Equalization of the Correlated Additive Gaussian Noises on the Wireless Links of Sensor Networks

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Outline

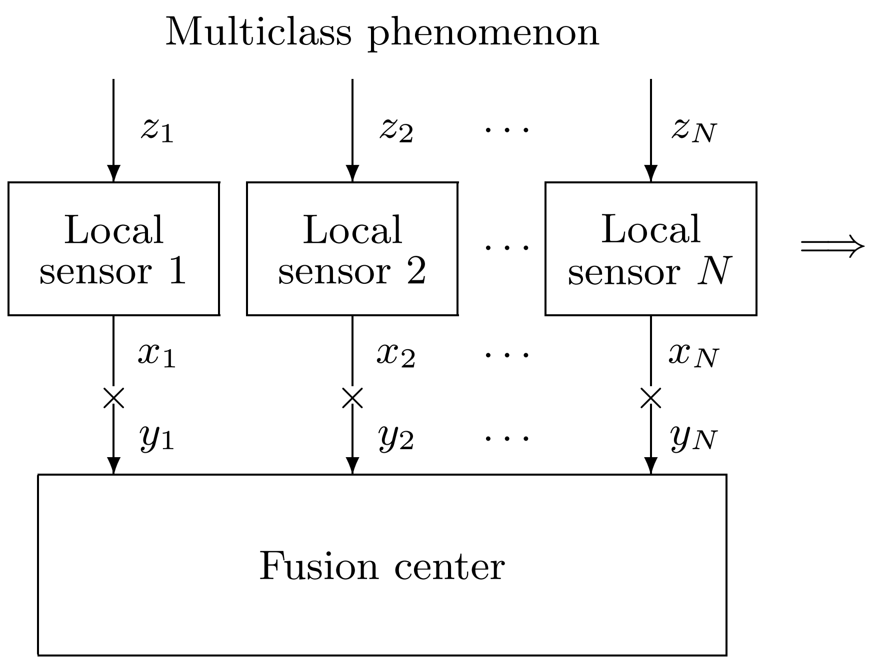
- Introduction
- Wireless Sensor Network and DCFECC
- System Model
- Soft-Decision Fusion Rule
- Channel Estimation
- Simple Code Search Criterion
- Simulation Results
- Conclusions

Introduction

- For wireless sensor networks operating in a harsh environment, distributed classification fusion approach using error correcting codes (DCFECC) has been shown to provide a desired robustness against sensor faults under limited energy support.
- In this thesis, we extend the DCFECC by relaxing the assumption of independently and identically distributed wireless link noises to correlated ones.
- Since the wireless link channel can be estimated and equalized by the help of the training sequence, we try to design a fault-tolerant soft decision fusion rule suitable for use under correlated link noises.

- As the code matrix using in DCFECC approach is hard to design when the number of sensor is large, we propose a simple code search criterion under AWGN wireless link noises.
- The simulation results are finally given under the AWGN channels, spatially correlated channels and non-identical uncorrelated channels.

System model



Code matrix $\mathbb{C}$

	sensor 1	$\cdots$	sensor N
H_0	$c_{0,1}$	$\cdots$	$c_{0,N}$
H_1	$c_{1,1}$	$\cdots$	$c_{1,N}$
$\vdots$	$\vdots$	$\cdots$	$\vdots$
H_{M-1}	$c_{M-1,1}$	$\cdots$	$c_{M-1,N}$

$x_j = (-1)^{c_{\ell,j}}$, if H_ℓ is declared true upon the reception of z_j

Figure 1: The distributed M-ary classification system model.

- $\{z_j\}_{j=1}^N$ are conditionally independent given each hypothesis.
- Denote by $h_{\ell|i}^{(j)}$ the probability of classifying H_ℓ given that H_i is the true hypothesis at sensor j.
- The prior probability of each hypothesis is assumed equal.
- $c_{\ell,j} \in \{0,1\}$ and $x_j = (-1)^{c_{\ell,j}}$.

- The communication channel between sensors and fusion center is assumed to be a spatially correlated additive Gaussian channel.
- The received vector $\boldsymbol{y}$ is given by

$$\boldsymbol{y} = \boldsymbol{x} + \mathbb{T}\boldsymbol{n},$$

where $\boldsymbol{y} = (y_1, \ldots, y_N) \in \Re^N$, $\quad \boldsymbol{x} = (x_1, \ldots, x_N) \in \{-1, 1\}^N$, and $\boldsymbol{n} = (n_1, \ldots, n_N) \in \Re^N$ is i.i.d. standard Gaussian distributed.

- The probability of receiving $\boldsymbol{y}$ given that $\boldsymbol{x}$ is transmitted is equal to:

$$\Pr(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{(2\pi)^{\frac{m}{2}} |\mathbb{C}|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\boldsymbol{y} - \boldsymbol{x})^T \mathbb{C}^{-1}(\boldsymbol{y} - \boldsymbol{x})\right\},$$

where $\mathbb{C} = \mathbb{T}\mathbb{T}^T$ is the covariance matrix of the correlated noises.

MAP fusion rule under AWGN links

- The MAP fusion rule under AWGN links can be derived as:

$$
\begin{aligned}
\hat{i} &= \arg \max_{0 \le \ell \le M-1} \Pr(H_\ell | \boldsymbol{y}) \\
&= \arg \max_{0 \le \ell \le M-1} \sum_{\boldsymbol{x} \in \{-1,1\}^N} \Pr(\boldsymbol{x} | H_\ell) \Pr(\boldsymbol{y} | \boldsymbol{x}) \\
&= \arg \max_{0 \le \ell \le M-1} \sum_{\boldsymbol{x} \in \{-1,1\}^N} \prod_{j=1}^{N} \Pr(x_j | H_\ell) \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(y_j - x_j)^2}{2\sigma^2} \right\} \\
&= \arg \max_{0 \le \ell \le M-1} \prod_{j=1}^{N} \left(\sum_{x_j \in \{-1,1\}} \Pr(x_j | H_\ell) \exp\left\{ \frac{x_j y_j}{\sigma^2} \right\} \right) \\
&= \arg \max_{0 \le \ell \le M-1} \sum_{j=1}^{N} \log \left[1 - \left(1 - e^{-\frac{2y_j}{\sigma^2}} \right) \left(\sum_{i=0}^{M-1} c_{i,j} h_{i|\ell}^{(j)} \right) \right].
\end{aligned}
$$

Minimum Euclidean distance fusion rule

- MED fusion rule under AWGN link noises is given by:

$$\hat{i} = \arg \min_{0 \leq \ell \leq M-1} \|\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\|^2,$$

  where $\boldsymbol{c}_\ell \triangleq (c_{\ell,1}, c_{\ell,2}, \ldots, c_{\ell,N})$ denotes the row of $\mathbb{C}$ corresponding to the hypothesis H_ℓ.

- Unlike the conventional approach that employs the optimal MAP rule, the MED fusion rule provides enough distance between all the decision regions corresponding to different hypotheses by using the code matrix, and hence, can achieve the desired robustness against sensor failures.

- Unsurprisingly, when the local classification is adequately accurate, the suboptimal MED fusion rule performs close to the optimal MAP fusion rule.
- Specifically, if $\Pr\{x_j = (-1)^{c_{\ell,j}}|H_\ell\} \gg \Pr\{x_j \neq (-1)^{c_{\ell,j}}|H_\ell\}$, then the MAP fusion rule can be refined as:

$$\begin{aligned}
\hat{i} &\approx \arg\max_{0\le\ell\le M-1} \prod_{j=1}^{N} \exp\left\{-\frac{(y_j-(-1)^{c_{\ell,j}})^2}{2\sigma^2}\right\} \\
&= \arg\min_{0\le\ell\le M-1} \sum_{j=1}^{N} (y_j-(-1)^{c_{\ell,j}})^2 \\
&= \arg\min_{0\le\ell\le M-1} \|\boldsymbol{y}-(-1)^{\boldsymbol{c}_\ell}\|^2.
\end{aligned}$$

MED fusion rule under correlated link noises

- Under spatially correlated wireless link noises, the distance between different hypothesis decision regions are distorted. The MED fusion rule should be adapted for different correlated link noises.
- The whitening matrix $\mathbb{W}$ with respect to colored noise $\mathbb{T}\boldsymbol{n}$ always exists.
- If $\mathbb{T}$ is invertible, $\mathbb{W} = \mathbb{T}^{-1}$. Otherwise, $\mathbb{W} = \Delta^{-1}\mathbb{B}^T$, where nonnegative definite $\mathbb{C} = \mathbb{T}\mathbb{T}^T$ can be represented as $\mathbb{B}\Lambda\mathbb{B}^T$ for $\mathbb{B}$ orthogonal and Λ diagonal with nonnegative entries, and Δ is a diagonal matrix with $\Delta_{kk} = \Lambda_{kk}^{1/2}$ if $\Lambda_{kk} > 0$ and $\Delta_{kk} = 1$ if $\Lambda_{kk} = 0$.

- After whitening, we can transform the correlated system into one with uncorrelated noises as:

$$\mathbb{W}\boldsymbol{y} = \mathbb{W}\boldsymbol{x} + \mathbb{W}\mathbb{T}\boldsymbol{n}.$$

- Then, the MED fusion rule under spatially correlated link noises is given by:

$$\begin{aligned}\hat{i} &= \arg\min_{0\le\ell\le M-1} \|\mathbb{W}\boldsymbol{y} - \mathbb{W}(-1)^{\boldsymbol{c}_\ell}\|^2 \\ &= \arg\min_{0\le\ell\le M-1} \left[\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\right]^T \mathbb{W}^T\mathbb{W}\left[\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\right].\end{aligned}$$

If $\mathbb{C} = \mathbb{T}\mathbb{T}^T$ is invertible, then

$$\hat{i} = \arg\min_{0\le\ell\le M-1} \left[\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\right]^T \mathbb{C}^{-1}\left[\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\right].$$

Training sequence

- The MED fusion rule requires the knowledge of the noise covariance matrix $\mathbb{C}$. It is necessary to estimate $\mathbb{C}$ beforehand.
- In our system setting, K training bits are initially and sequentially sent to the fusion center at each sensor.
- In notations, these training vectors are denoted by $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, ..., $\boldsymbol{u}_K$, where $\boldsymbol{u}_j = (u_{j,1}, u_{j,2}, \ldots, u_{j,N})$, and $u_{j,m} \in \{-1, 1\}$ is the training bit sent by sensor m at time j.

Channel estimation

- As $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_K$ are known to the fusion center, the channel covariance matrix $\mathbb{C} = \mathbb{T}\mathbb{T}^T$ can be estimated upon the reception of $\boldsymbol{y}_k = \boldsymbol{u}_k + \mathbb{T}\boldsymbol{n}_k$ for $1 \leq k \leq K$.
- It can be shown that in such case, the maximum-likelihood (ML) estimator of $\mathbb{C}$ is equal to

$$\hat{\mathbb{C}}_{ML} = \frac{1}{K}\sum_{k=1}^{K}(\boldsymbol{y}_k - \boldsymbol{u}_k)(\boldsymbol{y}_k - \boldsymbol{u}_k)^T.$$

Code matrix design

- The code matrix employed in our wireless sensor network plays an important role in the determination of system performance.
- The design objective of a good code matrix is to have the fusion system exhibit good performance in both fault-free and faulty situations.
- The minimum pairwise Hamming distance among codewords in the code matrix should be large so that the system can tolerate more sensor faults under faulty situation.
- The code matrix should achieve good misclassification error under fault-free situation, which seemingly favors the minimum-fusion-error criterion.

Simple code search criterion

- Using the MED fusion rule under AWGN wireless link noises, the probability of fusion error given that H_i is the true hypothesis is derived as:

$$
\begin{aligned}
& \Pr[\text{error}|H_i] \\
\leq\ & \Pr\left\{ \|\boldsymbol{y} - (-1)^{\boldsymbol{c}_i}\| \geq \min_{0\leq\ell\leq M-1,\ell\neq i} \|\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\| \,\middle|\, H_i \right\} \\
\leq\ & \sum_{0\leq\ell\leq M-1,\ell\neq i} \Pr\left\{ \|\boldsymbol{y} - (-1)^{\boldsymbol{c}_i}\|^2 \geq \|\boldsymbol{y} - (-1)^{\boldsymbol{c}_\ell}\|^2 \,\middle|\, H_i \right\} \\
=\ & \sum_{0\leq\ell\leq M-1,\ell\neq i} \Pr\left\{ \sum_{\{j\,:\,c_{\ell,j}=0,c_{i,j}=1\}} y_j - \sum_{\{j\,:\,c_{\ell,j}=1,c_{i,j}=0\}} y_j \geq 0 \,\middle|\, H_i \right\}.
\end{aligned}
$$

- Assume that $\Pr\{x_j = (-1)^{c_{\ell,j}} | H_\ell\} = 1$ for $1 \le j \le N$. Then, $\{(y_j|H_i)\}_{j=1}^{N}$ is independent Gaussian distributed, and

$$
\begin{aligned}
&\Pr[\text{error}] \\
\le\ & \frac{1}{M}\sum_{i=0}^{M-1}\sum_{\ell=0,\ell\ne i}^{M-1}\Pr\left\{\sum_{\{j\,:\,c_{\ell,j}=0,c_{i,j}=1\}} y_j - \sum_{\{j\,:\,c_{\ell,j}=1,c_{i,j}=0\}} y_j \ge 0 \,\middle|\, H_i\right\} \\
=\ & \frac{1}{M}\sum_{i=0}^{M-1}\sum_{\ell=0,\ell\ne i}^{M-1}\int_0^{\infty}\frac{1}{\sqrt{2\pi\sigma^2 d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)}}\exp\left\{-\frac{[y+d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)]^2}{2\sigma^2 d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)}\right\}dy \\
=\ & \frac{1}{M}\sum_{i=0}^{M-1}\sum_{\ell=0,\ell\ne i}^{M-1} Q\left(\frac{\sqrt{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)}}{\sigma}\right). \qquad (1)
\end{aligned}
$$

- Since that a larger minimum pairwise Hamming distance results in a smaller value in (1), we can achieve the code design objective by searching for the code matrix that minimizes (1).

Table 1: The code matrices that minimize (1), which are obtained by exhaustive computer search.

H_0	0	0	0	0	0	0	0	0	0	0
H_1	0	0	0	1	1	1	1	1	1	1
H_2	1	1	1	0	0	0	1	1	1	1

H_0	0	0	0	0	0	0	0	0	0	0
H_1	0	0	0	1	1	1	1	1	1	1
H_2	1	1	1	0	0	0	0	1	1	1
H_3	1	1	1	1	1	1	1	0	0	0

H_0	0	0	0	0	0	0	0	0	0	0
H_1	0	0	0	0	1	1	1	1	1	1
H_2	0	1	1	1	0	0	0	1	1	1
H_3	1	0	1	1	0	1	1	0	0	1
H_4	1	1	0	1	1	0	1	0	1	0

Simulation results

- All sensor observations have the same distribution given each hypothesis, and are randomly drawn from a σ^2-variance Gaussian distribution with means $m_0, m_1, \cdots, m_{M-1}$ corresponding to hypothesis $H_0, H_1, \cdots, H_{M-1}$, respectively.
- Each sensor makes the local classification based on the thresholds $\frac{m_0+m_1}{2}, \frac{m_1+m_2}{2}, \cdots, \frac{m_{M-2}+m_{M-1}}{2}$.
- OSNR is defined as $\frac{\left[(m_1-m_0)^2+(m_2-m_1)^2+\cdots+(m_{M-1}-m_{M-2})^2\right]/(M-1)}{\sigma^2}$, while CSNR is given by $\frac{1}{2\cdot\mathrm{tr}(\mathbb{C})/N}$.
- The faulty sensors are uniformly drawn from the N deployed sensor nodes when stuck-at-one fault or random fault is simulated.

AWGN wireless link noises

$$\mathbb{C} = \sigma^2 \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$

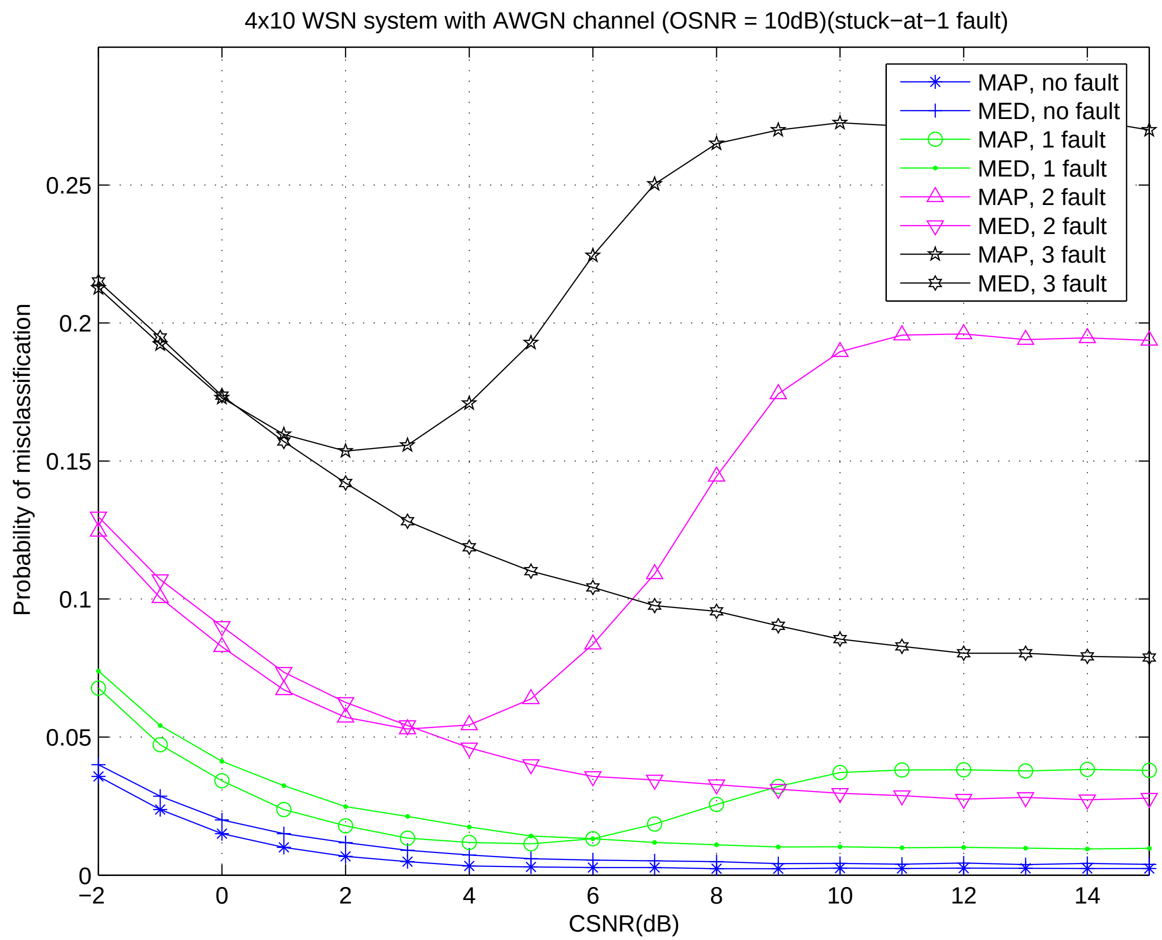


Figure 2: Performance of the MAP rule and the MED rule under AWGN channel at OSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

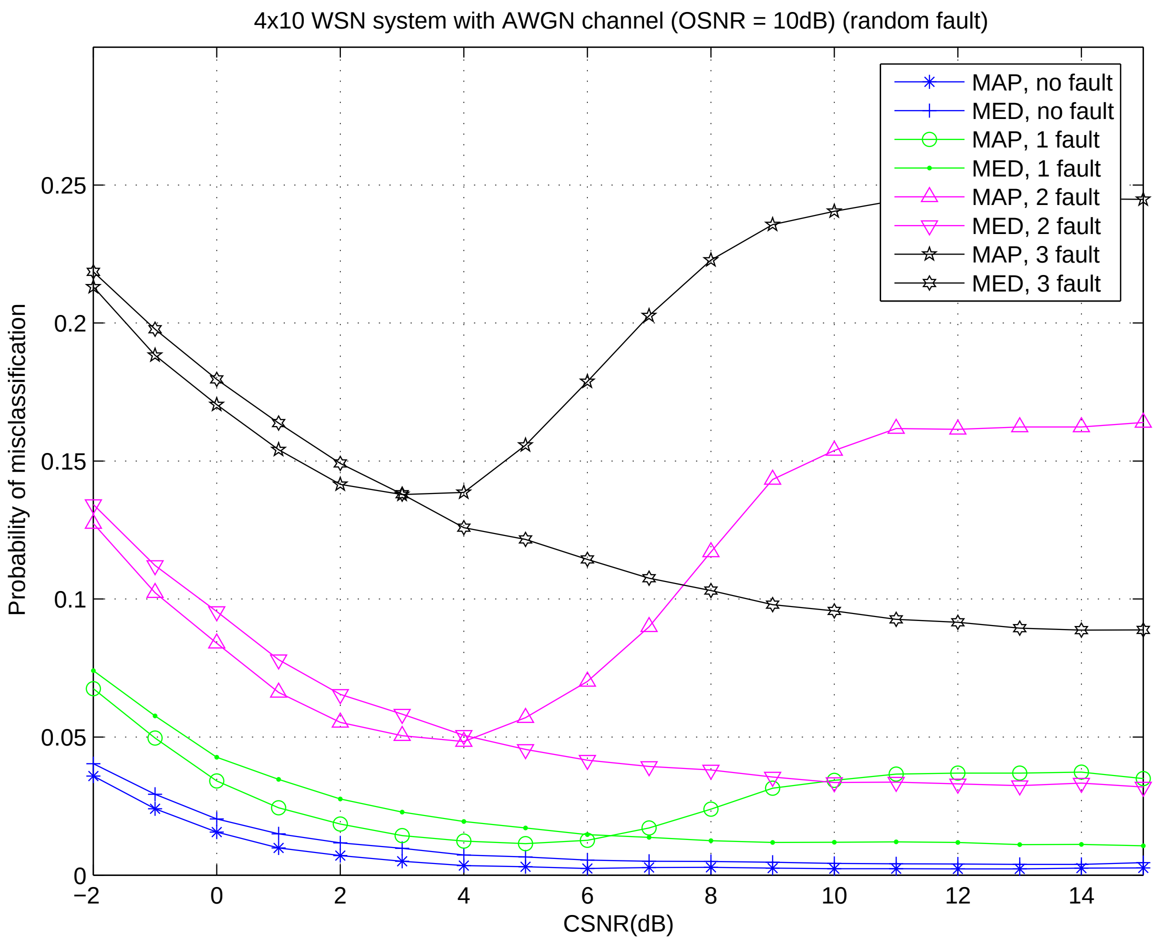


Figure 3: Performance of the MAP rule and the MED rule under AWGN channel at OSNR=10 dB for random faults when 4×10 code matrix is employed.

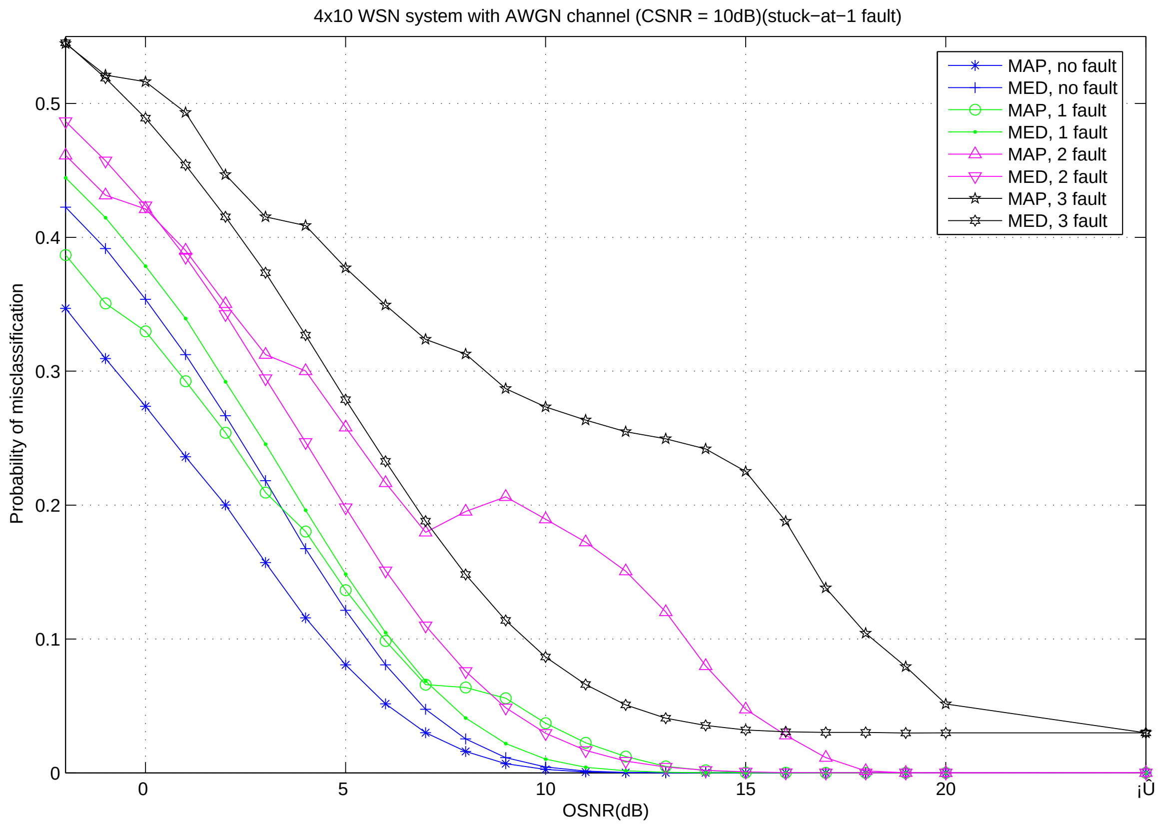


Figure 4: Performance of the MAP rule and the MED rule under AWGN channel at CSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

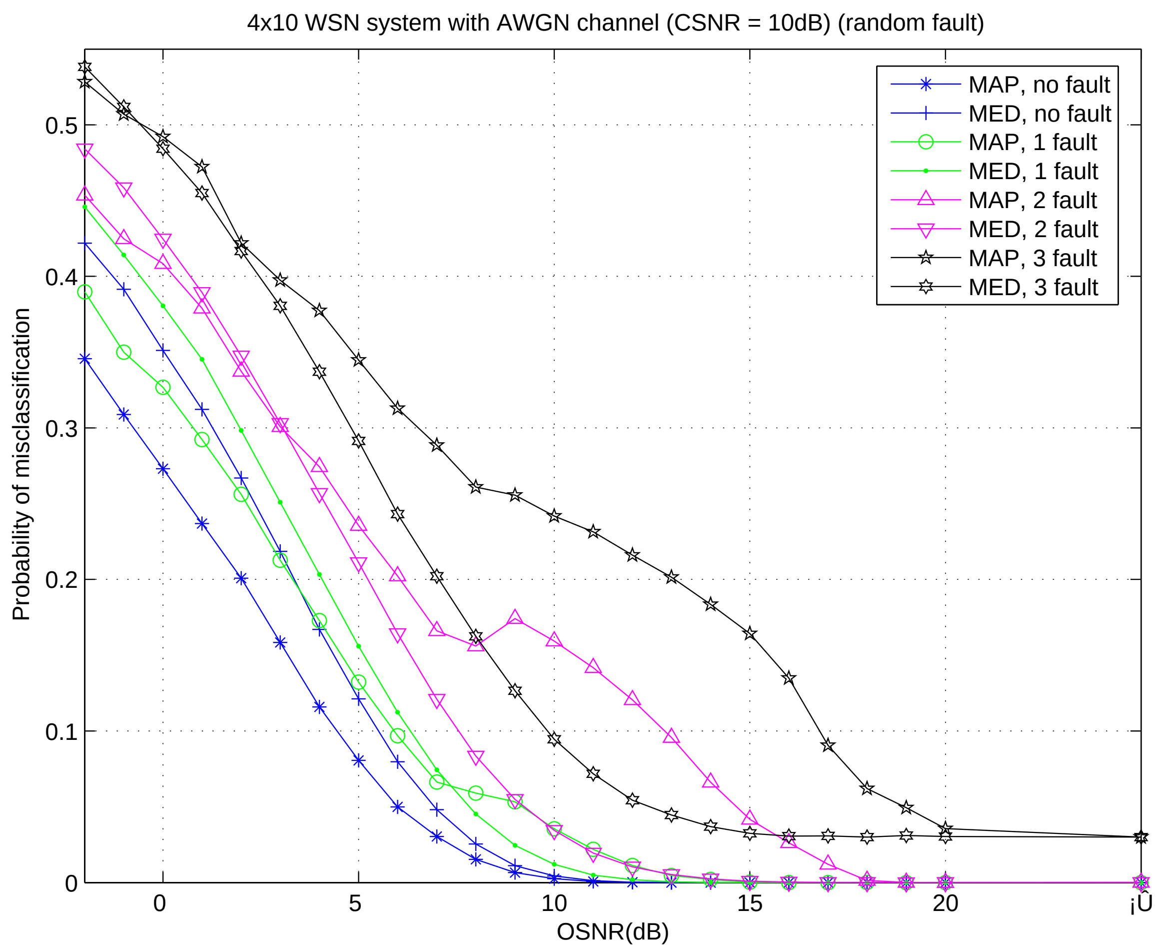


Figure 5: Performance of the MAP rule and the MED rule under AWGN channel at CSNR=10 dB for random faults when 4×10 code matrix is employed.

Spatially correlated wireless link noises

$$\mathbb{C} = \sigma^2 \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}.$$

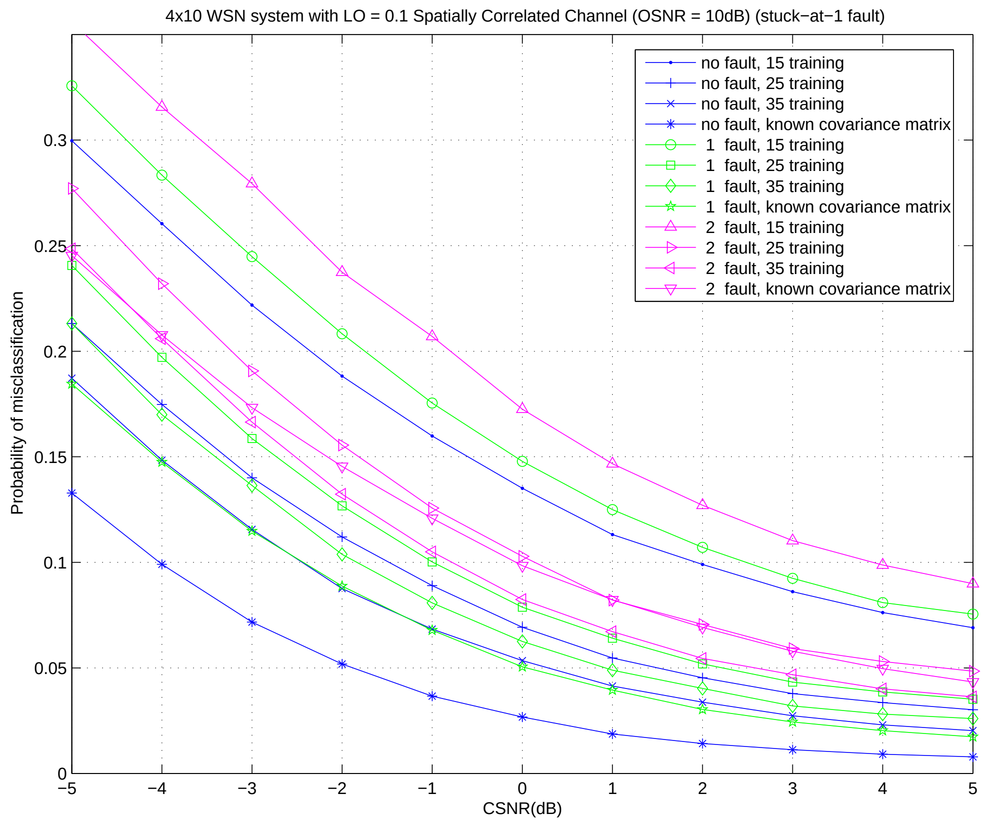


Figure 6: Performance of the MED rule under $\rho = 0.1$ spatially correlated channel at OSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

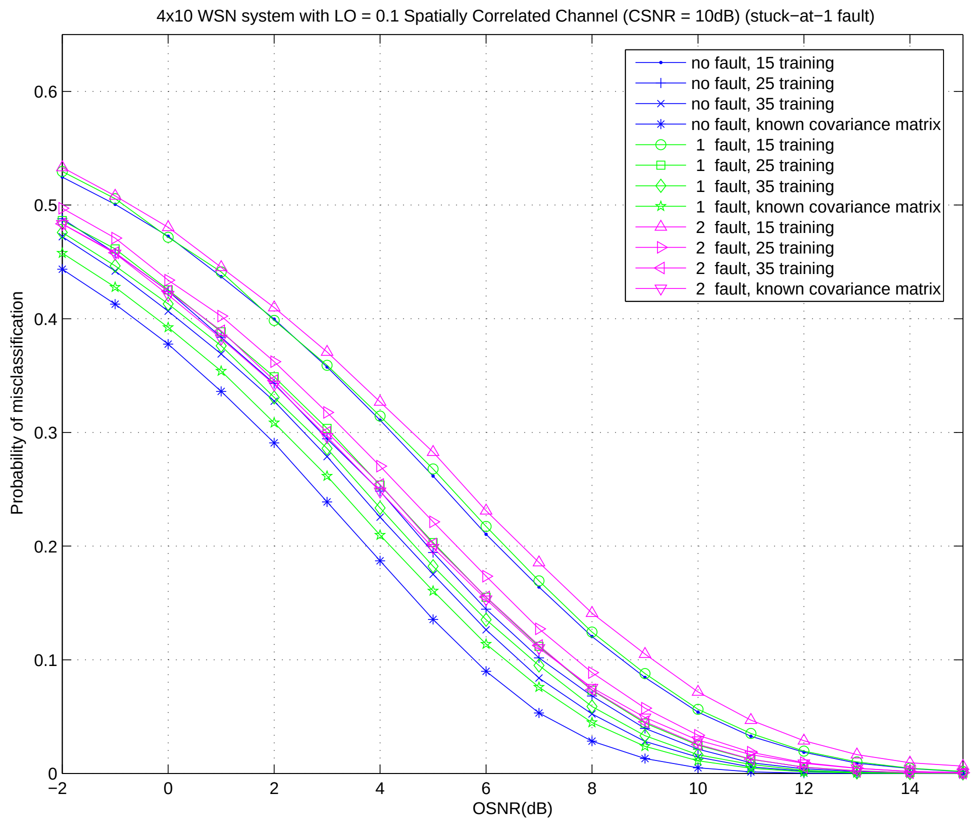


Figure 7: Performance of the MED rule under $\rho = 0.1$ spatially correlated channel at CSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

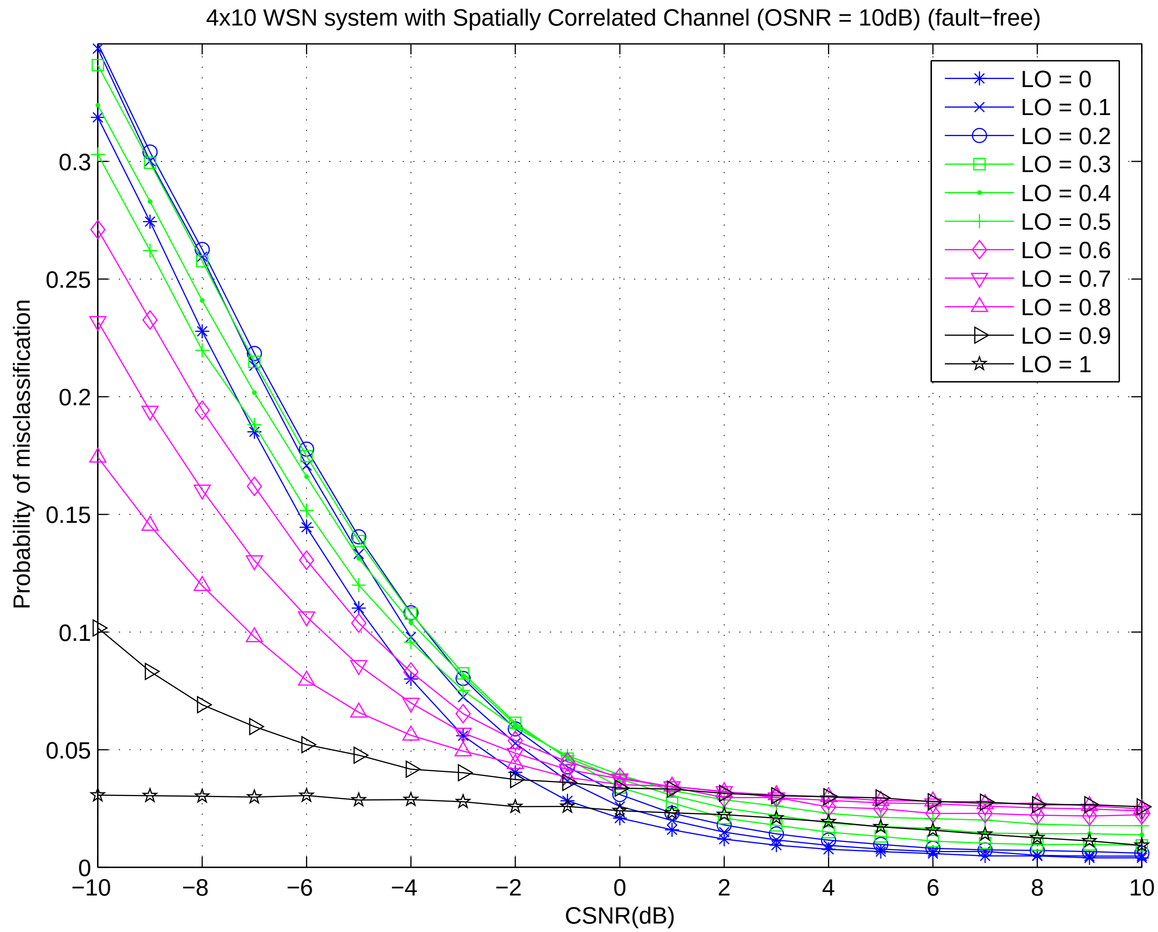


Figure 8: Performance of the MED rule with perfect channel estimation under spatially correlated channel at OSNR=10 dB in fault-free situation when 4×10 code matrix is employed.

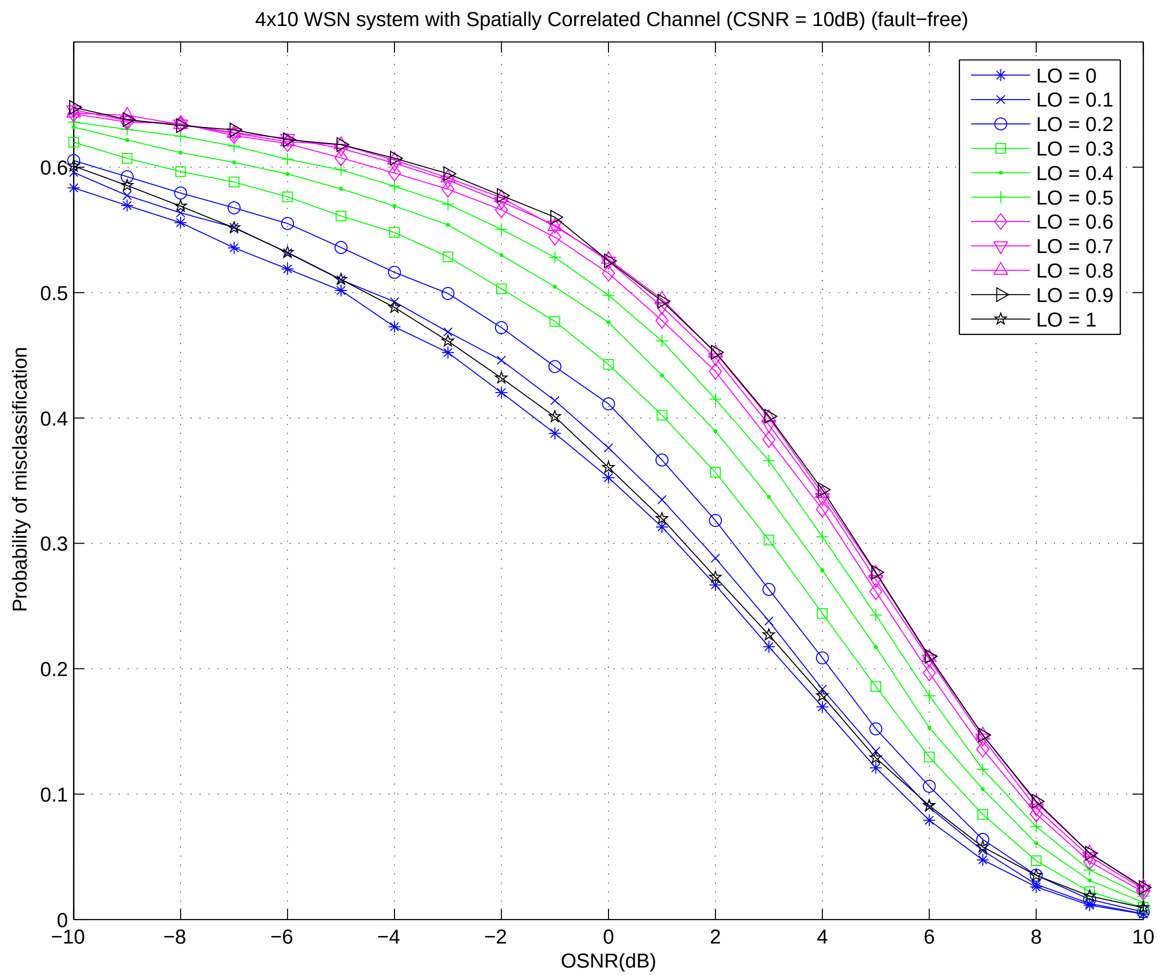


Figure 9: Performance of the MED rule with perfect channel estimation under spatially correlated channel at CSNR=10 dB in fault-free situation when 4×10 code matrix is employed.

Non-identical uncorrelated wireless link noises

$$\mathbb{C} = \sigma^2 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}.$$

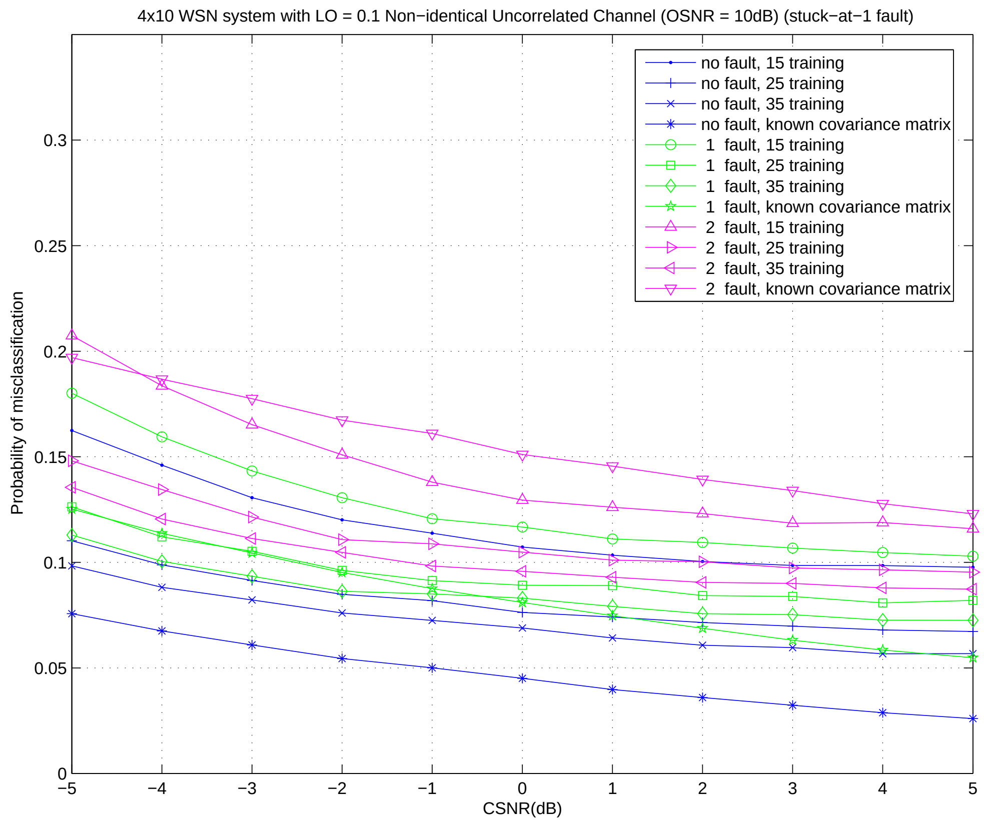


Figure 10: Performance of the MED rule under $\rho = 0.1$ non-identical uncorrelated channel at OSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

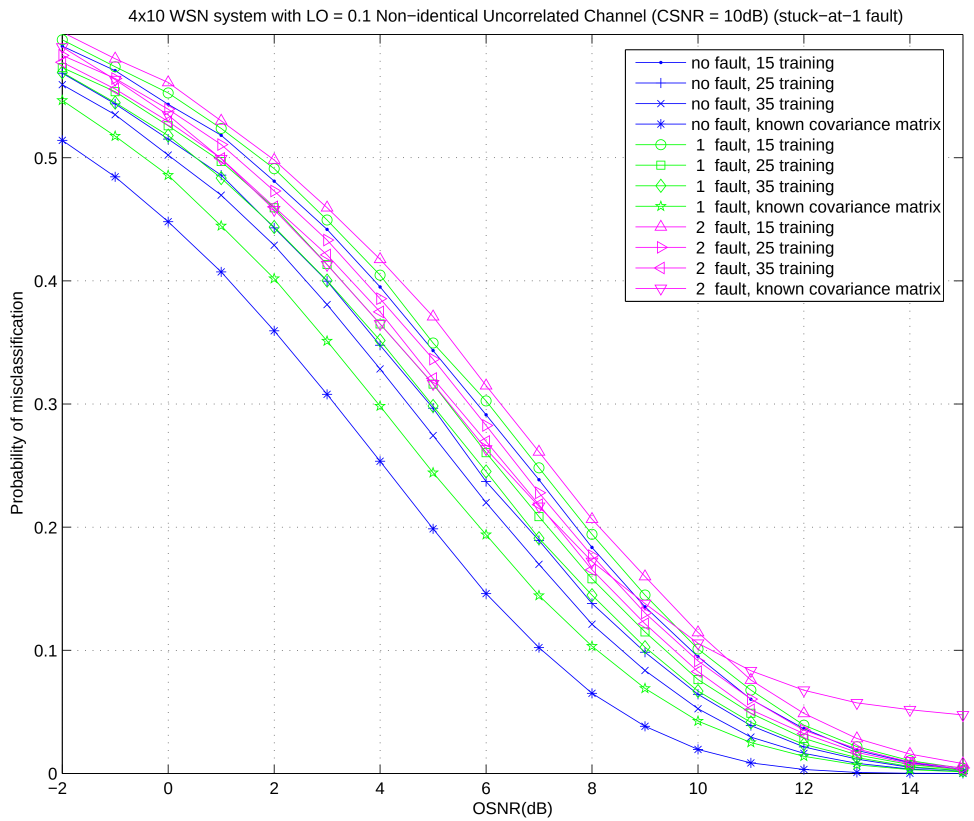


Figure 11: Performance of the MED rule under $\rho = 0.1$ non-identical uncorrelated channel at CSNR=10 dB for stuck-at-1 faults when 4×10 code matrix is employed.

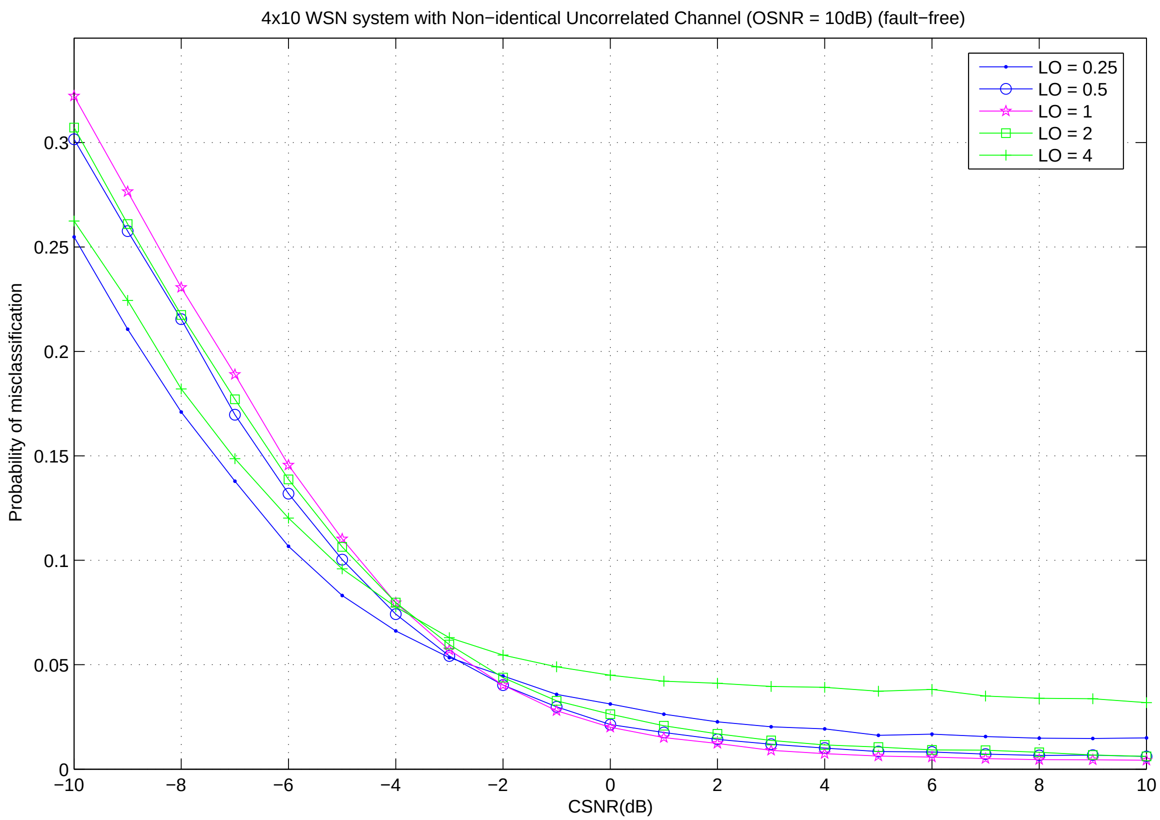


Figure 12: Performance of the MED rule with perfect channel estimation under non-identical uncorrelated channel at OSNR=10 dB in fault-free situation when 4×10 code matrix is employed.

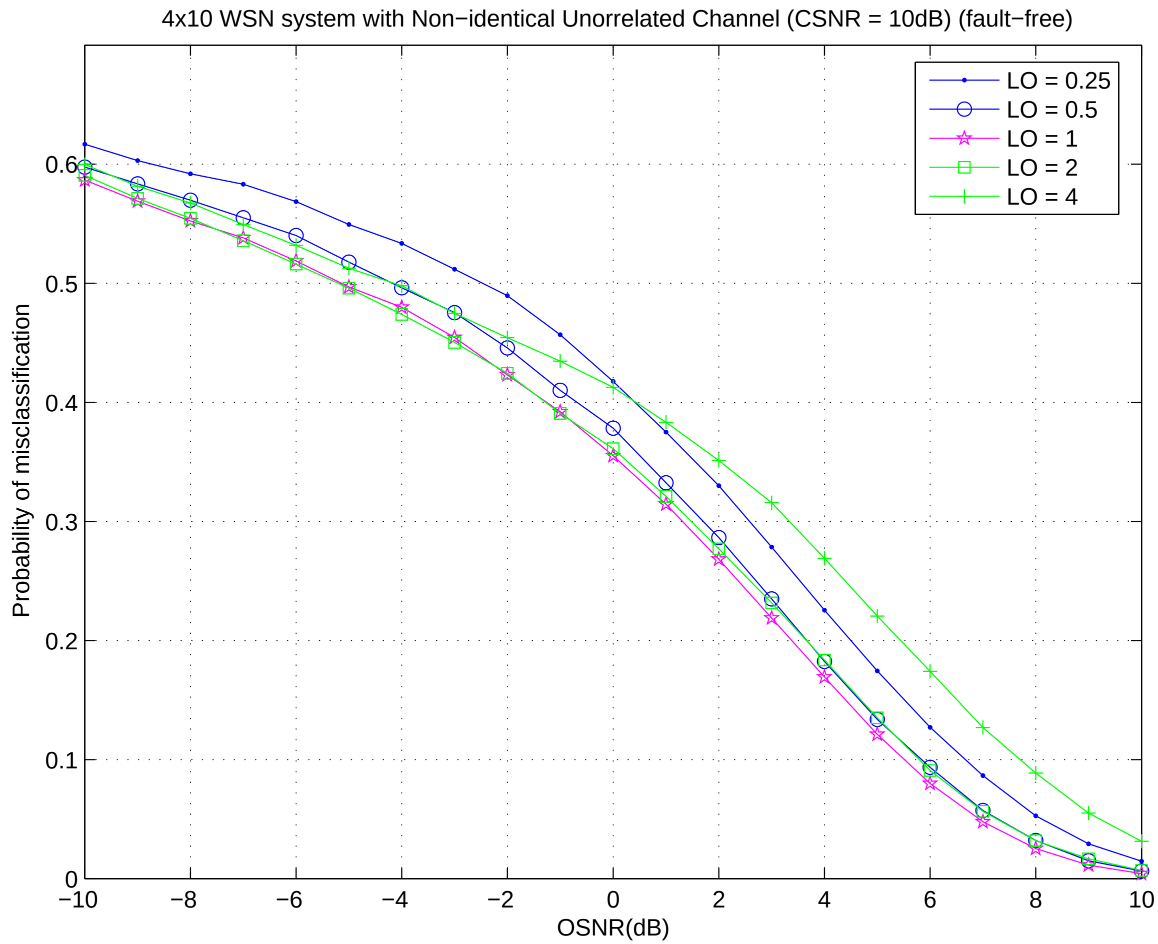


Figure 13: Performance of the MED rule with perfect channel estimation under non-identical uncorrelated channel at CSNR=10 dB in fault-free situation when 4×10 code matrix is employed.

Conclusions

- In this thesis, we propose a soft-decision fusion rule to be employed in the WSN classification system under correlated additive Gaussian link noises.
- With the help of the pre-sent training sequence, the channel covariance matrix can be maximum-likelihoodly estimated, and further optimally equalized by means of the modified MED fusion rule.
- In terms of the MED fusion rule, the code matrix search criterion can be simplified under AWGN channel assumption.
- In light of the DCFECC approach, the simulation results show that the MED fusion rule successfully provides a considerable fault-tolerance capability against both deterministic or random sensor faults.