Chapter 1

Introduction

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Introduction

- This volume will talk about some advanced topics in information theory.
- The mathematical background on which these topics are based can be found in Appendices A and B of the first volume.

Notations

- For a random variable X, we use P_X to denote its distribution.
- The probability of $X = x$ is denoted by

$$\text{either} \quad \Pr\{X = x\} \quad \text{or} \quad P_X(x).$$

- Similarly, the probability of a set characterizing through an inequality, such as $f(x) < a$, is expressed by

$$\text{either} \quad P_X\{x \in \mathcal{X} \ : \ f(x) < a\} \quad \text{or} \quad \Pr\{f(X) < a\}.$$

- In the second expression, $f(X)$ is a new random variable defined through X and a function $f(\cdot)$.
- Obviously, the above expressions can be applied to any legitimate function $f(\cdot)$ defined over $\mathcal{X}$, including any probability function $P_{\hat{X}}(\cdot)$ (or $\log P_{\hat{X}}(x)$) of a random variable $\hat{X}$.
- Therefore, the next two expressions denote the probability of $f(x) = P_{\hat{X}}(x) < a$ evaluated under distribution P_X:

$$P_X\{x \in \mathcal{X} \ : \ f(x) < a\} = P_X\left\{x \in \mathcal{X} \ : \ P_{\hat{X}}(x) < a\right\}$$

and

$$\Pr\{f(X) < a\} = \Pr\left\{P_{\hat{X}}(X) < a\right\}.$$

Notations

- As a result, if we write

$$\begin{aligned} P_{X,Y}&\left\{(x,y)\in\mathcal{X}\times\mathcal{Y} \;:\; \log\frac{P_{\hat{X},\hat{Y}}(x,y)}{P_{\hat{X}}(x)P_{\hat{Y}}(y)}<a\right\}\\ =\ \Pr&\left\{\log\frac{P_{\hat{X},\hat{Y}}(X,Y)}{P_{\hat{X}}(X)P_{\hat{Y}}(Y)}<a\right\},\end{aligned}$$

it means that we define a new function

$$f(x,y)\triangleq\log\frac{P_{\hat{X},\hat{Y}}(x,y)}{P_{\hat{X}}(x)P_{\hat{Y}}(y)}$$

in terms of the joint distribution $P_{\hat{X},\hat{Y}}$ and its two marginal distributions, and concern the probability of $f(x,y)<a$ where x and y have distribution $P_{X,Y}$.