QPSK-modulated Code Design for Combined Channel Estimation and Error Correction on a Frequency-selective Fading Channel

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  - code searching algorithm
- Impacts of Amplitude Distortions and Phase Distortions
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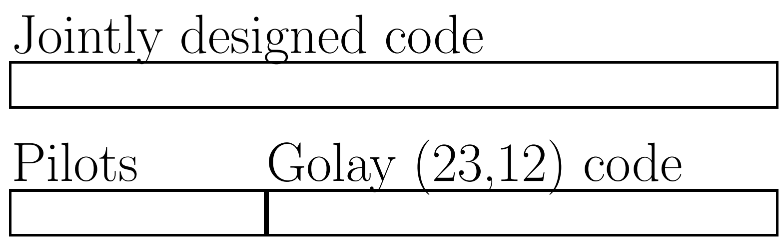
- Traditionally, in a communication system over a fading environment, **channel estimation**, **channel equalization** and **error correction** are carried out at the receiver in sequence.
- – In 2002, Skoglund *et al.*[1] proposed **combining channel estimation and equalization with the error correction**.
  - He found the optimal non-linear binary code for channels with unknown parameters by computer search.
  - Simulations showed that the computer searched code outperforms the **Golay (23,12) code** extended with one known pilot by **1.3 dB** at WER $= 10^{-2}$.
- In order to **improve the transmission rate**, we attempt to design the **QPSK modulated codes**, rather than the BPSK modulated codes.

[1]M. Skoglund, J. Giese and S. Parkvall, "Code design for combined channel estimation and error protection," *IEEE Trans. Inform. Theory*, vol. 48, no. 5, pp. 1162-1171, May 2002.

- As an emphasization, we do not send the pilots or training sequence.
- **All** the transmission energies are used to transmit the "information-bearing bits."



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Block Fading Channel Model

A single-input-single-output (SISO) time-discrete system can be modelled as

$$\boldsymbol{y} = \mathbb{B}\boldsymbol{h} + \boldsymbol{n}, \tag{1}$$

where

$$\mathbb{B} \triangleq \begin{bmatrix} b_1 & 0 & \cdots & 0 \\ \vdots & b_1 & \ddots & \vdots \\ b_N & \vdots & \ddots & 0 \\ 0 & b_N & \ddots & b_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & b_N \end{bmatrix}_{L\times P} .$$

- $\boldsymbol{b} = [b_1, \ldots, b_N]^T$ is the transmitted codeword, and $b_i \in \{\pm 1, \pm\imath\}$, where $1 \le i \le N$.
- $\mathbb{B}$ emulates the convolution operation with channel coefficient $\boldsymbol{h}$.
- $\boldsymbol{h}$ is the $P \times 1$ channel coefficient vector and remains constant within a coding block with length $L = N + P - 1$.
- $\boldsymbol{n}$ is the $L \times 1$ zero-mean complex Gaussian noise vector with covariance matrix $\sigma_n^2 \mathbb{I}_L$.

Since $\boldsymbol{h}$ is unknown and we assume the transmitted codeword matrix $\mathbb{B}$ is equal likely, the joint maximum-likelihood (JML) detection becomes the best option:

$$(\hat{\mathbb{B}}, \hat{\boldsymbol{h}}) = \arg\max_{\mathbb{B}} \max_{\boldsymbol{h}} \Pr(\boldsymbol{y}|\mathbb{B}, \boldsymbol{h}).$$

For a given $\mathbb{B}$, the channel coefficient $\boldsymbol{h}$ that maximizes $\Pr(\boldsymbol{y}|\mathbb{B}, \boldsymbol{h})$ can be pre-determined, i.e.,

$$\hat{\boldsymbol{h}} = (\mathbb{B}^H\mathbb{B})^{-1}\mathbb{B}^H\boldsymbol{y}. \tag{2}$$

Then, the JML estimate of the transmitted codeword is

$$\begin{aligned} \hat{\boldsymbol{b}} &= \arg\max_{\boldsymbol{b}} \Pr(\boldsymbol{y}|\mathbb{B}, \hat{\boldsymbol{h}}) \\ &= \arg\min_{\boldsymbol{b}} \|\boldsymbol{y} - \mathbb{B}(\mathbb{B}^H\mathbb{B})^{-1}\mathbb{B}^H\boldsymbol{y}\|^2 \\ &= \arg\min_{\boldsymbol{b}} \|\boldsymbol{y} - \mathbb{P}_B\boldsymbol{y}\|^2, \end{aligned} \tag{3}$$

where $\mathbb{P}_B \triangleq \mathbb{B}(\mathbb{B}^H\mathbb{B})^{-1}\mathbb{B}^H$. Noted that $\mathbb{B}$ and $\mathbb{P}_B$ are not one to one correspondence unless **the first bit b_1 is fixed** as, for example, -1.

Code Design Criterion

- The (N, K) code $\mathcal{C}$ design criterion is to **minimize the union bound of the average block error probability** .
- The average block error probability P_e is given as

$$P_e = 2^{-K} \sum_{i \in \mathcal{J}} \Pr\left(\hat{\boldsymbol{b}} \neq \boldsymbol{b}(i) \middle| \boldsymbol{b}(i) \text{ transmitted}\right), \tag{4}$$

where $\mathcal{J}$ is the set of indices of codewords in $\mathcal{C}$.

- The union bound of P_e is given as

$$P_e \leq 2^{-K} \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}, j \neq i} p_{j|i}, \tag{5}$$

where $p_{j|i}$ is the pairwise error probability (PEP).

- PEP is defined by

$$p_{j|i} \triangleq \Pr\left(\hat{\boldsymbol{b}} = \boldsymbol{b}(j) \middle| \boldsymbol{b}(i) \text{ transmitted}\right). \tag{6}$$

Pairwise Error Probability

- According to (3),

$$\begin{aligned} p_{j|i} &= \Pr\left(\|\boldsymbol{y}(i) - \mathbb{P}_B(j)\boldsymbol{y}(i)\|^2 < \|\boldsymbol{y}(i) - \mathbb{P}_B(i)\boldsymbol{y}(i)\|^2\right) \\ &\quad + \frac{1}{2}\Pr\left(\|\boldsymbol{y}(i) - \mathbb{P}_B(j)\boldsymbol{y}(i)\|^2 = \|\boldsymbol{y}(i) - \mathbb{P}_B(i)\boldsymbol{y}(i)\|^2\right) \\ &= \Pr\left(\|\boldsymbol{y}(i) - \mathbb{P}_B(j)\boldsymbol{y}(i)\|^2 < \|\boldsymbol{y}(i) - \mathbb{P}_B(i)\boldsymbol{y}(i)\|^2\right) \end{aligned} \tag{7}$$

- By defining $\mathbb{Q}(j,i) \triangleq \mathbb{P}_B(j) - \mathbb{P}_B(i)$,

$$p_{j|i} = \Pr\left(\boldsymbol{y}^H(i)\mathbb{Q}(j,i)\boldsymbol{y}(i) > 0\right). \tag{8}$$

- $\boldsymbol{y}(i) = \mathbb{B}(i)\boldsymbol{h} + \boldsymbol{n}$ is complex Gaussian distributed with mean $\boldsymbol{m}_y(i) = \mathbb{B}(i)\boldsymbol{m}_h$ and covariance matrix $\mathbb{S}_y(i) = \mathbb{B}(i)\mathbb{S}_h\mathbb{B}^H(i) + \sigma_{\mathrm{n}}^2\mathbb{I}_L$.
- Assuming that $\sigma_{\mathrm{n}}^2 > 0$, the real and symmetric matrix $\mathbb{S}_y(i)$ is positive definite and can be factorized to $\mathbb{S}_y(i) = \mathbb{S}_y^{1/2}(i)\mathbb{S}_y^{1/2}(i)$.
- Considering the real and symmetric $L \times L$ matrix

$$\mathbb{S}_y^{1/2}(i)\mathbb{Q}(j,i)\mathbb{S}_y^{1/2}(i) = \sum_{n=1}^{L} \lambda_n \boldsymbol{q}_n \boldsymbol{q}_n^T, \tag{9}$$

where $\{\boldsymbol{q}_n\}_{n=1}^{L}$ are orthonormal eigenvectors with eigenvalues as multiplicative coefficients $\{\lambda_n\}_{n=1}^{L}$, which is in descending order, namely,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L. \tag{10}$$

- Hence, we get

$$\begin{aligned} \boldsymbol{y}^H(i)\mathbb{Q}(j,i)\boldsymbol{y}(i) &= \left(\mathbb{S}_y^{-1/2}(i)\boldsymbol{y}(i)\right)^H \mathbb{S}_y^{1/2}(i)\mathbb{Q}(j,i)\mathbb{S}_y^{1/2}(i)\left(\mathbb{S}_y^{-1/2}(i)\boldsymbol{y}(i)\right) \\ &= \sum_{n=1}^{L} \lambda_n \left|\boldsymbol{q}_n^T \mathbb{S}_y^{-1/2}(i)\boldsymbol{y}(i)\right|^2 \\ &= \sum_{n=1}^{L} \lambda_n |X_n|^2. \end{aligned} \tag{11}$$

- $X_n \triangleq \boldsymbol{q}_n^T \mathbb{S}_y^{-1/2}(i)\boldsymbol{y}(i)$ is complex Gaussian distributed, with mean $m_x(n) = \boldsymbol{q}_n^T \mathbb{S}_y^{-1/2}(i)\mathbb{B}(i)\boldsymbol{m}_h$ and variance $\sigma_{X_n}^2 = 1$.
- $\{|X_n|^2\}_{n=1}^L$ are a set of independent noncentral χ^2-variables with two degrees of freedom.

Pairwise Error Probability

Under Rayleigh fading channel with $\boldsymbol{m}_h = 0$, (8) can be solved by Imhof [2] as

$$p_{j|i} = \sum_{l=1}^{p} \frac{1}{(k_l - 1)!} \left[\frac{\partial^{k_l - 1}}{\partial x^{k_l - 1}} F_l(x) \right]_{x=\overline{\lambda}_l}, \tag{12}$$

where

$$F_l(x) = x^{q-1} \prod_{1 \leq r \leq \bar{L}, r \neq l} (x - \overline{\lambda}_r)^{-k_r}.$$

Hence, the design criterion (5) can be evaluated via (12).

[2] J. P. Imhof, "Computing the distribution of quadratic forms in normal variables," *Biometrica*, vol. 48, no. 3–4, pp. 419–426, 1991.

Code Searching Algorithm

The below procedure is a typical simulated annealing algorithm :

`Choose initial code` $\mathcal{J}$ `and initial temperature` T.
REPEAT
  REPEAT
    `Chose another code` $\mathcal{J}'$.
    `Set` $\Delta\epsilon = \epsilon(\mathcal{J}') - \epsilon(\mathcal{J})$.
    IF $(\Delta\epsilon < 0)$
    THEN Set $\mathcal{J} = \mathcal{J}'$.
    ELSE `With probability` p, `set` $\mathcal{J} = \mathcal{J}'$.
  UNTIL `(Reach a certain number of energy drops or too many iterations.)`
  `Set` $T = \alpha T$.
UNTIL `(Reach the targeted freezing temperature.)`

Code Searching Algorithm

The detail of the above algorithm, specifically for our code search, is given below.

- The initial code $\mathcal{J}$ consists of the first 2^K elements of all possible candidates, listing in alphabetical order.
- The initial temperature is $T = 10^7$ and the targeted freezing temperature is 10^{-7}.
- Following Skoglund, we set $\alpha = 0.995$.
- The energy function $\epsilon(\cdot)$ of this system is the union bound in (5) without the multiplicative constant 2^{-K}, i.e.,

  $$\epsilon(\mathcal{J}) \triangleq \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}, j \neq i} p_{j|i}, \tag{13}$$

  which can be evaluated via (12).
- The encoder will map the uniformly distributed information messages to the codewords in alphabetical order.

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Impacts of Amplitude Distortions and Phase Distortions

- For simplicity and clarity, we let $P = 1$.
- Considering the first case :
  - The phase distortion θ is unknown, and define $\vartheta \triangleq e^{\imath(\angle h)} = e^{\imath\theta}$
  - The amplitude distortion $h_m \triangleq |h|$ is known.
  - $h = |h|e^{\imath\theta} = h_m\vartheta$ is the complex-valued channel coefficient.
- The joint maximum-likelihood (JML) decoder becomes

$$(\hat{\boldsymbol{b}}, \hat{\boldsymbol{h}}) = \arg \min_{(\boldsymbol{b},\vartheta)\in\mathcal{C}\times\mathbb{C}:|\vartheta|=1} \|\boldsymbol{y} - \mathbb{B}h_m\vartheta\|^2, \tag{14}$$

where $\mathcal{C}$ is the set of all codewords, and $\mathbb{C}$ consists of all complex numbers.

- Denoting the L-by-1 matrix $\mathbb{B}h_m$ by $\mathbb{A}$ and fixing $\mathbb{B}$, we get

$$\begin{aligned}\hat{\vartheta} &= \arg \min_{\vartheta\in\mathbb{C}:|\vartheta|=1} \|\boldsymbol{y} - \mathbb{A}\vartheta\|^2 \\ &= \arg \min_{\vartheta\in\mathbb{C}:|\vartheta|=1} (\boldsymbol{y} - \mathbb{A}\vartheta)^H(\boldsymbol{y} - \mathbb{A}\vartheta) \\ &= \arg \min_{\vartheta\in\mathbb{C}:|\vartheta|=1} \|\boldsymbol{y}\|^2 - 2\mathrm{Re}\left\{\vartheta\boldsymbol{y}^H\mathbb{A}\right\} + \mathbb{A}^H\mathbb{A}. \qquad (15)\end{aligned}$$

- This results is

$$\hat{\vartheta} = \frac{\mathbb{A}^H\boldsymbol{y}}{|\mathbb{A}^H\boldsymbol{y}|} = \frac{\mathbb{B}^H\boldsymbol{y}}{|\mathbb{B}^H\boldsymbol{y}|}. \qquad (16)$$

- Via (16), the decoder in (14) turns to

$$\begin{aligned}\hat{\boldsymbol{b}} &= \arg\min_{\boldsymbol{b}\in\mathcal{C}} \|\boldsymbol{y} - \mathbb{B}h_m\hat{\vartheta}\|^2 \\ &= \arg\min_{\boldsymbol{b}\in\mathcal{C}} \left\|\boldsymbol{y} - \mathbb{B}h_m\frac{\mathbb{B}^H\boldsymbol{y}}{|\mathbb{B}^H\boldsymbol{y}|}\right\|^2 \qquad (17)\end{aligned}$$

- It will be shown in simulation results that the (17) decoder, knowing the amplitude distortion, has no performance gain.

- Considering the other case :
  h_m is unknown but ϑ is known at the receiver.
- Again, denoting $\mathbb{B}\vartheta$ by $\mathbb{D}$, we get

$$\begin{aligned}\|\boldsymbol{y}-\mathbb{D}h_m\|^2 &= (\boldsymbol{y}-\mathbb{D}h_m)^H(\boldsymbol{y}-\mathbb{D}h_m) \\ &= \|\boldsymbol{y}\|^2-(\boldsymbol{y}^H\mathbb{D}+\mathbb{D}^H\boldsymbol{y})h_m+\|\mathbb{D}\|^2h_m^2. \quad (18)\end{aligned}$$

- Fixing $\mathbb{B}$, the estimate of the amplitude distortion, which minimizes (18) subject to $h_m \geq 0$, is given by

$$\begin{aligned}\hat{h}_m &= \left\{\frac{\boldsymbol{y}^H\mathbb{D}+\mathbb{D}^H\boldsymbol{y}}{2\|\mathbb{D}\|^2}\right\}^+ \\ &= \left\{\frac{e^{\imath\theta}\boldsymbol{y}^H\mathbb{B}+e^{-\imath\theta}\mathbb{B}^H\boldsymbol{y}}{2N}\right\}^+, \quad (19)\end{aligned}$$

  where $\{x\}^+ \triangleq \max\{x,0\}$, and the last step follows from $|\vartheta|^2=1$ and $\|\mathbb{B}\|^2=N$.

- Following (19), we obtain

$$\begin{aligned}\hat{\boldsymbol{b}} &= \arg\min_{\boldsymbol{b}\in\mathcal{C}} \|\boldsymbol{y} - \mathbb{B}\vartheta\hat{h}_m\|^2 \\ &= \arg\min_{\boldsymbol{b}\in\mathcal{C}} \left\|\boldsymbol{y} - \frac{1}{2N}\mathbb{B}(e^{\imath 2\theta}\boldsymbol{y}^H\mathbb{B} + \mathbb{B}^H\boldsymbol{y})\right\|^2. \qquad (20)\end{aligned}$$

- (20) will lead to a performance improvement over the criterion in (3), and the gain will be shown in the simulation results.

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The Modified System Model

- Given a channel with two taps, denote by h_i the channel coefficients of path i. The reception at time n can be obtained as

$$y_n = h_1 \cdot b_n + h_2 \cdot b_{n-1}. \tag{21}$$

- Their phases should have constant difference. For simplicity, assume these differences are all zero, i.e. the phases of two channel taps are synchronized.

PDF of the Channel Coefficients of the Modified System Model

- – The amplitude distortion h_{m_1} and h_{m_2} are independent Rayleigh distributed with mean $\sqrt{\frac{\pi\tau^2}{2}}$, where $\tau^2 = \frac{\sigma^2}{2}$.
  – The marginal PDfs of them are

$$
\begin{aligned}
f_{h_{m_1}}(u) &= \frac{u}{\tau^2} e^{-\frac{u^2}{2\tau^2}}, \quad u \geq 0 \\
f_{h_{m_2}}(v) &= \frac{v}{\tau^2} e^{-\frac{v^2}{2\tau^2}}, \quad v \geq 0
\end{aligned} \tag{22}
$$

- – The phase distortion θ is uniformly distributed over $[-\pi, \pi)$ and independent of both h_{m_1} and h_{m_2}.
  – The marginal PDfs of it becomes

$$
f_\theta(\theta) = \frac{1}{2\pi}, \quad -\pi \leq \theta \leq \pi \tag{23}
$$

- Denoting $h_1 = x_1 + \imath y_1$ and $h_2 = x_2 + \imath y_2$, we get

$$\begin{aligned} x_1 &= h_{m_1} \cos\theta_1, \qquad y_1 = h_{m_1} \sin\theta_1, \\ x_2 &= h_{m_2} \cos\theta_2, \qquad y_2 = h_{m_2} \sin\theta_2, \end{aligned} \tag{24}$$

where $\theta_1 = \theta_2 = \theta$ with probability one.

- By the independence of h_{m_1}, h_{m_2} and θ, we derive

$$\begin{aligned} f_{h_1,h_2}(ue^{\imath\theta_1}, ve^{\imath\theta_2}) &= f_{h_{m_1},h_{m_2},\theta_1,\theta_2}(u, v, \theta_1, \theta_2) \\ &= \frac{u}{\tau^2} e^{-\frac{u^2}{2\tau^2}} \cdot \frac{v}{\tau^2} e^{-\frac{v^2}{2\tau^2}} \cdot \frac{1}{2\pi} \delta(\theta_1 - \theta_2). \end{aligned} \tag{25}$$

- Through the Jacobian transformation, we get

$$\begin{aligned} & f_{x_1,y_1,x_2,y_2}(x_1, y_1, x_2, y_2) \\ = & \frac{1}{2\pi\tau^4} e^{-\frac{x_1^2+y_1^2+x_2^2+y_2^2}{2\tau^2}} \delta(\tan\frac{y_1}{x_1} - \tan\frac{y_2}{x_2}). \end{aligned} \tag{26}$$

Approximate PDF of the Channel Coefficients of the Modified System M

- A zero-mean Gaussian $\mathfrak{f}_{h_1,h_2}$ having the same covariance matrix as f_{h_1,h_2} is a simple approximate to f_{h_1,h_2} in (25).
- Based on f_{h_1,h_2}, we derive that

$$\begin{aligned} E\left[h_1 h_2^*\right] &= E\left[h_{m_1} \cdot e^{j\theta} \cdot h_{m_2} \cdot e^{-j\theta}\right] \\ &= E\left[h_{m_1} \cdot h_{m_2}\right] \\ &= E\left[h_{m_1}\right] E\left[h_{m_2}\right]. \end{aligned} \tag{27}$$

- The covariance matrix due to f_{h_1,h_2} is given by

$$\mathbb{S}_h = \begin{bmatrix} \sigma^2 & \frac{\pi\sigma^2}{4} \\ \frac{\pi\sigma^2}{4} & \sigma^2 \end{bmatrix}. \tag{28}$$

- Accordingly, we adopt $\mathfrak{f}_{h_1,h_2} \sim N(0, \mathbb{S}_h)$ as the approximate PDF to f_{h_1,h_2}.

Approximate PDF of the Channel Coefficients of the Modified System M

- Apparently, the second moments of X_n due to both f_{h_1,h_2} and $\mathfrak{f}_{h_1,h_2}$ are the same.
- An elementary result for a zero-mean complex Gaussian random variable Z is that [3]

$$E\left[Z^{2m}\right] = m! \cdot E^m\left[|Z|^2\right]. \tag{29}$$

- The fourth moment of X_n from (11) with respect to the zero-mean Gaussian $\mathfrak{f}_{h_1,h_2}$ is

$$E\left[X_n^4\right] = 2, \tag{30}$$

since $m_{X_n} = 0$ and $\sigma_{X_n^2} = 1$ under PDF $\mathfrak{f}_{h_1,h_2}$.

[3] I.S. Reed, "On a moment theorem for complex Gaussian processes," *IRE Trans. Inform. Theory*, vol. IT-8, pp. 194V195, Apr. 1962.

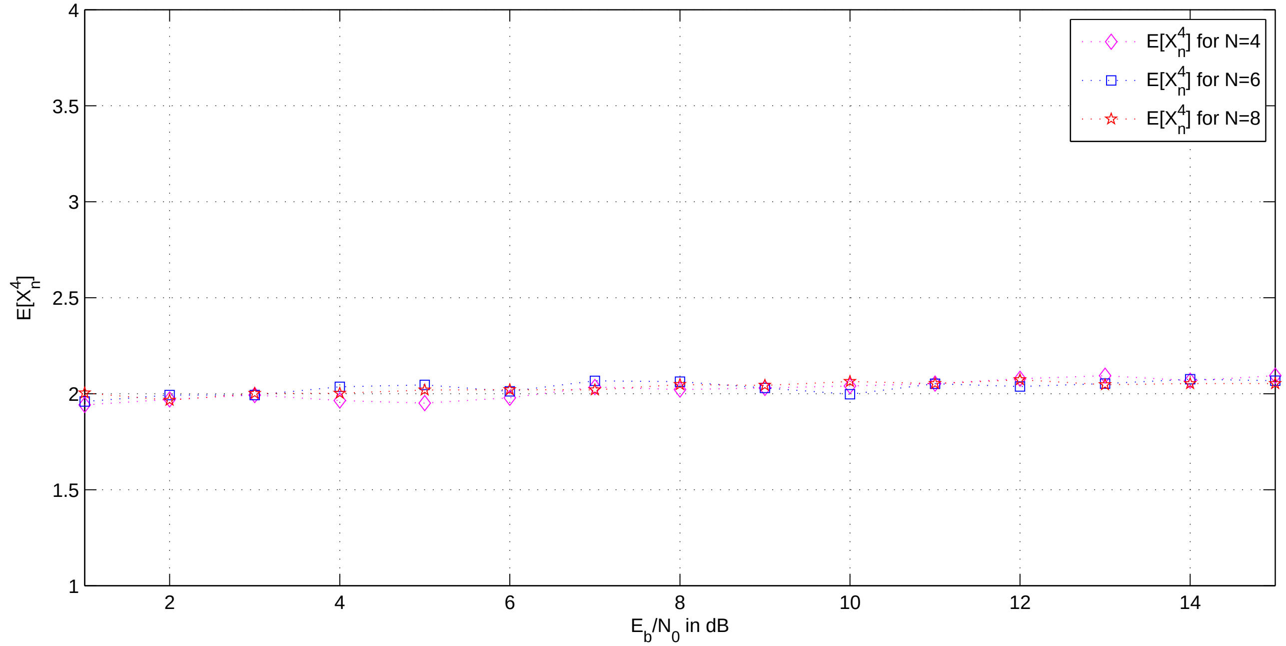

Figure 1: $E[X_n^4]$ with respect to f_{h_1,h_2} via Monte Carlo simulation.

The Decoding Criterion of the Modified System Model

- The channel coefficient vector $\boldsymbol{h} = [h_1 \ h_2]^T$ of the modified system model is given by

$$\boldsymbol{h} = \boldsymbol{h}_m \vartheta, \tag{31}$$

where $\boldsymbol{h}_m = [h_{m_1} \ h_{m_2}]^T$ is amplitude distortion vector and $\vartheta \triangleq e^{\imath(\angle h_1)} = e^{\imath(\angle h_2)}$ is phase distortion.

- Given $\mathbb{B}$, by the formula below,

$$\begin{aligned} \hat{\boldsymbol{h}} &= \min_{\boldsymbol{h} \in \mathbb{C}^P} \|\boldsymbol{y} - \mathbb{B}\boldsymbol{h}\|^2 \\ &= \min_{\boldsymbol{h}_m \geq 0} \min_{|\vartheta|=1} \|\boldsymbol{y} - \mathbb{B}\boldsymbol{h}_m \vartheta\|^2, \end{aligned} \tag{32}$$

The Decoding Criterion of the Modified System Model

- The minimizers for $\boldsymbol{h}_m$ and ϑ are as follows.

$$\hat{\vartheta} = (\boldsymbol{h}_m^H \mathbb{B}^H \mathbb{B} \boldsymbol{h}_m)^{-1} \boldsymbol{h}_m^H \mathbb{B}^H \boldsymbol{y}, \tag{33}$$

$$\hat{h}_{m_1} = \frac{1}{N} \left\{ \sum_{n=1}^{L} (y_n^* g_{n,1} + y_n g_{n,1}^*) - \hat{h}_{m_2} \sum_{n=1}^{L} (g_{n,1} g_{n,2}^* + g_{n,1}^* g_{n,2}) \right\}^+,$$

$$\hat{h}_{m_2} = \frac{1}{N} \left\{ \sum_{n=1}^{L} (y_n^* g_{n,2} + y_n g_{n,2}^*) - \hat{h}_{m_1} \sum_{n=1}^{L} (g_{n,1} g_{n,2}^* + g_{n,1}^* g_{n,2}) \right\}^+,$$

  where $g_{i,j}$ represents the element located at ith row and jth column of the matrix $\mathbb{G} = \mathbb{B}\hat{\vartheta}$.

- The optimal decoding criterion of the modified system model is straightforwardly given by

$$\arg\min_{\boldsymbol{b} \in \mathcal{C}} \|\boldsymbol{y} - \mathbb{B}\hat{\boldsymbol{h}}_m \hat{\vartheta}\|^2. \tag{34}$$

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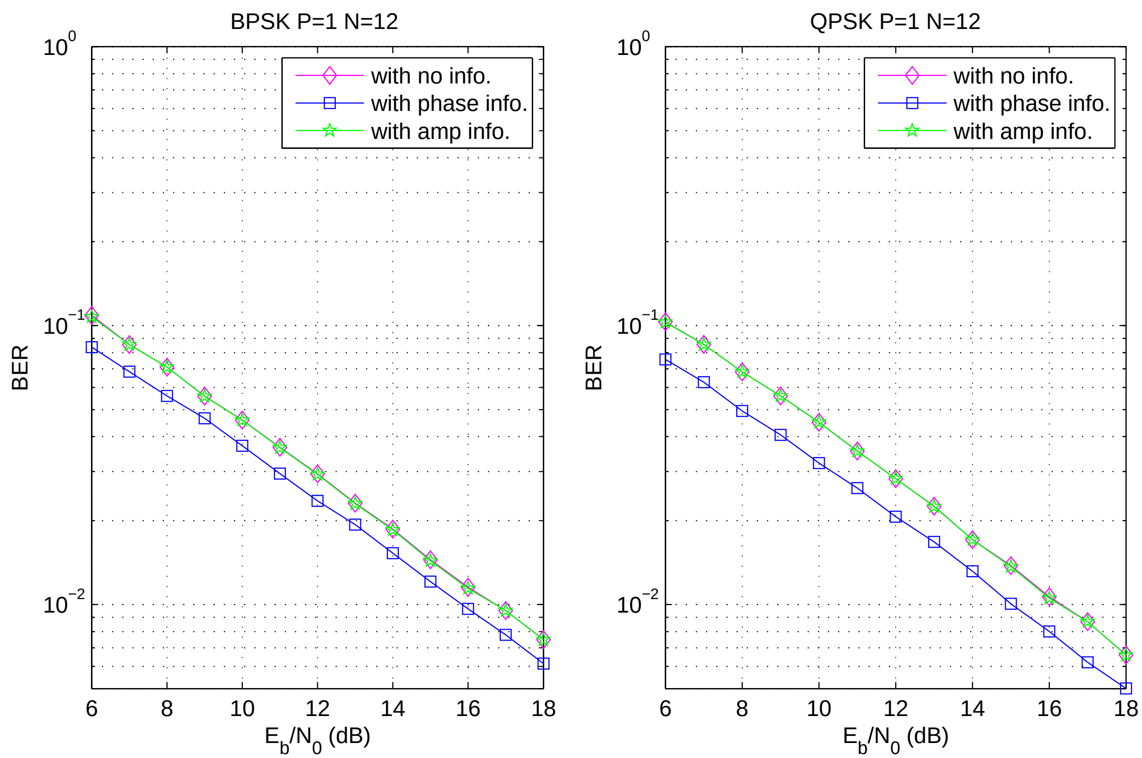

Figure 2: The BERs of the $N = 12$ codes. The channel simulated is the Rayleigh-fading channel with $P = 1$, and decoder is with no/phase distortion/amplitude distortion information. Here, the codes are BPSK-modulated and QPSK-modulated, respectively.

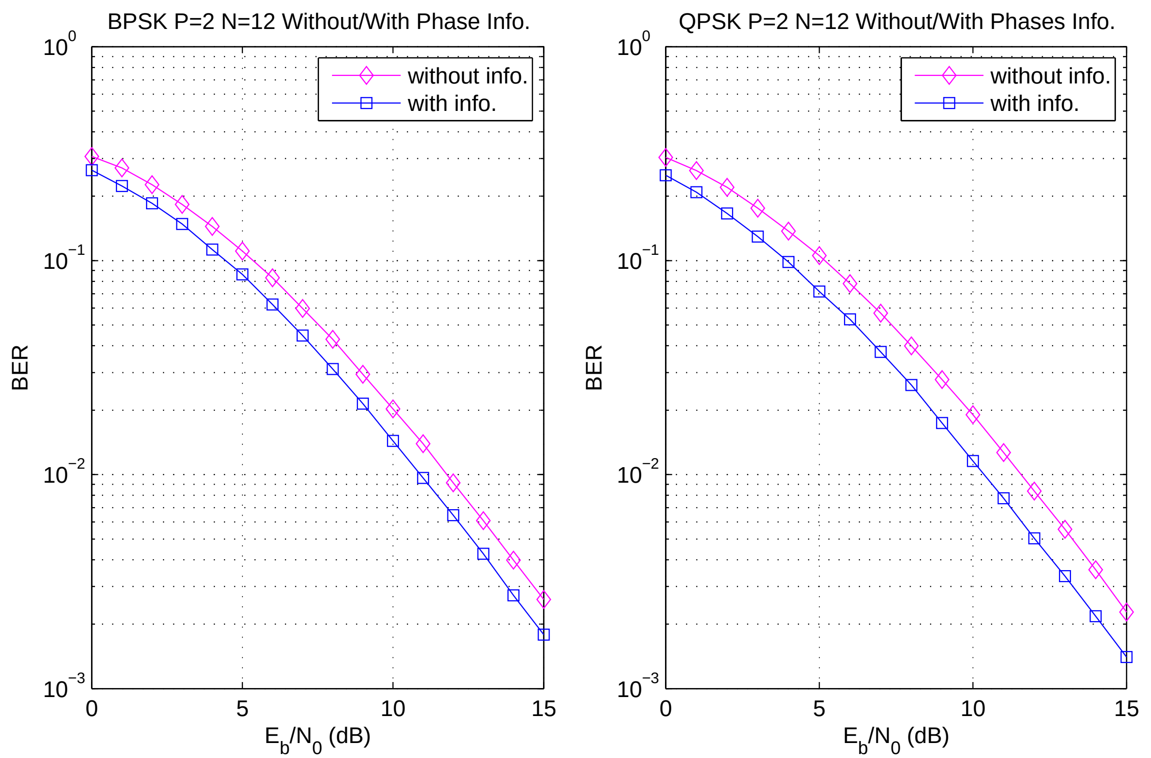

Figure 3: The BERs of the $N = 12$ codes. The channel simulated is the Rayleigh-fading channel with $P = 2$, and decoder is without/with phase distortion information. Here, the codes are BPSK-modulated and QPSK-modulated, respectively.

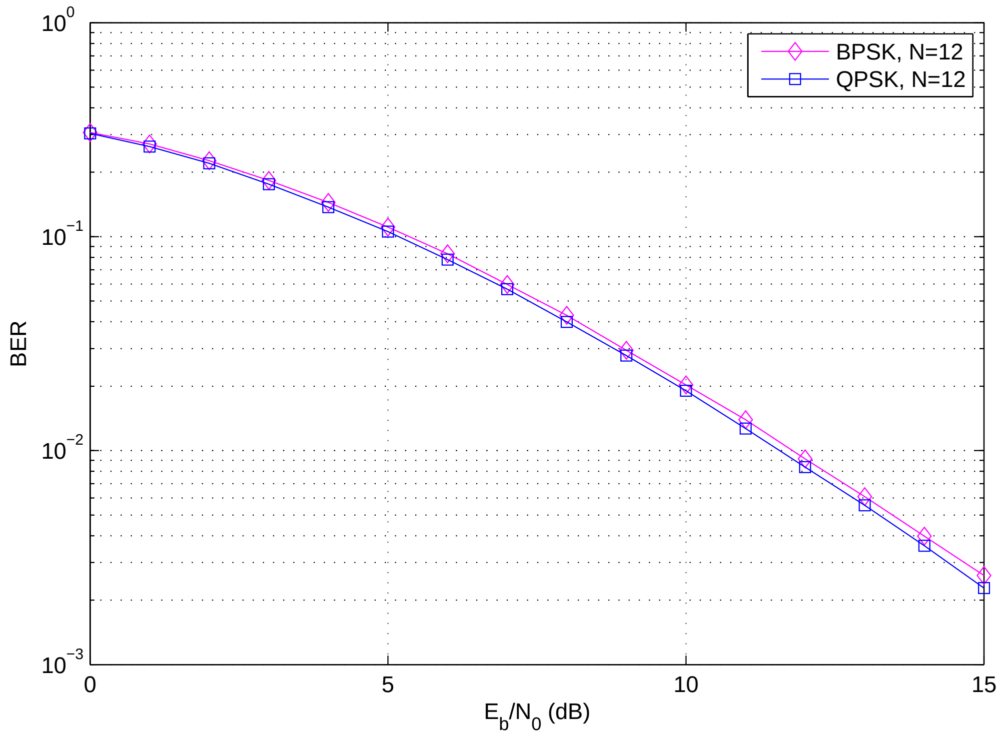

Figure 4: The BERs of BPSK-modulated and QPSK-modulated codes using decoder without any information on the channels. The channel simulated is the multi-path Rayleigh-fading channel with $P = 2$, for which the phases of two channel taps are independent. Here, the codeword lengths examined are $N = 12$.

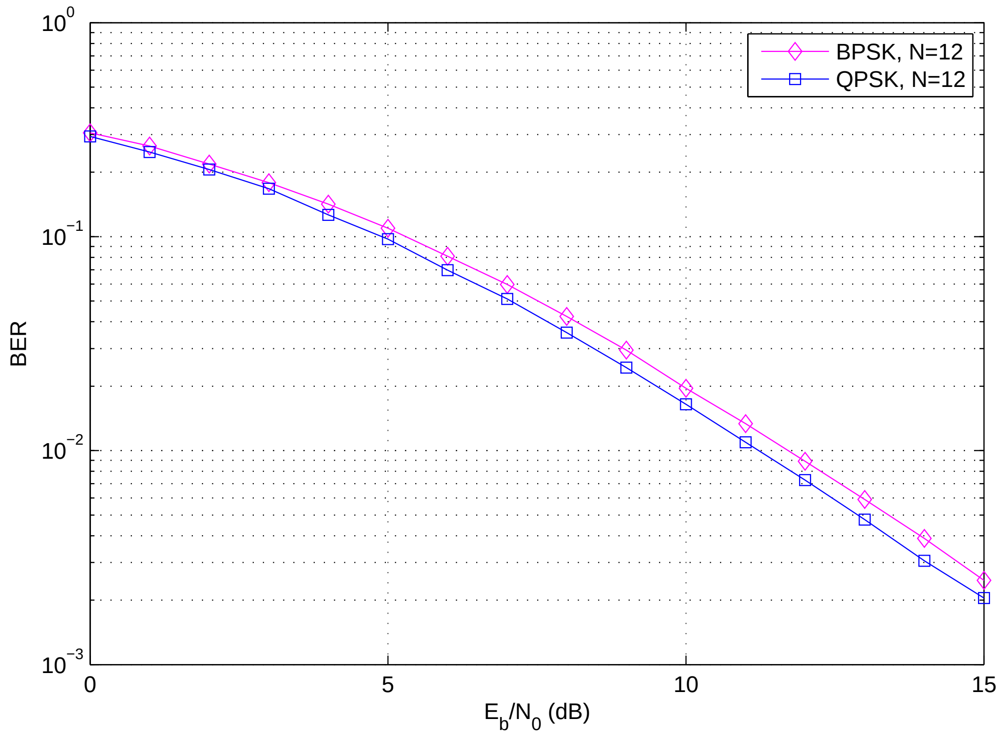

Figure 5: The BERs of BPSK-modulated and QPSK-modulated codes using decoder with the information that the phases of two channel taps are synchronized. The channel simulated is the multi-path fading with $P = 2$, for which the phases of two channel taps are also synchronized. Here, the codeword lengths examined are $N = 12$.

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Conclusion

- Simulations show that the blind receiver of QPSK-modulated codes may need to know the phase information of the channel taps in order to obtain an acceptable coding gain over the BPSK-modulated codes.
- We assume that phases among different channel taps are synchronized even if they are unknown.
- In this thesis, we derived an approximation close-form formula for the union bound of the error performance for QPSK-modulated codes transmitted over a frequency-selective fading channel and demodulated by a blind receiver.
- Based on the criterion, we then searched for good QPSK-modulated codes for combined channel estimation and error protection by computers.

Future Work

- Find a systematic code design for our simulated-annealing-based computer-searched codes.
- Establish a decoding scheme for such codes with low decoding complexity.
- Extend to the QAM modulation, in order to increase the transmission rate.

Thank you for your attention.