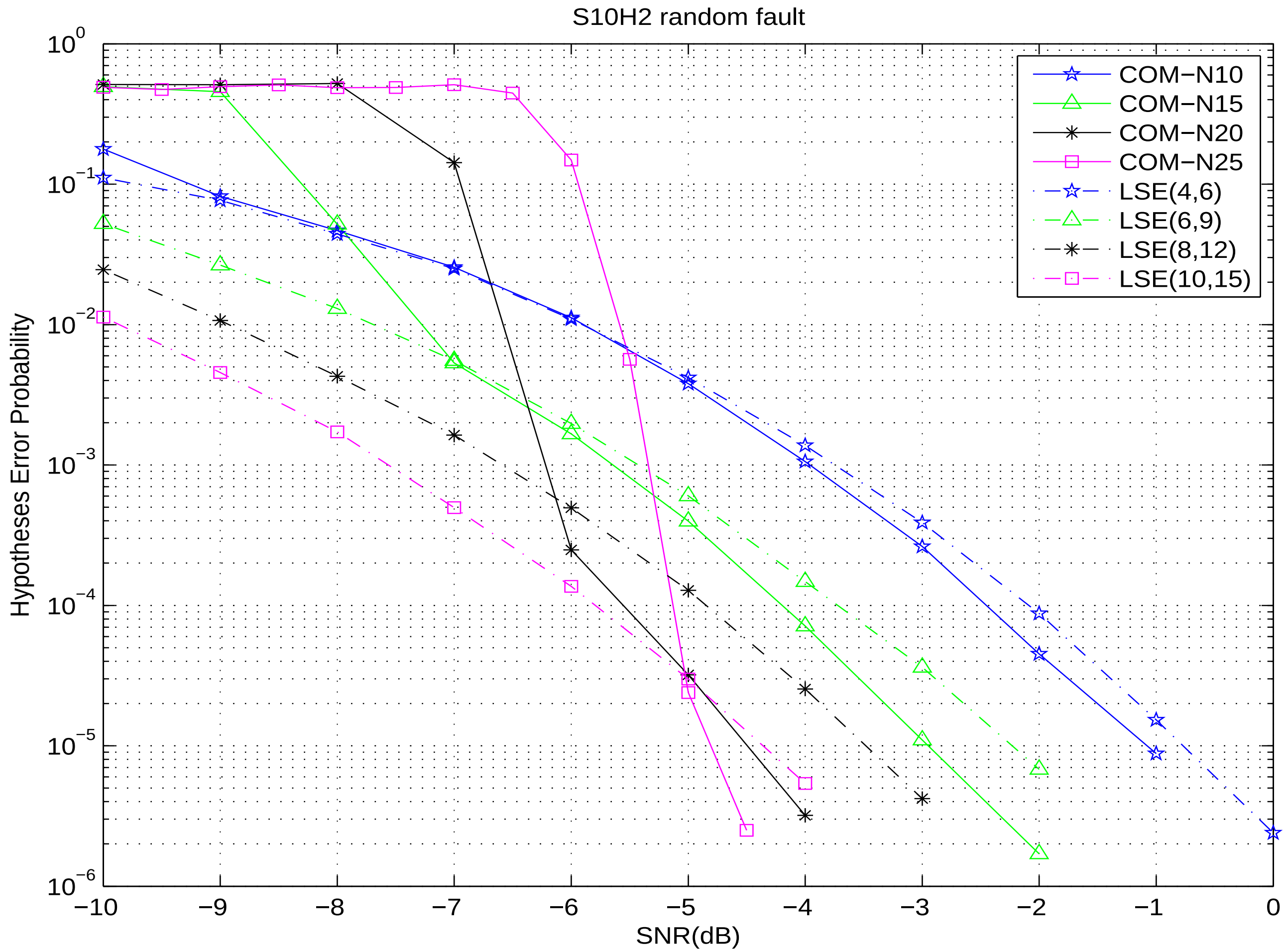

Figure 6: Performance of hypothesis detection. "S10H2 random fault" represents ten sensors, two hypotheses and one random faulty sensor are assumed in this figure.

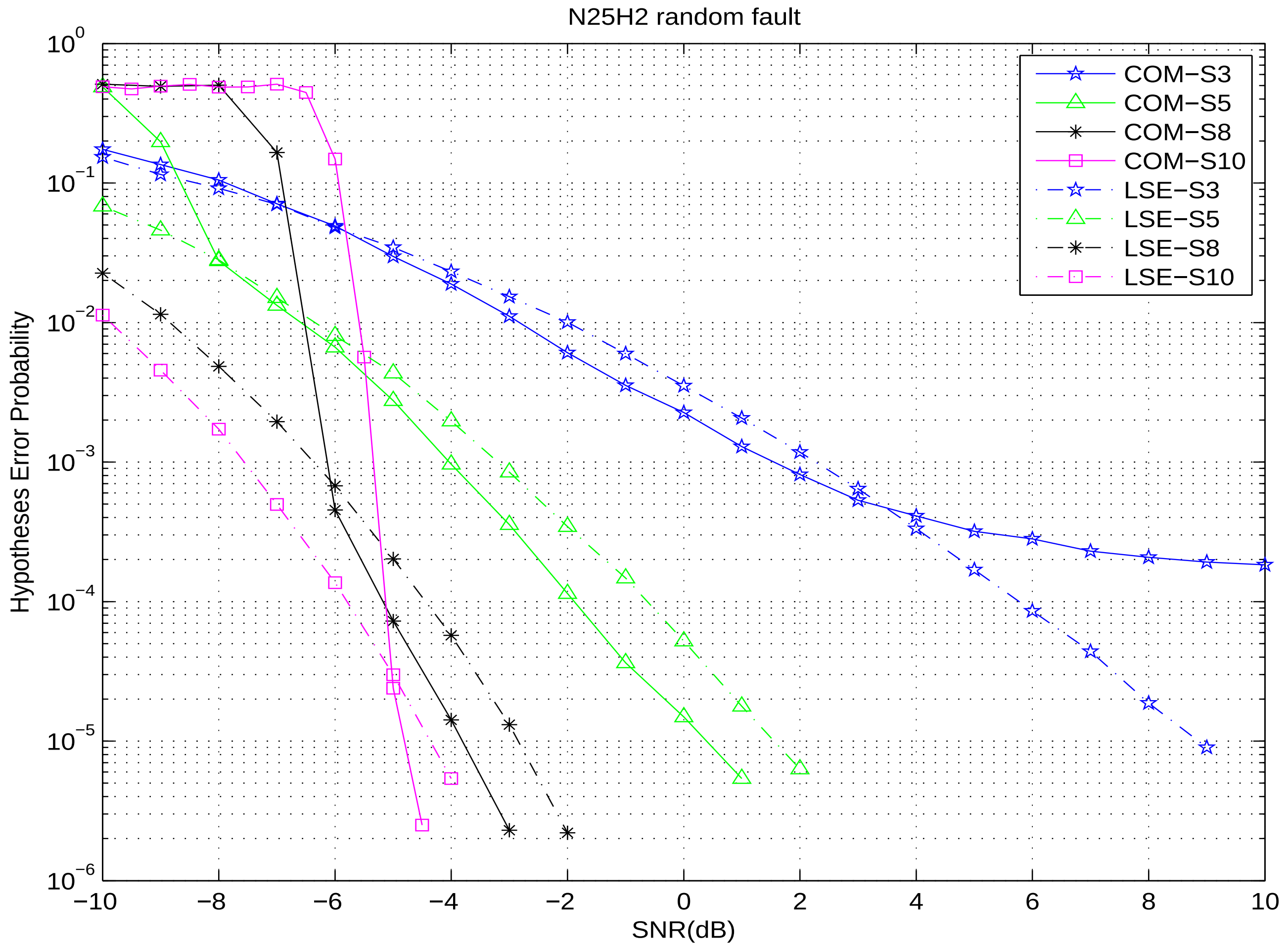

Figure 7: Performance of hypothesis detection. "N25H2 random fault" represents 25 bits per sensor, two hypotheses and one random faulty sensor are assumed in this figure.

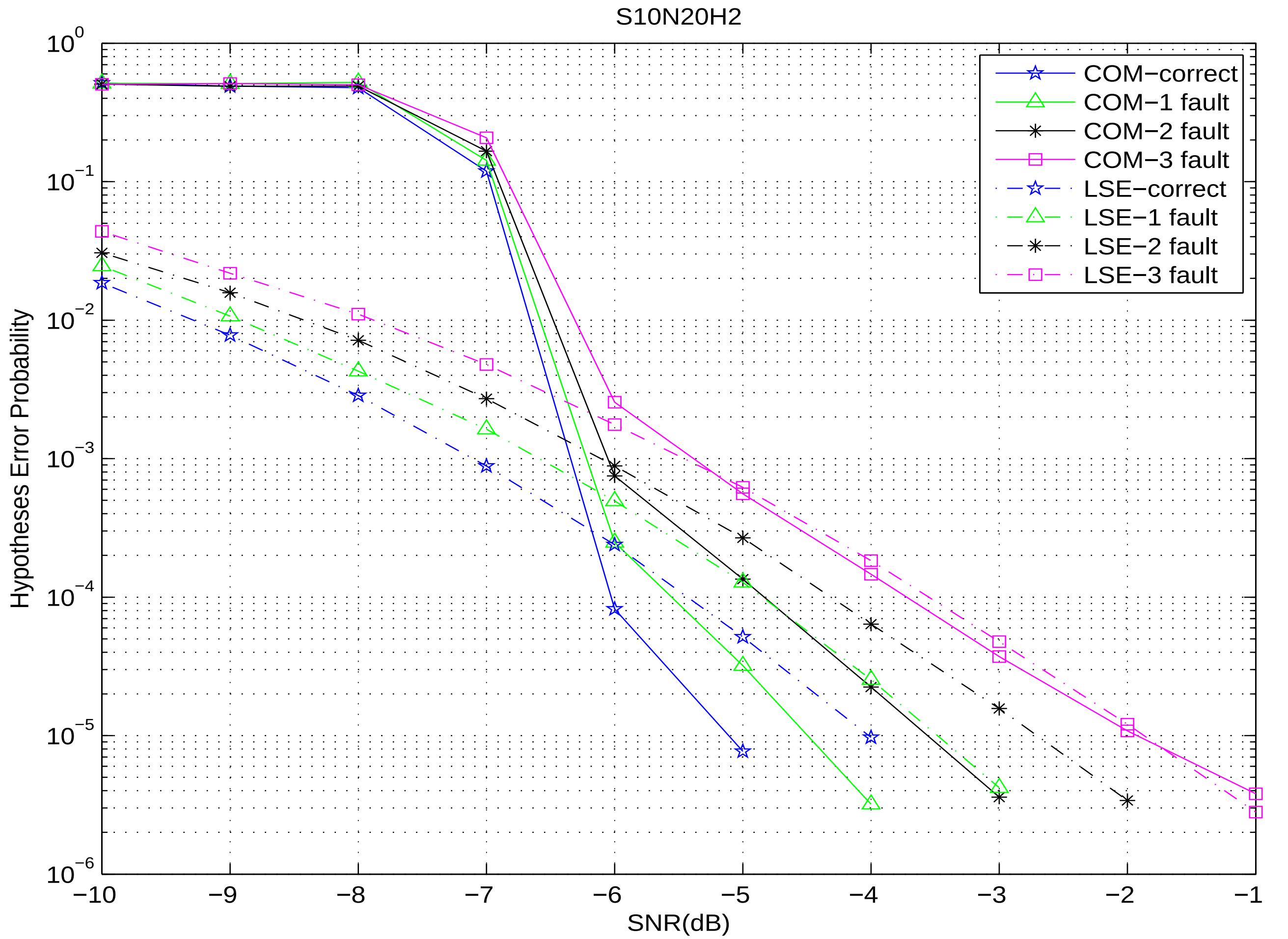

Figure 8: Performance of hypothesis detection. "S10N20H2" represents ten sensors, 20 bits per sensor and two hypotheses are assumed in this figure.

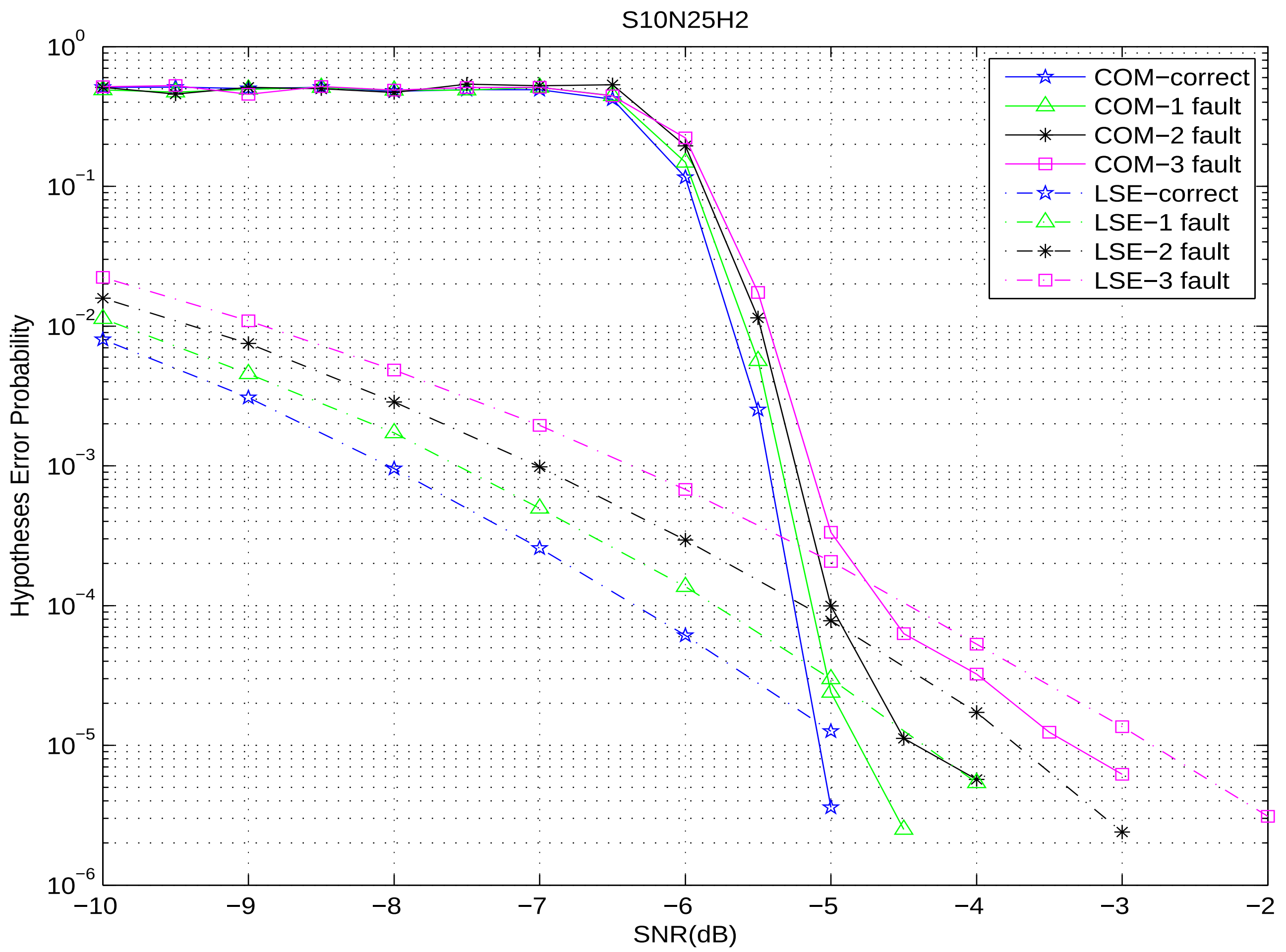

Figure 9: Performance of hypothesis detection. "S10N25H2" represents ten sensors, 25 bits per sensor and two hypotheses are assumed in this figure.

Deep Fading Effect Due to Soft-Decision

Table 1: The SNRs corresponding to sudden performance degradation of the COM scheme.

S	10	5	10	10	10
N	10	25	15	20	25
S×N	100	125	150	200	250
Sudden degraded SNR break point (dB)	-9	-8	-7	-6	-5
See Fig.	4	5	4	4	4

Quantization

- The deep fading of the COM soft-decision.
- We use the 3-bits quantizater to quantize the received information.
- The step size is the optimum step size for gaussian random variable.

Table 2: The optimum step size for gaussian random variable

Number of quantization bits	1	2	3	4	5
Number of output levels	2	4	8	16	32
Optimum step size	1.596	0.9957	0.586	0.3352	0.1881

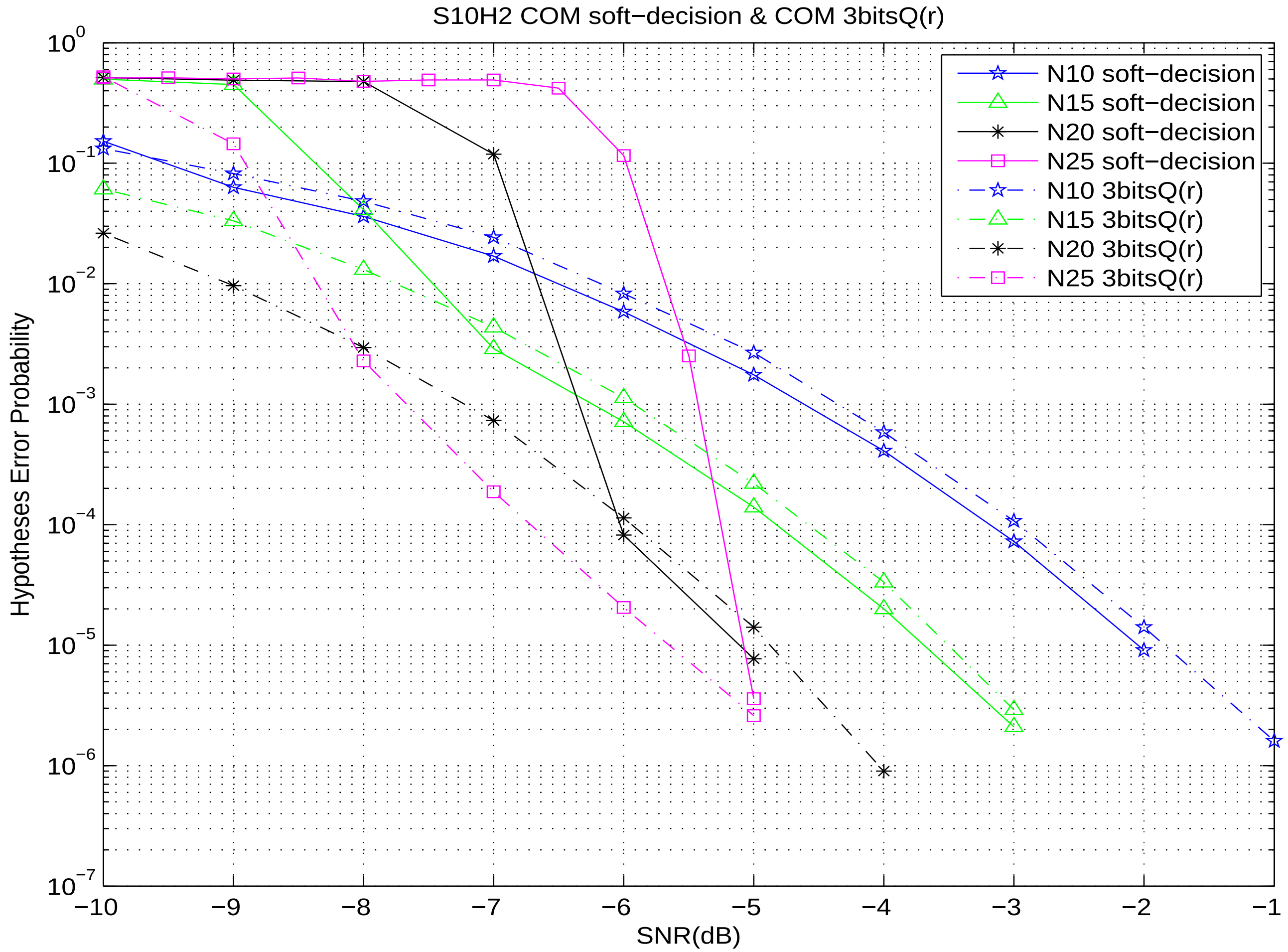

Figure 10: Performance of hypothesis detection. "S10H2" represents ten sensors and two hypotheses are assumed in this figure. "3bitsQ(r)" means using 3bits quantizer to quantize r.

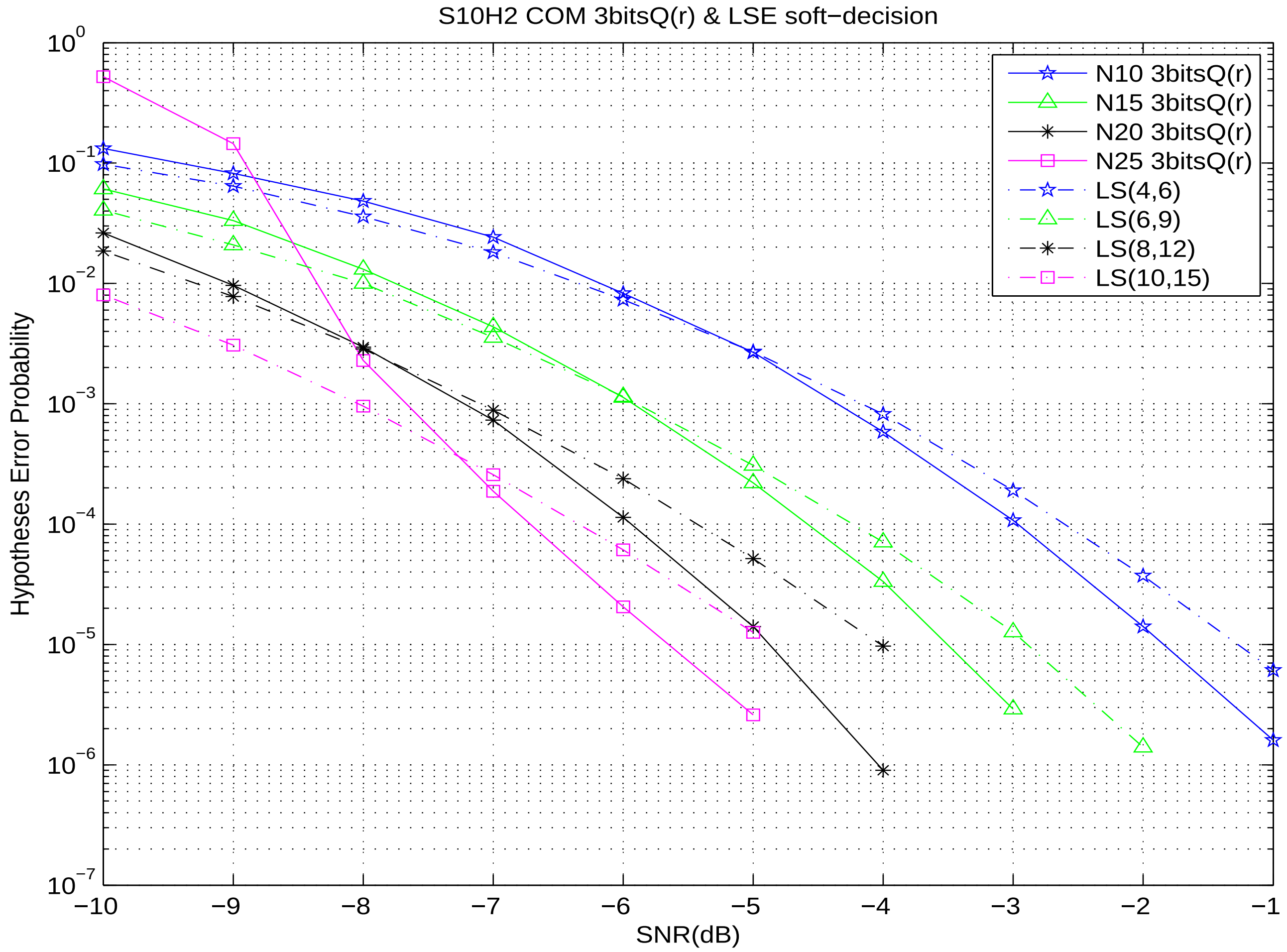

Figure 11: Performance of hypothesis detection. "S10H2" represents ten sensors and two hypotheses are assumed in this figure. "3bitsQ(r)" means using 3bits quantizer to quantize r.

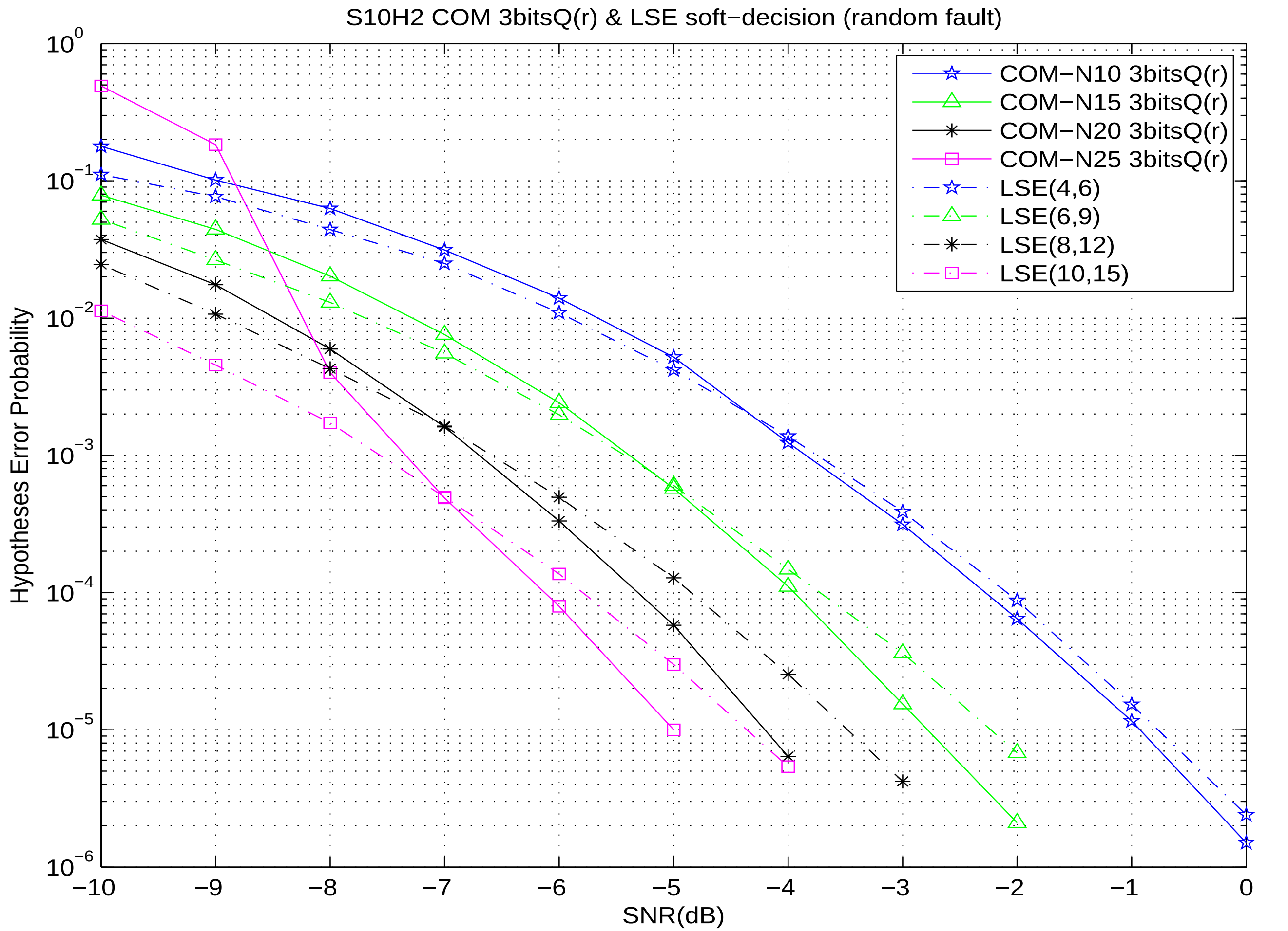

Figure 12: Performance of 3-bit quantized COM fusion and LSE fusion. “S10H2 $\cdots$ (random fault)” represents ten sensors, two hypotheses and one random faulty sensor are assumed in this figure. “3bitsQ(r)” means 3-bit quantizer is used in the quantization of reception $\boldsymbol{r}$.

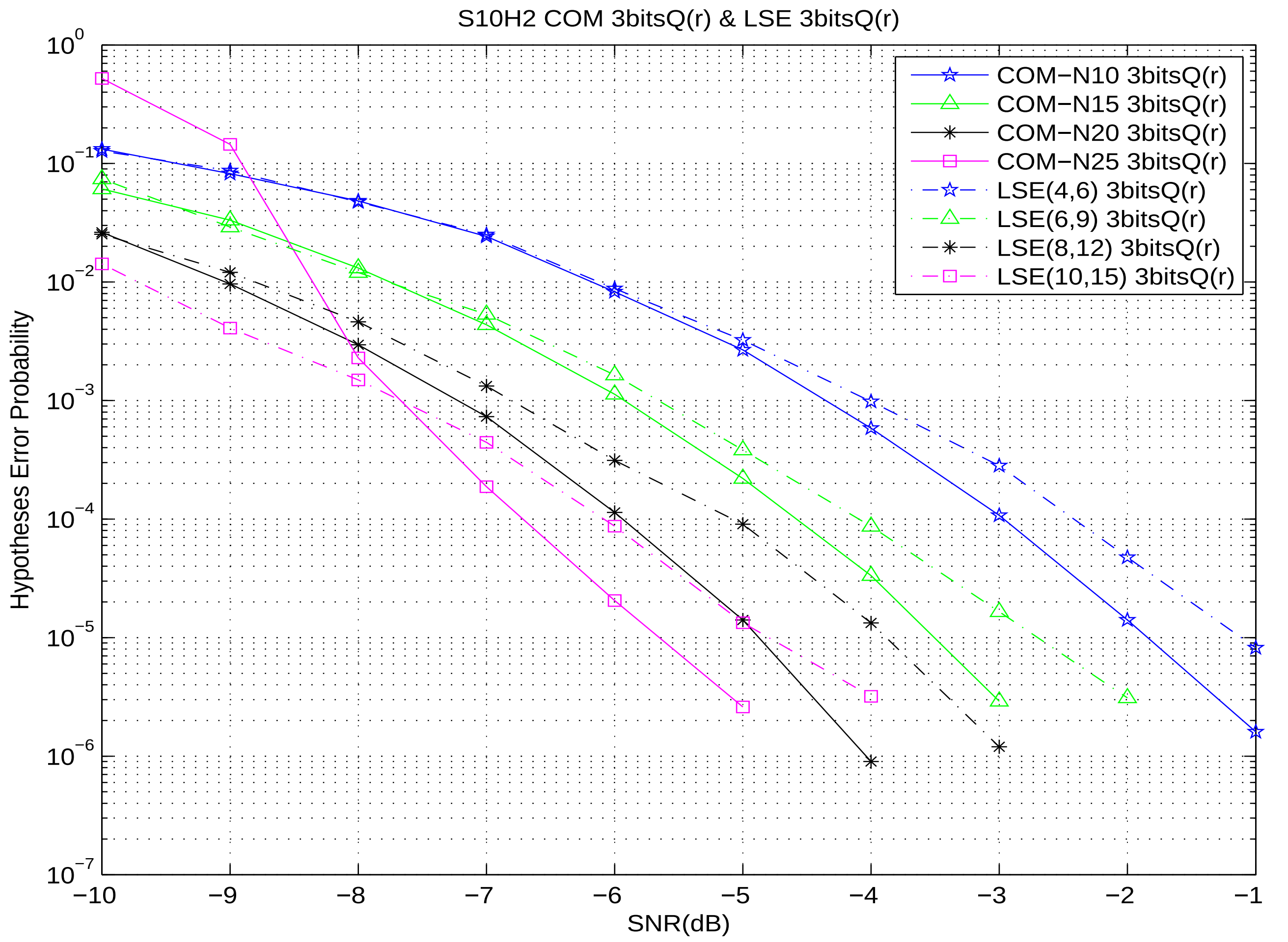

Figure 13: Performance of 3-bit quantized COM fusion and 3-bit quantized LSE fusion. "S10H2" represents ten sensors and two hypotheses are assumed in this figure. "3bitsQ(r)" means 3-bit quantizer is used in the quantization of reception $\boldsymbol{r}$.

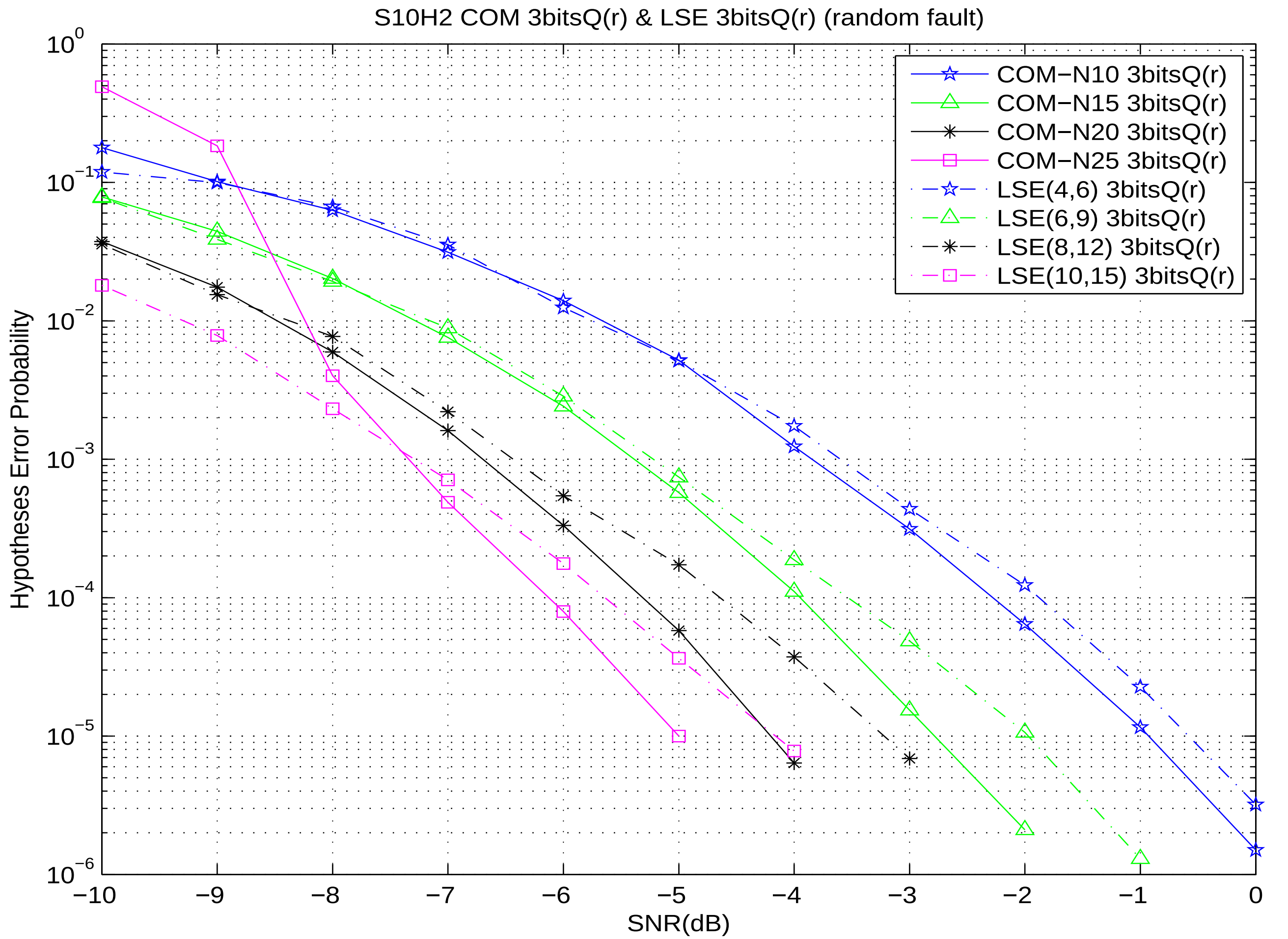

Figure 14: Performance of 3-bit quantized COM fusion and 3-bit quantized LSE fusion. "S10H2 $\cdots$ (random fault)" represents ten sensors, two hypotheses and one random faulty sensor are assumed in this figure. "3bitsQ(r)" means 3-bit quantizer is used in the quantization of reception $\boldsymbol{r}$.

$$
\begin{aligned}
\hat{\ell} &= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}\|\boldsymbol{r}_k-\mathbb{P}_B^{(\ell)}\boldsymbol{r}_k\|^2\\
&= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}(\boldsymbol{r}_k-\mathbb{P}_B^{(\ell)}\boldsymbol{r}_k)^H(\boldsymbol{r}_k-\mathbb{P}_B^{(\ell)}\boldsymbol{r}_k)\\
&= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}(\boldsymbol{r}_k^H\boldsymbol{r}_k-\boldsymbol{r}_k^H\mathbb{P}_B^{(\ell)}\boldsymbol{r}_k)\\
&= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}-tr(\mathbb{P}_B^{(\ell)}\boldsymbol{r}_k\boldsymbol{r}_k^H)\\
&= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}\Big(\|vec(\boldsymbol{r}_k\boldsymbol{r}_k^H)\|^2-vec(\mathbb{P}_B^{(\ell)})^T vec(\boldsymbol{r}_k\boldsymbol{r}_k^H)\\
&\qquad\qquad -vec(\boldsymbol{r}_k\boldsymbol{r}_k^H)^H vec(\mathbb{P}_B^{(\ell)})+\|vec(\mathbb{P}_B^{(\ell)})\|^2\Big)\\
&= \arg\min_{1\le\ell\le M}\sum_{k=1}^{K}\|vec(\boldsymbol{r}_k\boldsymbol{r}_k^H)-vec(\mathbb{P}_B^{(\ell)})\|^2
\end{aligned}
$$

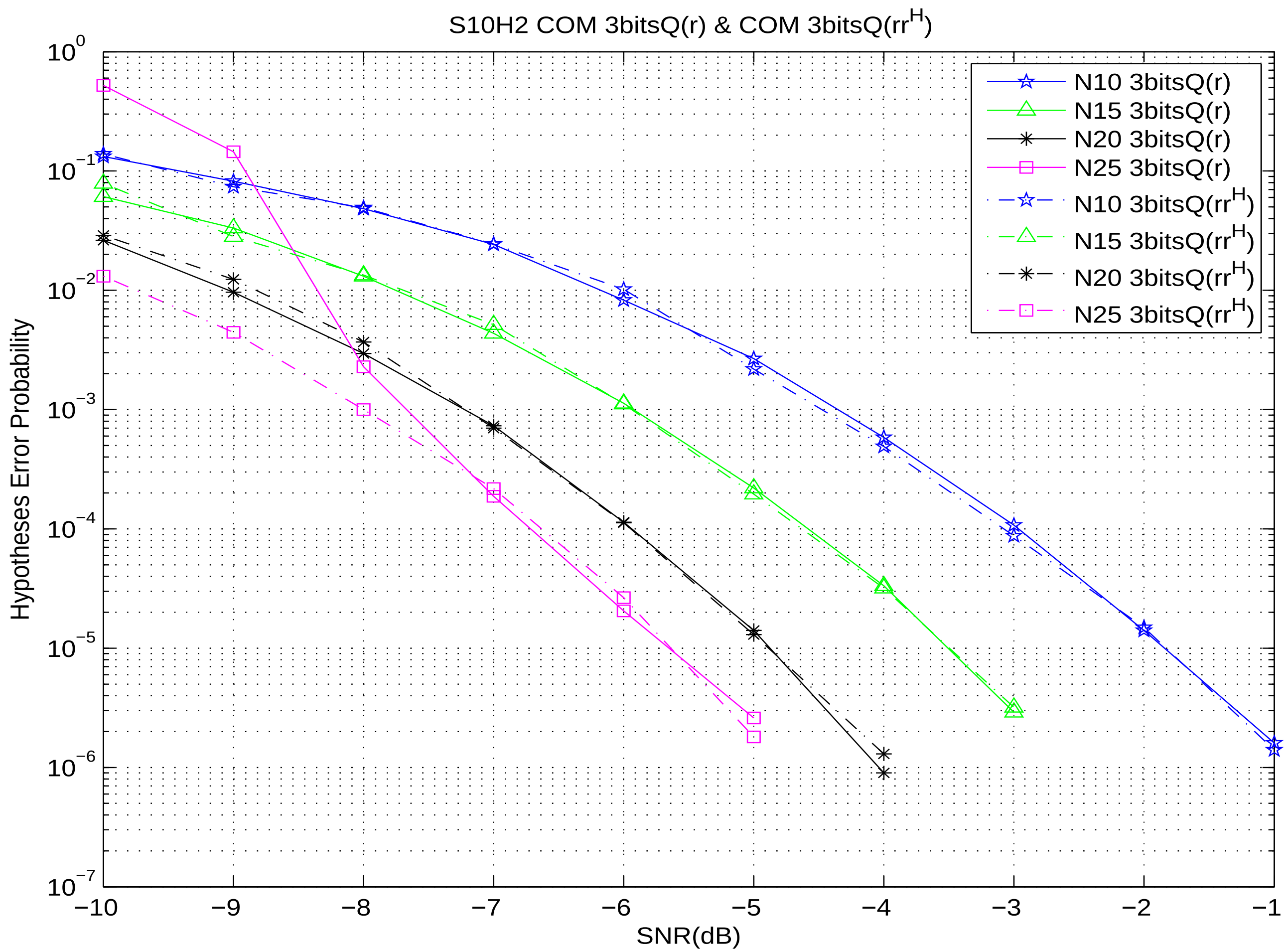

Figure 15: Performance of hypothesis detection. "S10H2" represents ten sensors and two hypotheses are assumed in this figure. "3bitsQ(r)" means using 3bits quantizer to quantize r.

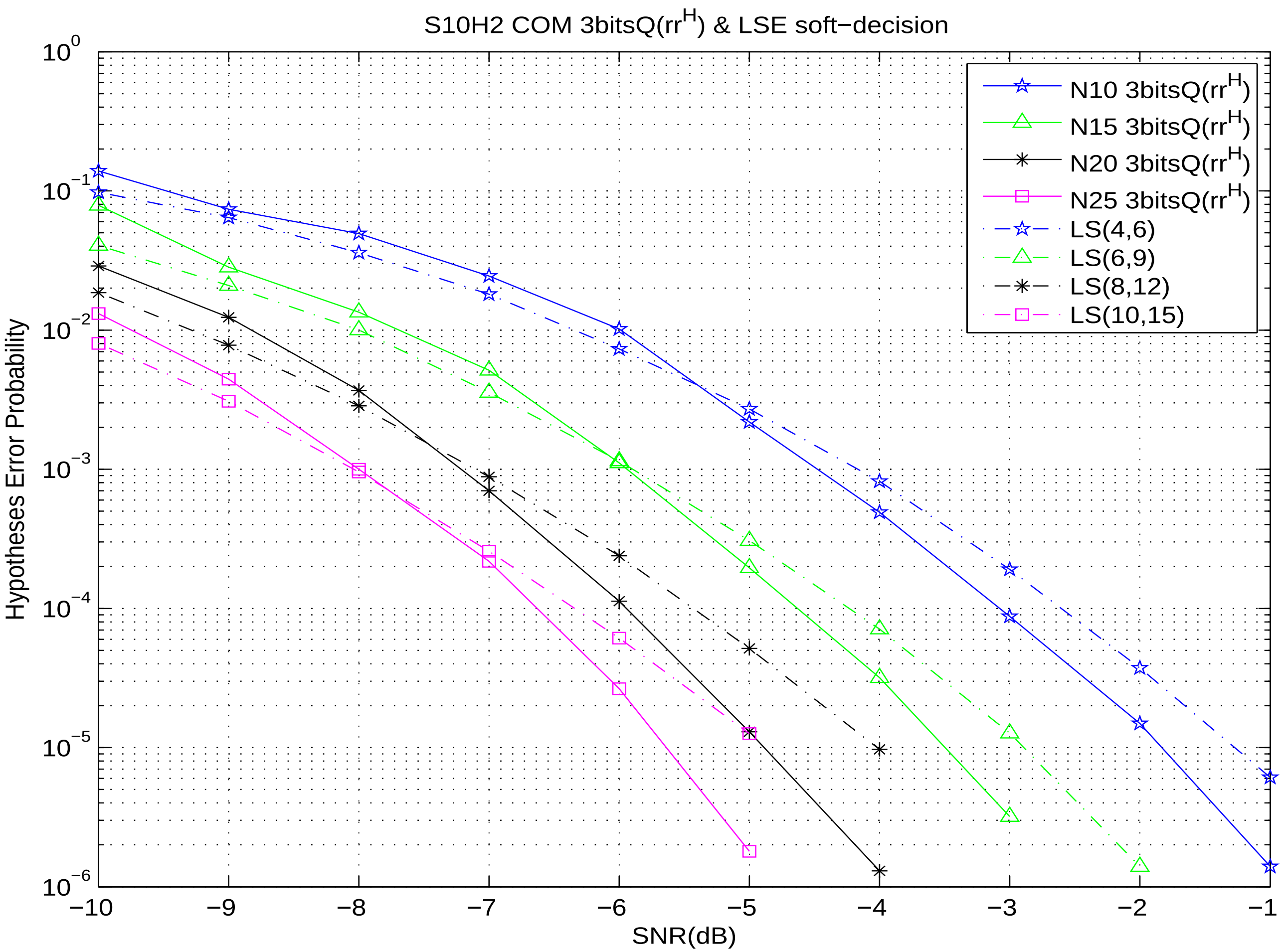

Figure 16: Performance of hypothesis detection. “S10H2” represents ten sensors and two hypotheses are assumed in this figure. “3bitsQ(r)” means using 3bits quantizer to quantize r.

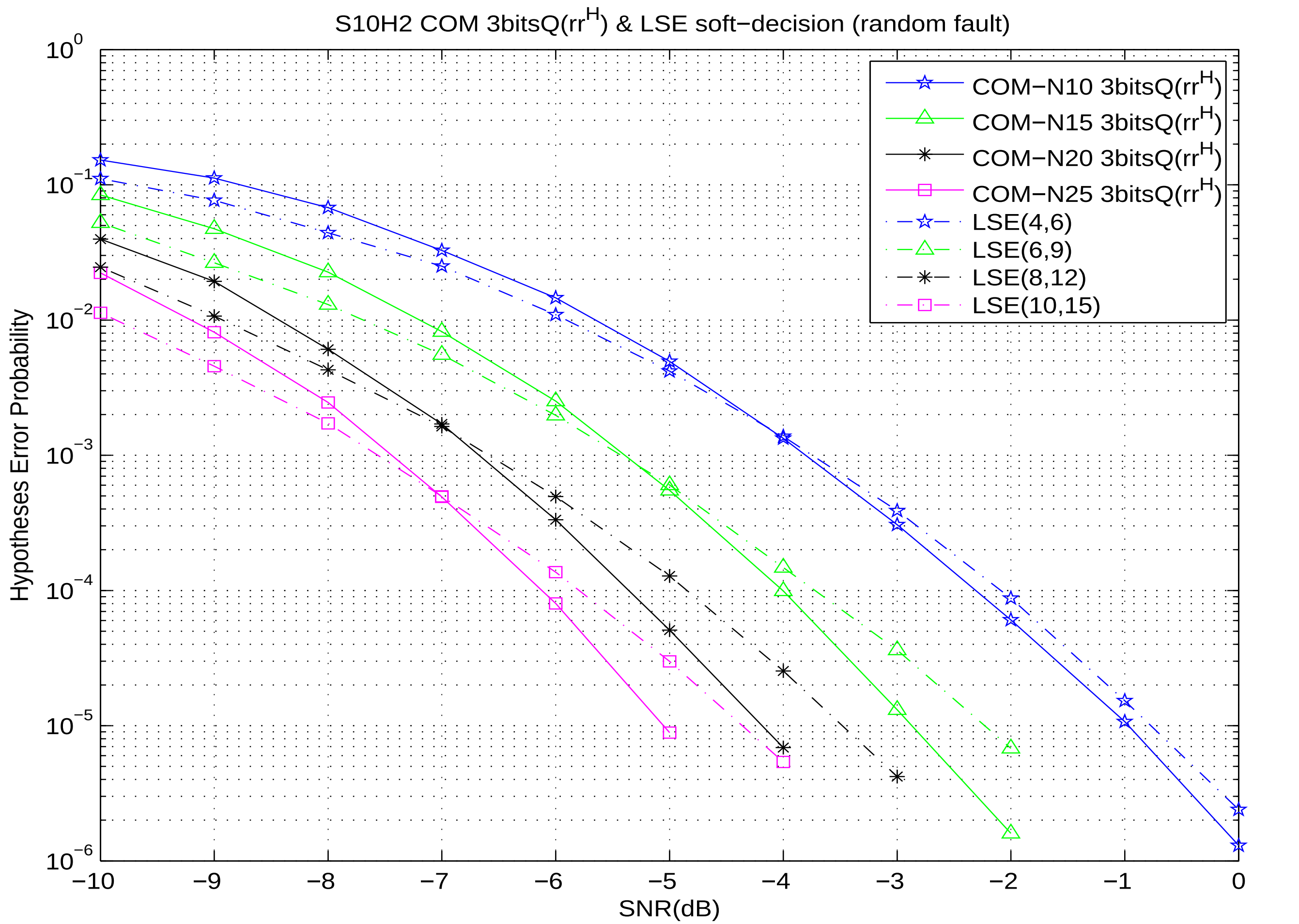

Figure 17: Performance of 3-bit product-quantized COM fusion and LSE fusion. "S10H2 $\cdots$ (random fault)" represents ten sensors, two hypotheses and one random faulty sensor are assumed in this figure. "3bitsQ(rr^H)" means 3-bit quantizer is used in the quantization of product reception $\boldsymbol{rr}^H$.

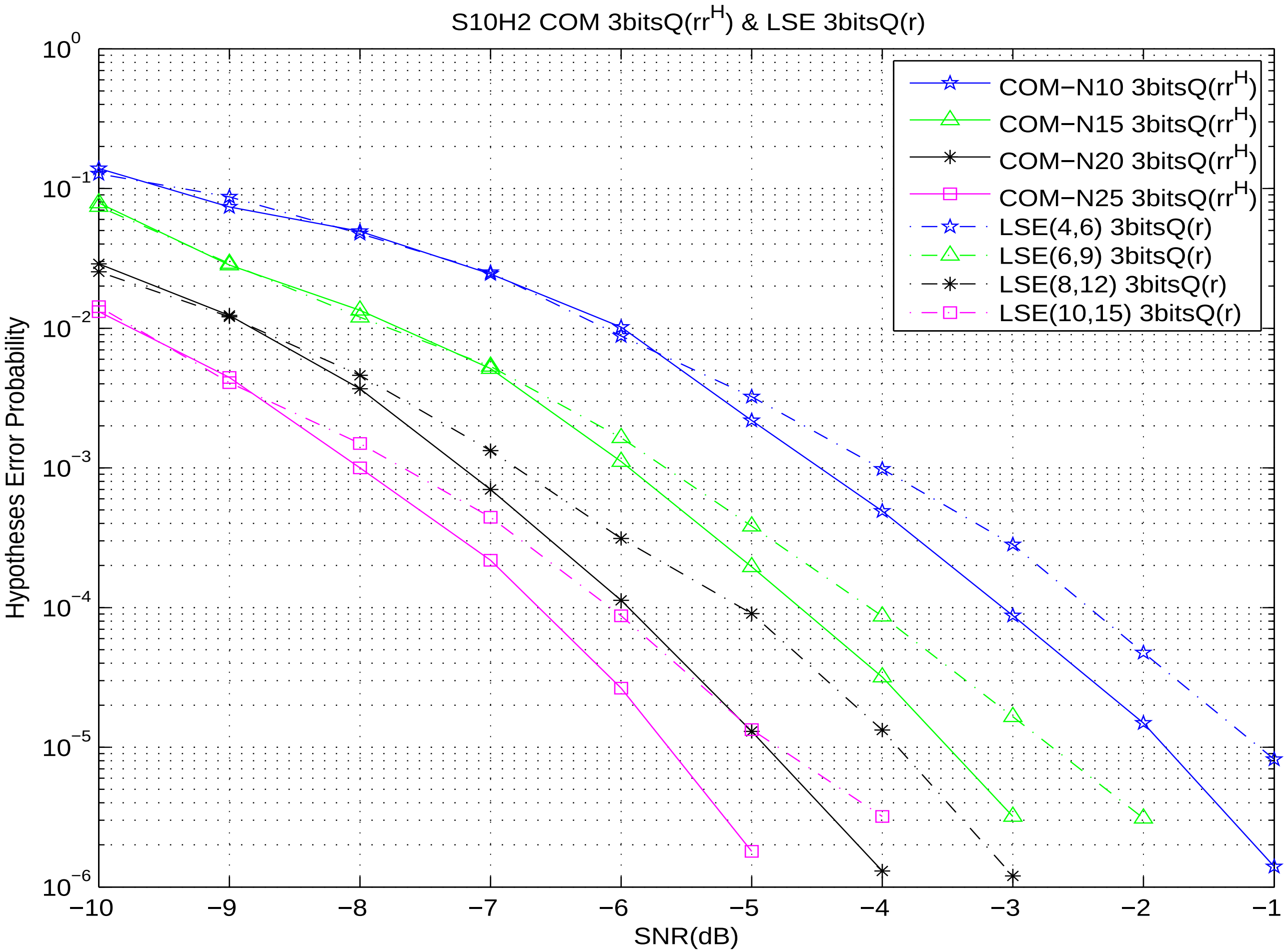

Figure 18: Performance of 3-bit reception-product-quantized COM fusion and 3-bit quantized LSE fusion. "S10H2" represents ten sensors and two hypotheses are assumed in this figure. "3bitsQ(rr^H)" means 3-bit quantizer is used in the quantization of product reception $\boldsymbol{rr}^H$.

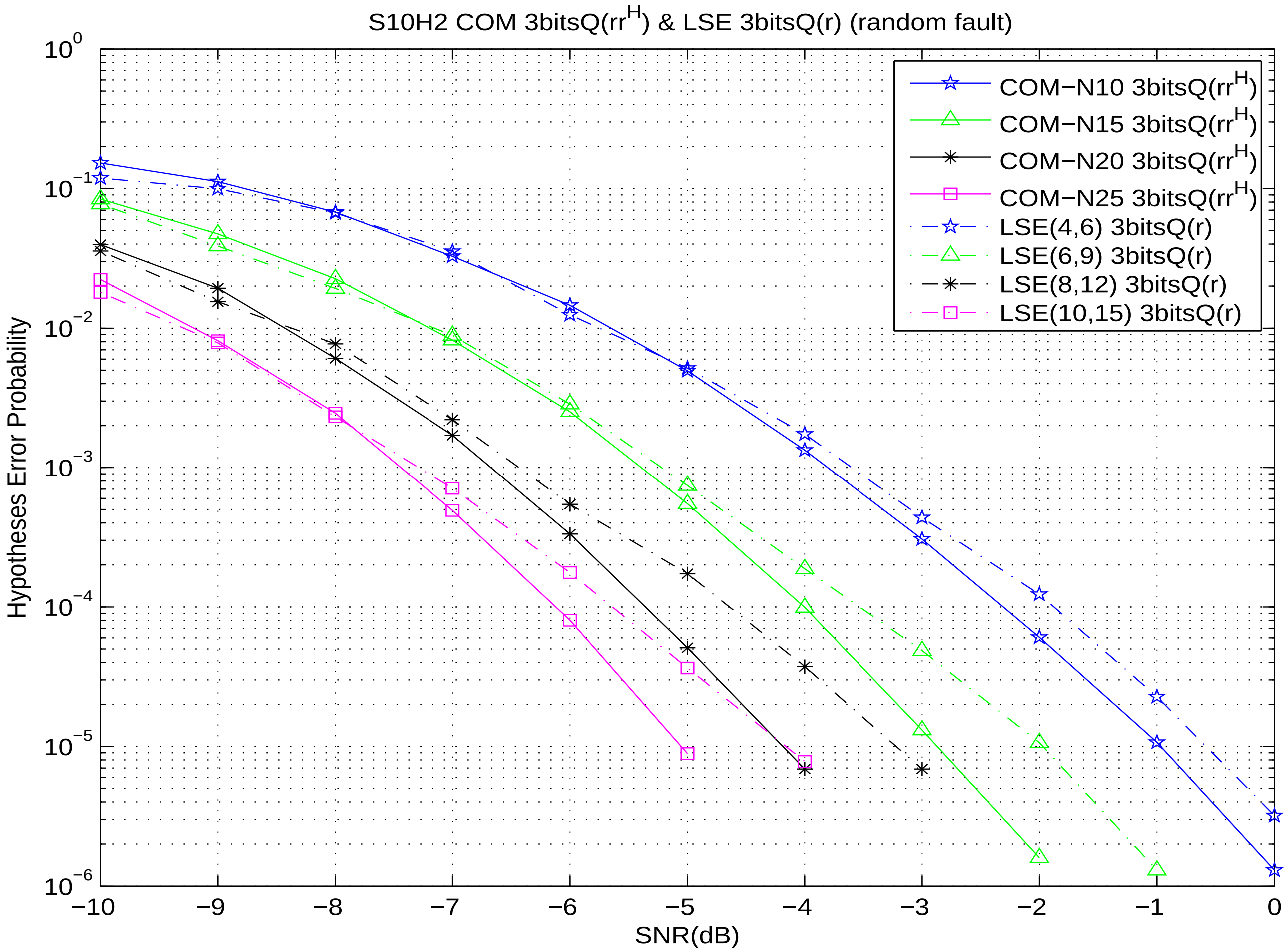

Figure 19: Performance of 3-bit reception-product-quantized COM fusion and 3-bit quantized LSE fusion. "S10H2 $\cdots$ (random fault)" represents ten sensors, two hypotheses and one random faulty sensor are assumed in this figure. "3bitsQ(rr^H)" means 3-bit quantizer is used in the quantization of product reception $\boldsymbol{rr}^H$.

Conclusions

- The training sequence is retained for information-bearing, the simulations indicate that a better performance over the conventional scheme with training-sequence-based channel estimation is resulted.
- The error floor levels owing to random sensor faults are less severe in the scheme of combined channel estimation and sensor fault protection.
- The sudden performance degradation due to deep fading suggests that hard-decision at the fusion is more robust than the soft-decision fusion.
- Quantize rr^H is more robust than quantize r.