Lecture Notes in Information Theory
Volume II: Problems

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Chapter 3

Problems

3-1 Let $U_1, U_2, \ldots$ be an i.i.d. binary sequence with marginal distribution

$$P_U(0) = P_U(1) = \frac{1}{2}.$$

Define the source $X_1, X_2, \ldots$ by

$$X_n = U_1 \times \ U_2 \times \cdots \times U_n.$$

Find the sup-entropy rate and inf-entropy rate of the source $\mathbf{X} = \{X^n = (X_1, \ldots, X_n)\}$.

3-2 Show that when $t \to 0$, the exponential cost function

$$L(t) \triangleq \frac{1}{t} \log_2 \left(\sum_{x \in \mathcal{X}} P_X(x) 2^{t\ell(\boldsymbol{c}_x)} \right)$$

reduce to

$$L(0) = \log_2(e) \cdot \sum_{x \in \mathcal{X}} P_X(x)\ell(\boldsymbol{c}_x),$$

which is the average codeword length. Also show that in the case of $t \to \infty$,

$$L(\infty) = \max_{x \in \mathcal{X}} \ell(\boldsymbol{c}_x),$$

which is the maximum codeword length for all binary codewords.

Chapter 4

Problems

4-1 Prove that the worst-case complexity of a random variable X is no less than $\lceil R(X) \rceil$. Also prove that the average-case complexity of a random variable X lies between $H(X)$ and $H(X) + 1$.
(Hint: To write an algorithm with desired worst-case complexity for source X is similar to establish a binary data compaction block code with desired codeword length for source X. Likewise, to write an algorithm with desired average-case complexity for random variable X is similar to design a binary variable-length data compaction code with desired average codeword length for source X.)