Iterative MAP Algorithm for Gauss-Markov Channel

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Outline

- Introduction
- System model
- Iterative MAP algorithm for Gauss-Markov channel
- Simulation results for iterative MAP decoder
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Introduction

- In this work, we speculated that the received vector due to *time-varying channel* can be broken into nearly *time-independent blocks* by the insertive transmission of random bits of length no less than the channel memory.
- We further speculated that these interblock random bits can be designed to be the parity-check bits of the interleaved information bits, in which interleaving can provide the required randomness, and parity-check bits can provide additional coding information for further improvement of system performance.
- The parallel concatenated convolutional code (PCCC) is the simplest case satisfying the aforementioned system view.
- The channel model we used is time-varying channel with first-order Gauss-Markov fading.
- Thus, we experimented on the speculated system view through the transmission of PCCC code over a time-varying channel with first-order Gauss-Markov fading and its respective iterative MAP decoder.

- Simulation results hint that the iterative MAP decoder that is derived based on blockwise independence assumption not only performs close to the CSI(channel state information)-aided decoding scheme but is at most 0.9 dB away from the Shannon limit, thereby confirms the feasibility of our proposal.

System Model(1/7)

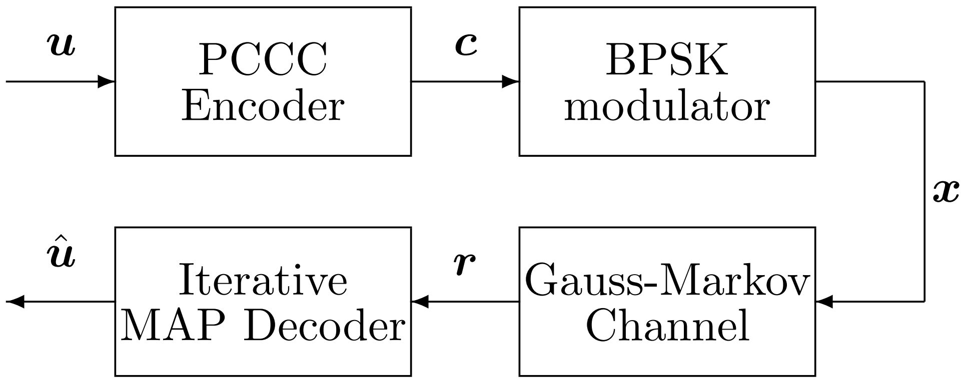

Figure 1: System model for coded transmission over Gauss-Markov channel.

System Model(2/7)

- $\boldsymbol{u} = [u_1, u_2, \cdots, u_K] \in (0,1)^K$ is the information bit sequence. Each bit is independent and identically distributed (i.i.d.) bit and with equal probable marginal.
- $\boldsymbol{c} = [c_1, c_2, \cdots, c_N] = [u_1, p_1, \bar{p}_1, u_2, p_2, \bar{p}_2, \cdots, u_K, p_K, \bar{p}_K] \in (0,1)^N$ is the coded bit sequence. Here, $\boldsymbol{p} = [p_1, p_2, \cdots, p_K]$ and $\bar{\boldsymbol{p}} = [\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K]$ are respectively the parity bit sequences generated by the first and the second component RSC encoders.
- $\boldsymbol{x} = [x_1, x_2, \cdots, x_N] \in (-1,1)^K$ is BPSK-modulated sequence and each x_j is generated according to $x_j = 2c_j - 1$.
- $\boldsymbol{r} = [r_1, r_2, \cdots, r_N]$ is the received sequence.
- $\hat{\boldsymbol{u}} = [\hat{u}_1, \hat{u}_2, \cdots, \hat{u}_K] \in (0,1)^K$ is the decoded bit sequence.

System Model(3/7)

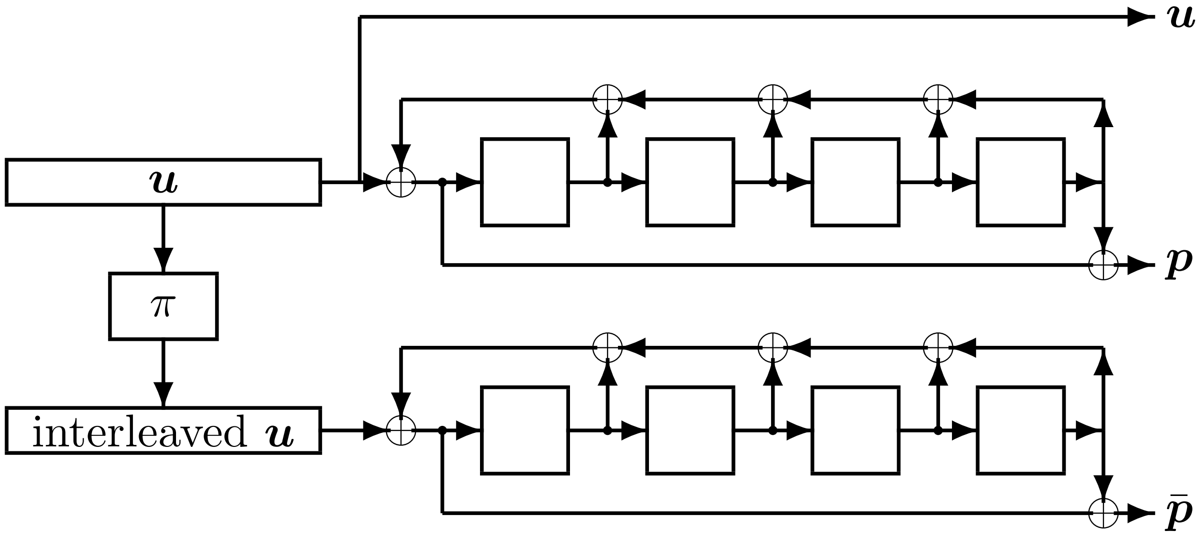

Figure 2: The PCCC encoder with $(37, 21)$ component RSC encoders.

System Model(4/7)

- At each time j, the received signal is $r_j = h_j x_j + z_j$, where $[z_1, z_2, \cdots, z_N]$ is an i.i.d. complex-valued Gaussian-distributed noise sequence with zero marginal mean and marginal variance $E[z_j z_j^*] = \sigma^2$.
- The channel coefficient h_j is Gauss-Markov distributed, satisfying that $h_j = \alpha h_{j-1} + v_j$ for complex-valued scaling constant α, complex-valued initial value h_0 and i.i.d. complex-valued Gaussian-distributed process $[v_1, v_2, \cdots, v_N]$ with $E[v_j] = 0$ and $E[v_j v_j^*] = \sigma_v^2$.

System Model(5/7)

- x_i^j denotes the portion $[x_i, \cdots, x_j]$ of sequence $\boldsymbol{x}$. Similar notations are used for r_i^j and h_i^j. Also, use $\bar{h}_i = \alpha^i h_0$ and $\bar{\sigma}_i^2 = \sigma_v^2(1-|\alpha|^{2i})/(1-|\alpha|^2)$ to represent the mean and variance of Gaussian variable h_i, respectively.
- Since h_i^j follows the Gauss-Markov distribution,

$$f\left\{h_i^j\right\} = f\{h_i\}\prod_{k=i+1}^{j} f\{h_k|h_{k-1}\},$$

and

$$f\left\{r_i^j \middle| h_i^j, x_i^j\right\} = \prod_{k=i}^{j} f\{r_k|h_k, x_k\}.$$

System Model(6/7)

- Thus,

$$
\begin{aligned}
& f\left\{r_i^j \middle| x_i^j\right\} \\
= & \int_{\mathcal{C}^{j-i+1}} f\left\{r_i^j, h_i^j \middle| x_i^j\right\} dh_i^j \\
= & \int_{\mathcal{C}^{j-i+1}} f\left\{r_i^j \middle| x_i^j, h_i^j\right\} f\left\{h_i^j\right\} dh_i^j \\
= & \frac{e^{-\frac{|\bar{h}_i|^2}{\bar{\sigma}_i^2}}}{\pi^{j-i+1}\sigma^{2(j-i+1)}\sigma_v^{2(j-i)}\bar{\sigma}_i^2}\left(\prod_{k=i}^{j} e^{-|r_k|^2/\sigma^2} G_k e^{G_k|q_k|^2}\right),
\end{aligned}
$$

where $\mathcal{C} \triangleq \Re \times \Re$ is the entire domain for complex numbers, and

$$
G_k^{-1} \triangleq \begin{cases}
\frac{1}{\sigma^2} + \frac{|\alpha|^2}{\sigma_v^2} + \frac{1}{\bar{\sigma}_k^2}, & \text{if } k = i; \\
\frac{1}{\sigma^2} + \frac{|\alpha|^2}{\sigma_v^2} + \frac{1}{\sigma_v^2} - \frac{|\alpha|^2 G_{k-1}}{\sigma_v^4}, & \text{if } i < k < j; \\
\frac{1}{\sigma^2} + \frac{1}{\sigma_v^2} - \frac{|\alpha|^2 G_{k-1}}{\sigma_v^4}, & \text{if } k = j,
\end{cases}
$$

and

$$q_k \triangleq \begin{cases} \dfrac{r_k x_k}{\sigma^2} + \dfrac{\bar{h}_k}{\bar{\sigma}_k^2}, & \text{if } k = i; \\ \dfrac{r_k x_k}{\sigma^2} + \dfrac{\alpha G_{k-1} q_{k-1}}{\sigma_v^2}, & \text{if } i < k \leq j. \end{cases}$$

Iterative MAP algorithm for Gauss-Markov channel(1/17)

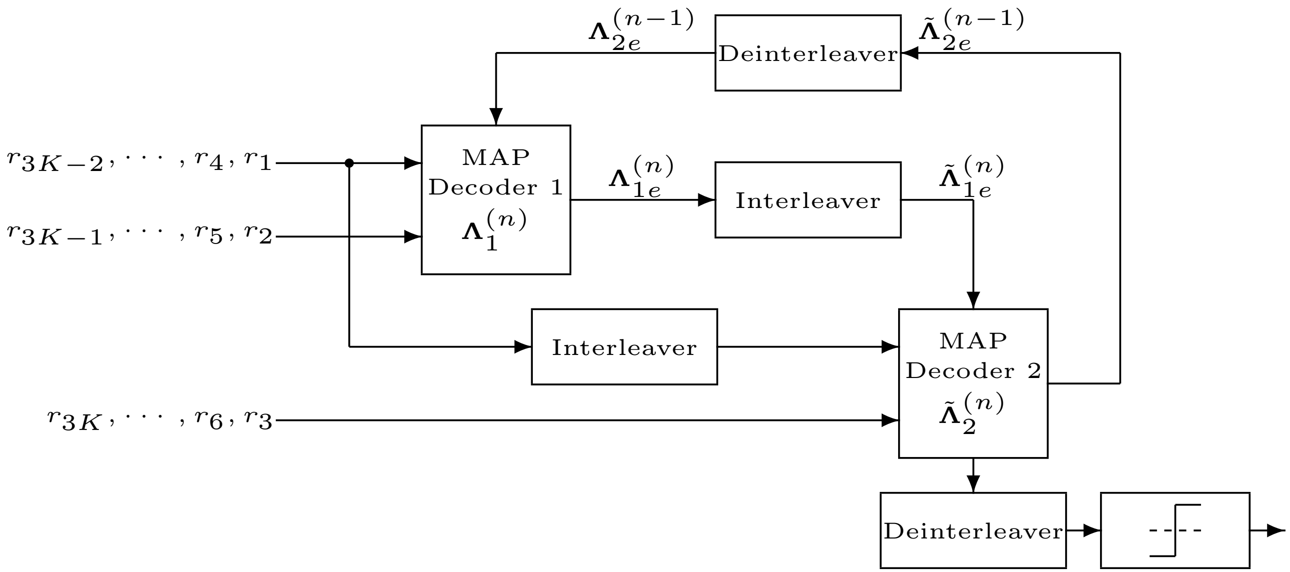

Figure 3: Block diagram of the iterative MAP decoder. A tilde over the vector represents its interleaved version.

Iterative MAP algorithm for Gauss-Markov channel(2/17)

- First component decoder
  - Based on the fact that the parity check bits $[x_3, x_6, x_9, \cdots]$ due to the interleaved $\boldsymbol{u}$ is almost bit-wisely independent of the uninterleaved $\boldsymbol{u} = [x_1, x_4, x_7, \cdots]$, as well as the parity check bit sequence $[x_2, x_5, x_8, \cdots]$ generated according to the uninterleaved $\boldsymbol{u}$, the first-order channel memory within $r_{3(i-1)+1}, r_{3(i-1)+2}, r_{3i}, r_{3i+1}, r_{3i+2}$ is somewhat weakened by the insersion of r_{3i} (due to the transmission of independent x_{3i}).
  - Thus, we assume that the received scalers r_{3i+1}^{3i+2} are independent of previous received scalers $r_{3(i-1)+1}^{3(i-1)+2}$.

Iterative MAP algorithm for Gauss-Markov channel(3/17)

- First component decoder
  - Recombine the received sequence as $\boldsymbol{d} = [d_1, d_2, \cdots, d_{2K}] = [r_1, r_2, r_4, r_5, \cdots, r_{3K-2}, r_{3K-1}]$.
  - Denote by t_s^i the node at level i with state s over a convolutional code trellis, and let $\mathcal{B}_i^{(u)}$ be the set of trellis edges such that the edge transition from node t_s^{i-1} to node $t_{\bar{s}}^i$ is due to information bit $u_i = u$.
  - Let T^i be the event of possible visited node at level i and abbreviate the event $[T^i = t_s^i]$ as T_s^i.

Iterative MAP algorithm for Gauss-Markov channel(4/17)

- First component decoder
  - The a posteriori probability (APP) of u_i upon the reception of $\boldsymbol{d}$ can be represented as:

$$\Pr\{u_i = u|\boldsymbol{d}\} = \sum_{(t_s^{i-1}, t_{\bar{s}}^i) \in \mathcal{B}_i^{(u)}} \frac{f\left\{T^{i-1} = t_s^{i-1}, T^i = t_{\bar{s}}^i, \boldsymbol{d}\right\}}{f\{\boldsymbol{d}\}},$$

and

$$\begin{aligned} f\left\{T_s^{i-1}, T_{\bar{s}}^i, \boldsymbol{d}\right\} &= f\left\{d_{2i+1}^{2K} \left| T_s^{i-1}, T_{\bar{s}}^i, d_1^{2i}\right.\right\} f\left\{T_s^{i-1}, T_{\bar{s}}^i, d_1^{2i}\right\} \\ &= f\left\{d_{2i+1}^{2K} \left| T_{\bar{s}}^i\right.\right\} f\left\{T_s^{i-1}, d_1^{2(i-1)}\right\} f\left\{T_{\bar{s}}^i, d_{2(i-1)+1}^{2i} \left| T_s^{i-1}\right.\right\} \\ &= \beta(T_{\bar{s}}^i)\alpha(T_s^{i-1})\gamma(T_s^{i-1}, T_{\bar{s}}^i), \end{aligned}$$

where $\alpha(T_s^{i-1}) \triangleq f\left\{T_s^{i-1}, d_1^{2(i-1)}\right\}$, $\beta(T_{\bar{s}}^i) \triangleq f\left\{d_{2i+1}^{2K} \left| T_{\bar{s}}^i\right.\right\}$, and

$$\gamma(T_s^{i-1}, T_{\bar{s}}^i) \triangleq f\left\{T_{\bar{s}}^i, d_{2(i-1)+1}^{2i} \left| T_s^{i-1}\right.\right\}.$$

Iterative MAP algorithm for Gauss-Markov channel(6/17)

- First component decoder
  - Function $\alpha(.)$ and $\beta(.)$ can be changed into recursive forms as:

$$\begin{aligned}
\alpha(T_s^i) &= f\left\{T_s^i, d_1^{2i}\right\} \\
&= \sum_{\bar{s}=0}^{15} f\left\{T_{\bar{s}}^{i-1}, T_s^i, d_1^{2i}\right\} \\
&= \sum_{\bar{s}=0}^{15} f\left\{T_{\bar{s}}^{i-1}, d_1^{2(i-1)}\right\} f\left\{T_s^i, d_{2(i-1)+1}^{2i} \left| T_{\bar{s}}^{i-1}\right.\right\} \\
&= \sum_{\bar{s}=0}^{15} \alpha(T_{\bar{s}}^{i-1}) \gamma(T_{\bar{s}}^{i-1}, T_s^i),
\end{aligned}$$

and

$$\begin{aligned}\beta(T_{\bar{s}}^{i}) &= f\left\{ d_{2i+1}^{2K} \middle| T_{\bar{s}}^{i} \right\} \\ &= \sum_{s=0}^{15} f\left\{ T_{s}^{i+1}, d_{2i+1}^{2K} \middle| T_{\bar{s}}^{i} \right\} \\ &= \sum_{s=0}^{15} f\left\{ d_{2(i+1)+1}^{2K} \middle| T_{s}^{i+1} \right\} f\left\{ T_{s}^{i+1} \middle| T_{\bar{s}}^{i} \right\} \\ &= \sum_{s=0}^{15} \beta(T_{s}^{i+1})\gamma(T_{\bar{s}}^{i}, T_{s}^{i+1}).\end{aligned}$$

Iterative MAP algorithm for Gauss-Markov channel(8/17)

- First component decoder
  - As derived previously, we get

$$f\left\{r_{3(i-1)+1}^{3(i-1)+2} \middle| x_{3(i-1)+1}^{3(i-1)+2}\right\}$$
$$= \frac{e^{-\frac{|\bar{h}_{3(i-1)+1}|^2}{\bar{\sigma}^2_{3(i-1)+1}}}}{\pi^2\sigma^4\sigma_v^2\bar{\sigma}^2_{3(i-1)+1}} \left(\prod_{k=3(i-1)+1}^{3(i-1)+2} e^{-|r_k|^2/\sigma^2} G_k e^{G_k|q_k|^2}\right),$$

where

$$G_k^{-1} \triangleq \begin{cases} \frac{1}{\sigma^2} + \frac{|\alpha|^2}{\sigma_v^2} + \frac{1}{\bar{\sigma}_k^2}, & \text{if } k = 3(i-1)+1; \\ \frac{1}{\sigma^2} + \frac{1}{\sigma_v^2} - \frac{|\alpha|^2 G_{k-1}}{\sigma_v^4}, & \text{if } k = 3(i-1)+2, \end{cases}$$

and

$$q_k \triangleq \begin{cases} \dfrac{r_k x_k}{\sigma^2} + \dfrac{\bar{h}_k}{\bar{\sigma}_k^2}, & \text{if } k = 3(i-1)+1; \\ \dfrac{r_k x_k}{\sigma^2} + \dfrac{\alpha G_{k-1} q_{k-1}}{\sigma_v^2}, & \text{if } k = 3(i-1)+2. \end{cases}$$

– Hence, function $\gamma(.)$ (with proper scaling without affecting the log-likelihood ratio) can be represented as

$$\begin{aligned} \gamma(T_s^{i-1}, T_{\bar{s}}^i) &= f\left\{ T_{\bar{s}}^i, d_{2(i-1)+1}^{2i} \middle| T_s^{i-1} \right\} \\ &= f\left\{ d_{2(i-1)+1}^{2i} \middle| T_s^{i-1}, T_{\bar{s}}^i \right\} \Pr\left\{ T_{\bar{s}}^i \middle| T_s^{i-1} \right\} \\ &= f\left\{ r_{3(i-1)+1}^{3(i-1)+2} \middle| x_{3(i-1)+1}^{3(i-1)+2} \right\} \Pr\left\{ T_{\bar{s}}^i \middle| T_s^{i-1} \right\} \\ &= \begin{cases} \Pr\{u_i = 0\} \prod_{k=1}^{2} e^{G_{3(i-1)+k} \left| q_{3(i-1)+k} \right|^2}, \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \in \mathcal{B}_i^{(0)}; \\ \Pr\{u_i = 1\} \prod_{k=1}^{2} e^{G_{3(i-1)+k} \left| q_{3(i-1)+k} \right|^2}, \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \in \mathcal{B}_i^{(1)}; \\ 0, \qquad \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \notin \mathcal{B}_i^{(0)} \cup \mathcal{B}_i^{(1)}. \end{cases} \end{aligned}$$

Iterative MAP algorithm for Gauss-Markov channel(10/17)

- Second component decoder
  - Assume the channel memory has been neutralized by the interleaver.
  - Let $\ell(\cdot)$ be the index of interleaved $\boldsymbol{u}$.
  - Then, we obtain

$$
\begin{aligned}
& f\left\{r_{3(l(i)-1)+1}, r_{3(i-1)+3} \,\middle|\, x_{3(l(i)-1)+1}, x_{3(i-1)+3}\right\} \\
= \; & f\left\{r_{3(l(i)-1)+1} \,\middle|\, x_{3(l(i)-1)+1}\right\} f\left\{r_{3(i-1)+3} \,\middle|\, x_{3(i-1)+3}\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
f\{r_k \,|\, x_k\} &= \frac{1}{\pi(\sigma^2+\bar{\sigma}_k^2)} e^{-\frac{|r_k - x_k \bar{h}_k|^2}{\sigma^2+\bar{\sigma}_k^2}} \\
&= \frac{e^{-|\bar{h}_k|^2/\bar{\sigma}_k^2}}{\pi\sigma^2\bar{\sigma}_k^2} e^{-|r_k|^2/\sigma^2} \bar{G}_k e^{\bar{G}_k |\bar{q}_k|^2},
\end{aligned}
$$

and

$$
\bar{G}_k^{-1} \triangleq \frac{1}{\sigma^2} + \frac{1}{\bar{\sigma}_k^2} \quad \text{and} \quad \bar{q}_k \triangleq \frac{r_k x_k}{\sigma^2} + \frac{\bar{h}_k}{\bar{\sigma}_k^2}.
$$

– Thus, function $\gamma(.)$ (with proper scaling without affecting the log-likelihood ratio) in the second component decoder is simplified to:

$$\gamma(T_s^{i-1}, T_{\bar{s}}^i) = \begin{cases} \Pr\left\{u_{\ell(i)} = 0\right\} e^{\bar{G}_{3(\ell(i)-1)+1}|\bar{q}_{3(\ell(i)-1)+1}|^2} e^{\bar{G}_{3i}|\bar{q}_{3i}|^2}, \\ \hfill \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \in \mathcal{B}_i^{(0)}; \\ \Pr\left\{u_{\ell(i)} = 1\right\} e^{\bar{G}_{3(\ell(i)-1)+1}|\bar{q}_{3(\ell(i)-1)+1}|^2} e^{\bar{G}_{3i}|\bar{q}_{3i}|^2}, \\ \hfill \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \in \mathcal{B}_i^{(1)}; \\ 0, \hfill \text{if } (T_s^{i-1}, T_{\bar{s}}^i) \notin \mathcal{B}_i^{(0)} \cup \mathcal{B}_i^{(1)}. \end{cases}$$

Iterative MAP algorithm for Gauss-Markov channel(12/17)

- Iterative MAP decoder
  - For the first component MAP decoder, the log-likelihood ratio is

$$\begin{aligned}\Lambda_1^{(n)}(i) &= \Lambda_{2e}^{(n-1)}(i) + \Lambda_{1e}^{(n)}(i) \\ &+ \frac{4G_{3(i-1)+1}}{\sigma^2 \bar{\sigma}^2_{3(i-1)+1}} \mathrm{Re}\left\{ r_{3(i-1)+1} \bar{h}^*_{3(i-1)+1} \right\},\end{aligned}$$

where $\Lambda_{2e}^{(n-1)}(i) = \log[\Pr\{u_i = 1\}/\Pr\{u_i = 0\}]$ is the *a priori probability estimate* from the previous stage $(n-1)$, and the extrinsic information that is used to improve the a priori probability estimate for the next decoding stage n is

$$
\begin{aligned}
\Lambda_{1e}^{(n)}(i) &\triangleq \log \frac{\sum\limits_{(T_s^{i-1},T_{\bar{s}}^{i})\in\mathcal{B}_i^{(1)}} \alpha(T_s^{i-1})\beta(T_{\bar{s}}^{i})e^{G_{3(i-1)+2}\left|q_{3(i-1)+2}\right|^2}}{\sum\limits_{(T_s^{i-1},T_{\bar{s}}^{i})\in\mathcal{B}_i^{(0)}} \alpha(T_s^{i-1})\beta(T_{\bar{s}}^{i})e^{G_{3(i-1)+2}\left|q_{3(i-1)+2}\right|^2}} \\
&= \Lambda_1^{(n)}(i) - \Lambda_{2e}^{(n-1)}(i) \\
&\quad -\frac{4G_{3(i-1)+1}}{\sigma^2\bar{\sigma}_{3(i-1)+1}^2}\mathrm{Re}\left\{r_{3(i-1)+1}\bar{h}_{3(i-1)+1}^{*}\right\}.
\end{aligned}
$$

– For the second component MAP decoder, the log-likelihood ratio is

$$\begin{aligned}\Lambda_2^{(n)}(\ell(i)) &= \Lambda_{1e}^{(n)}(\ell(i)) + \Lambda_{2e}^{(n)}(\ell(i)) \\ &+ \frac{4\bar{G}_{3(\ell(i)-1)+1}}{\sigma^2\bar{\sigma}^2_{3(\ell(i)-1)+1}}\mathrm{Re}\left\{r_{3(\ell(i)-1)+1}\bar{h}^*_{3(\ell(i)-1)+1}\right\},\end{aligned}$$

and the extrinsic information is

$$\begin{aligned}\Lambda_{2e}^{(n)}(\ell(i)) &\triangleq \log\frac{\sum_{(T_s^{i-1},T_{\bar{s}}^i)\in\mathcal{B}_i^{(1)}}\alpha(T_s^{i-1})\beta(T_{\bar{s}}^i)e^{\bar{G}_{3i}|\bar{q}_{3i}|^2}}{\sum_{(T_s^{i-1},T_{\bar{s}}^i)\in\mathcal{B}_i^{(0)}}\alpha(T_s^{i-1})\beta(T_{\bar{s}}^i)e^{\bar{G}_{3i}|\bar{q}_{3i}|^2}} \\ &= \Lambda_2^{(n)}(\ell(i)) - \Lambda_{1e}^{(n)}(\ell(i)) \\ &\quad - \frac{4\bar{G}_{3(\ell(i)-1)+1}}{\sigma^2\bar{\sigma}^2_{3(\ell(i)-1)+1}}\mathrm{Re}\left\{r_{3(\ell(i)-1)+1}\bar{h}^*_{3(\ell(i)-1)+1}\right\}.\end{aligned}$$

Iterative MAP algorithm for Gauss-Markov channel(15/17)

- Summary of iterative MAP decoder:

  Step 1: Set $\Lambda_{2e}^{(0)} = 0$, and set $n = 1$.

  Step 2: Calculate $\mathbf{\Lambda}_1^{(n)}$ and $\mathbf{\Lambda}_{1e}^{(n)}$

  1. *Initialization:*
     - For $i = 1, \ldots, K$, $\Pr\{u_i = 0\} = 1/(1 + e^{\Lambda_{2e}^{(n-1)}(i)})$ and $\Pr\{u_i = 1\} = 1 - \Pr\{u_i = 0\}$.
     - For $i = 1, \ldots, K$, $s = 0, \ldots, 15$ and $\bar{s} = 0, \ldots, 15$, derive $\gamma(T_s^{i-1}, T_{\bar{s}}^i)$.
  2. *Forward recursion:*
     - Set $\alpha(T_0^0) = 1$ and for $s = 1, \ldots, 15$, $\alpha(T_s^0) = 0$.
     - For $i = 1, \ldots, K$ and $s = 0, \ldots, 15$, perform $\alpha(T_s^i) = \sum_{\bar{s}=0}^{15} \alpha(T_{\bar{s}}^{i-1})\gamma(T_{\bar{s}}^{i-1}, T_s^i)$.
  3. *Backward recursion:*
     - Set $\beta(T_s^{K+1}) = \alpha(T_s^K)$ for $s = 0, \ldots, 15$.
     - For $i = K, \ldots, 1$ and $\bar{s} = 0, \ldots, 15$, perform $\beta(T_{\bar{s}}^i) = \sum_{s=0}^{15} \beta(T_s^{i+1})\gamma(T_{\bar{s}}^i, T_s^{i+1})$.

4. *Soft output:*
   - For $i = 1, \ldots, K$, update $\Lambda_1^{(n)}(i)$ and $\Lambda_{1e}^{(n)}(i)$.

Step 3: Calculate $\mathbf{\Lambda}_2^{(n)}$ and $\mathbf{\Lambda}_{2e}^{(n)}$.

1. *Initialization:*
   - For $i = 1, \ldots, K$, $\Pr\{u_i = 0\} = 1/(1 + e^{\Lambda_{1e}^{(n)}(i)})$ and $\Pr\{u_i = 1\} = 1 - \Pr\{u_i = 0\}$.
   - For $i = 1, \ldots, K$, $s = 0, \ldots, 15$ and $\bar{s} = 0, \ldots, 15$, derive $\gamma(T_s^{i-1}, T_{\bar{s}}^i)$.
2. *Forward recursion:*
   - Set $\alpha(T_0^0) = 1$ and for $s = 1, \ldots, 15$, $\alpha(T_s^0) = 0$.
   - For $i = 1, \ldots, K$ and $s = 0, \ldots, 15$, perform $\alpha(T_s^i) = \sum_{\bar{s}=0}^{15} \alpha(T_{\bar{s}}^{i-1})\gamma(T_{\bar{s}}^{i-1}, T_s^i)$.
3. *Backward recursion:*
   - Set $\beta(T_s^{K+1}) = \alpha(T_s^K)$ for $s = 0, \ldots, 15$.
   - For $i = K, \ldots, 1$ and $\bar{s} = 0, \ldots, 15$, perform $\beta(T_{\bar{s}}^i) = \sum_{s=0}^{15} \beta(T_s^{i+1})\gamma(T_{\bar{s}}^i, T_s^{i+1})$.
4. *Soft output:*
   - For $i = 1, \ldots, K$, update $\Lambda_2^{(n)}(\ell(i))$ and $\Lambda_{2e}^{(n)}(\ell(i))$.

Step 4: Repeat **Step 2** and **Step 3** (by setting $n = n + 1$) until the number of desired iterations is reached, and make final hard-decision based on the last $\mathbf{\Lambda_2}$.

Simulation results for iterative MAP decoder(1/6)

- Parameters setting:
  - The Berrou-Glavieux interleaver with size 256×256 is employed.
  - Length of information bit sequence $K = 65536$ and Length of coded bit sequence $N = 3 \times 65536$.
  - Same as [Chen *et al.* 2000], set $\alpha = 0.995$, $h_0 = (0.5 \text{ or } 1)$ and $\sigma_v^2 = (0.001 \text{ or } 0.01)$.
  - Reset the channel fading every 99 symbols; as a result,

$$\bar{h}_i = \alpha^{[(i-1) \bmod 99]+1} h_0$$

and

$$\bar{\sigma}_i^2 = \sigma_v^2 \frac{(1 - |\alpha|^{2([(i-1) \bmod 99]+1)})}{(1 - |\alpha|^2)}.$$

Simulation results for iterative MAP decoder(2/6)

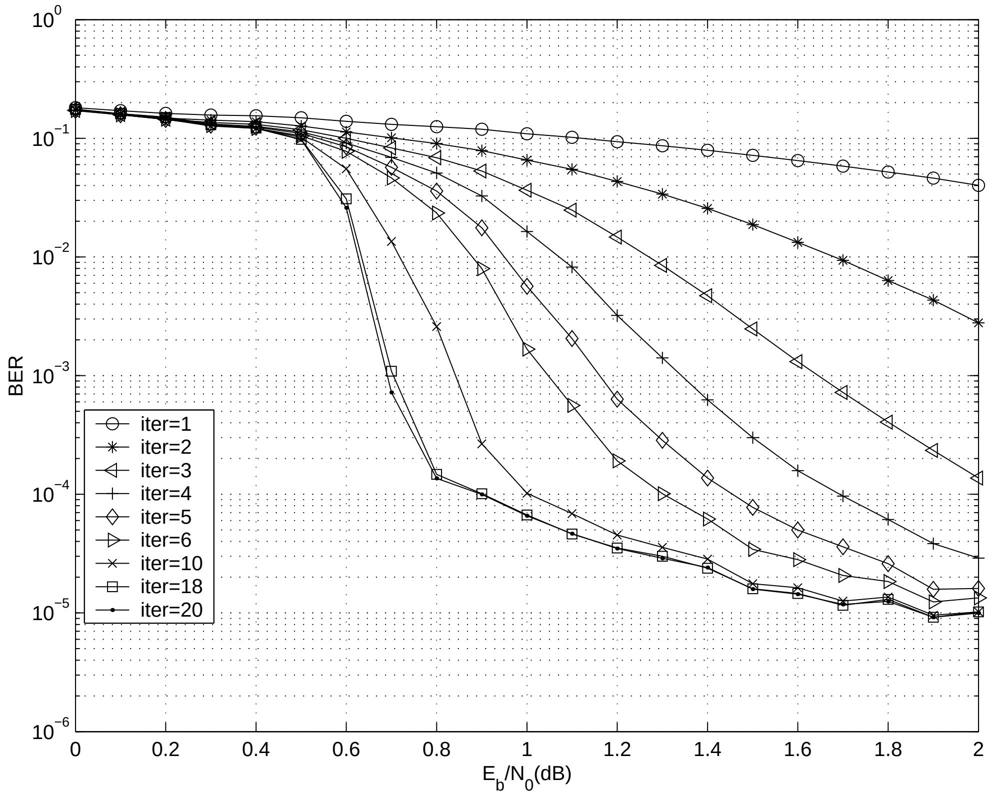

Figure 4: Performance curves of the proposed iterative MAP decoder. Parameters of Gauss-Markov channel are $\alpha = 0.995$, $\sigma_v^2 = 0.001$ and $h_0 = 1$.

Simulation results for iterative MAP decoder(3/6)

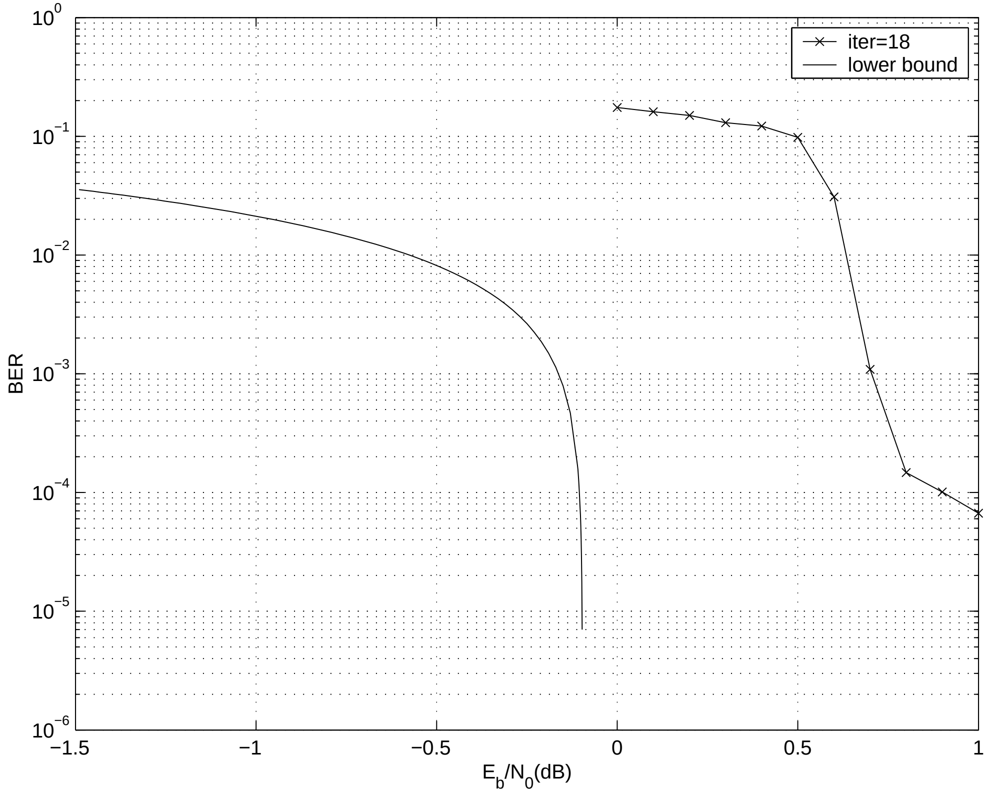

Figure 5: Performance comparison between the iterative MAP decoder with 18 iterations and a lower bound of the Shannon limit. Parameters of Gauss-Markov channel are $\alpha = 0.995$, $\sigma_v^2 = 0.001$ and $h_0 = 1$.

Simulation results for iterative MAP decoder(4/6)

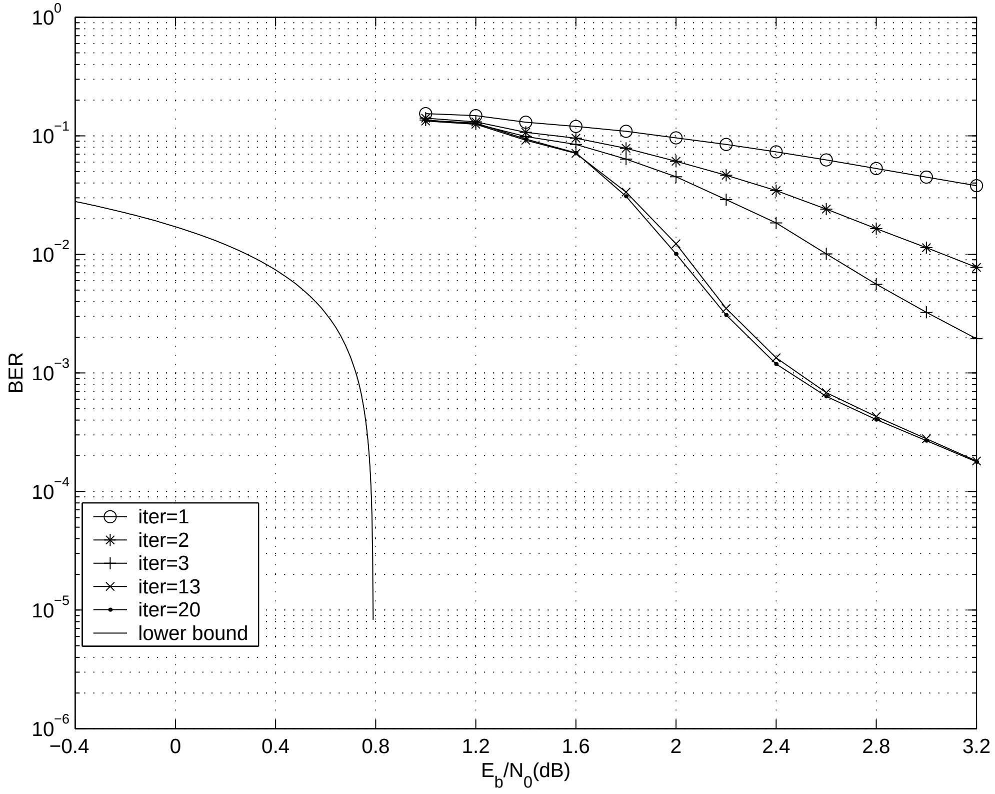

Figure 6: Performance comparison between the iterative MAP decoding and a lower bound of the Shannon limit. Parameters of Gauss-Markov channel are $\alpha = 0.995$, $\sigma_v^2 = 0.001$ and $h_0 = 0.5$.

Simulation results for iterative MAP decoder(5/6)

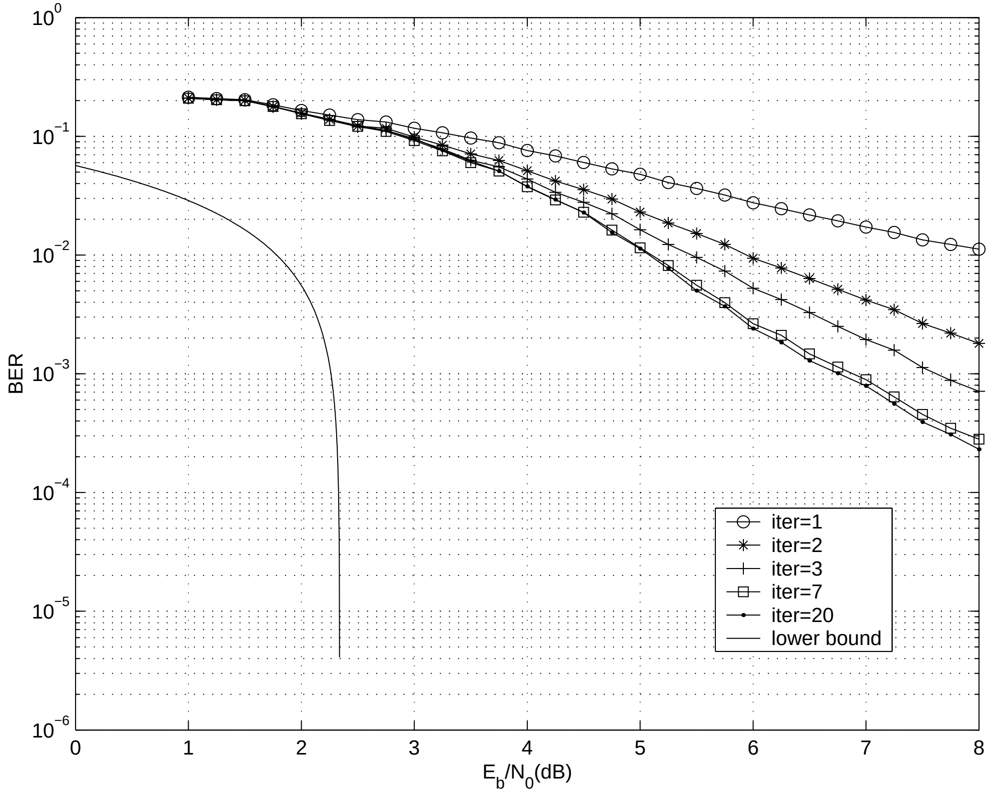

Figure 7: Performance comparison between the iterative MAP decoding and a lower bound of the Shannon limit. Parameters of Gauss-Markov channel are $\alpha = 0.995$, $\sigma_v^2 = 0.01$ and $h_0 = 1$.

Simulation results for iterative MAP decoder(6/6)

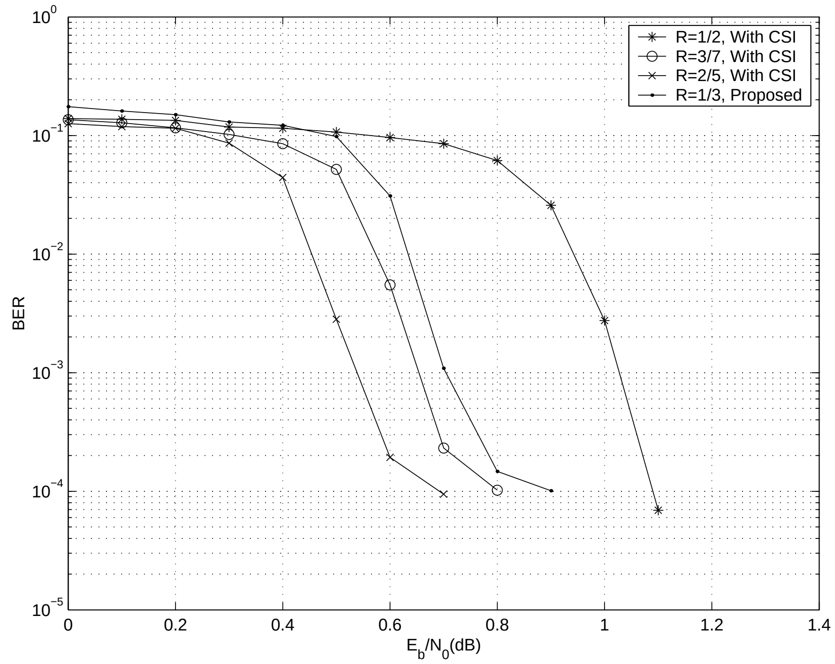

Figure 8: Performances of punctured PCCC codes with code rates 1/2, 3/7 and 2/5. The CSIs are assumed known for the iterative MAP decoder of these punctured code. For comparison, the performance of the proposed blind-CSI iterative MAP algorithm is also depicted. All of them are decoded with 18 iterations. Parameters of Gauss-Markov channel are $\alpha = 0.995$, $\sigma_v^2 = 0.001$ and $h_0 = 1$.

Conclusion

- In PCCC coding system, the parity check bits generated due to interleaved information bits can be treated "independent" of the parity check bits generated due to information bits and information bits themselves.
- We take the PCCC code and its respective iterative MAP decoder as a test vehicle to experiment on the idea that the temporal channel memory can be weakened to nearly blockwise time-independence by the insertive transmission of "random bits" of sufficient length between two consecutive blocks.
- The simulation results show that the metrics derived based on *completely blockwise independence* with 2-bit blocks periodically separated by single parity-check bit from the second component RSC encoder perform close to the CSI-aided decoding scheme, and is at most 0.9 dB away from the Shannon limit at BER $= 2 \times 10^{-4}$ when $h_0 = 1$ and $\sigma_v^2 = 0.001$.