Optimal Quantization for Finite-State Channels

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IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 43, NO. 2, pp.758-765, MARCH 1997

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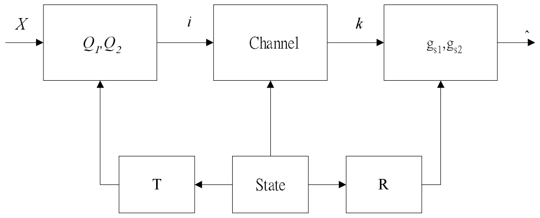
Goal

- Design the quantizer such that the MSE when the channel is in the good state is minimized subject to a constraint on the MSE when the channel is in the bad state.



Goal

- Design the quantizer to minimize

$$D_{s_1} = E\left[\left(X - \hat{X}\right)^2 \mid \text{channel is in state } s_1\right]$$

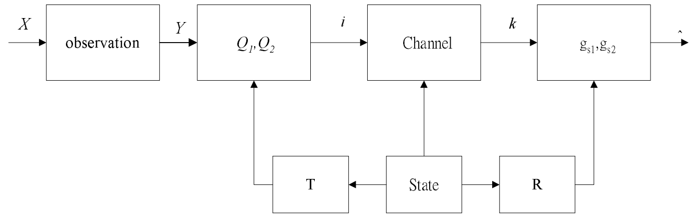
subject to the constraint

$$D_{s_2} = E\left[\left(X - \hat{X}\right)^2 \mid \text{channel is in state } s_2\right] \leq D$$

where D is the maximum allowed distortion when channel is in state s_2

Goal

- The system with noisy observation



Numerical Results

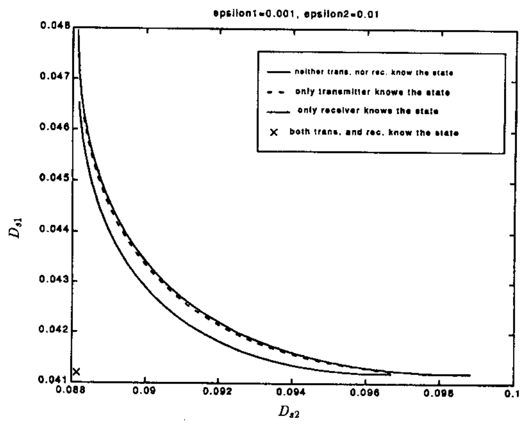


Fig. 4. The achievable set of distortions for NBC.

Numerical Result

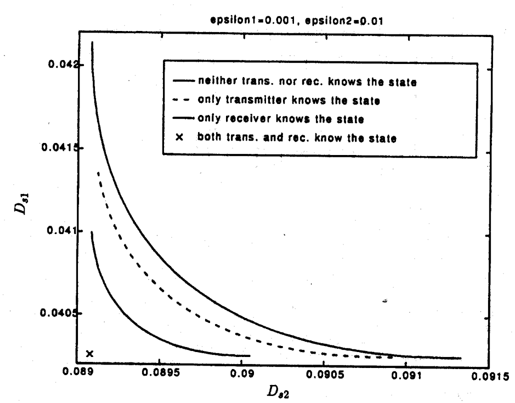


Fig. 5. The achievable set of distortions for FBC.

Numerical Result

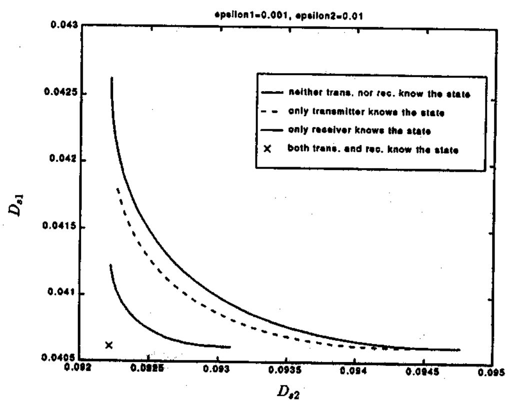


Fig. 6. The achievable set of distortions for GC.

Definitions

- Channel transition matrices

$$T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \qquad R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

- Distortion for each state

$$D_{s_j} = \sum_{m=1}^{2} \sum_{n=1}^{2} t_{mj} r_{nj} \sum_{i=1}^{N_1} \sum_{k=1}^{N_2} p_{s_j}(k \mid i) \int_{A_{mi}} p(x)(x - g_n(k))^2 \, dx$$

- Lagrangian

$$L = D_{s_1} + \lambda (D_{s_2} - D)$$
$$= \sum_{j=1}^{2} \lambda_j \sum_{m=1}^{2} \sum_{n=1}^{2} t_{mj} r_{nj} \sum_{i=1}^{N_1} \sum_{k=1}^{N_2} p_{s_j}(k \mid i) \int_{A_{mi}} p(x)(x - g_n(k))^2 \, dx - \lambda D$$

Definition

- Optimum Encoder:

$$A_{s_m i} = \{x : 2\alpha_{mi} x - \beta_{mi} \geq 2\alpha_{mi} x - \beta_{mi'}, \forall i' \neq i\}$$

$$\alpha_{mi} = \sum_{j=1}^{2} \lambda_j \sum_{n=1}^{2} t_{mj} r_{nj} \sum_{k=1}^{N_2} p_{s_j}(k \mid i) g_n(k)$$

$$\beta_{mi} = \sum_{j=1}^{2} \lambda_j \sum_{n=1}^{2} t_{mj} r_{nj} \sum_{k=1}^{N_2} p_{s_j}(k \mid i) g_n^2(k)$$

- Optimum Decoder:

$$g_{s_n}(k) = \frac{\sum_{j=1}^{2} \lambda_j \sum_{m=1}^{2} t_{mj} r_{nj} \sum_{i=1}^{N_1} P_{s_j}(k \mid i) \int_{A_{s_m i}} x p(x) dx}{\sum_{j=1}^{2} \lambda_j \sum_{m=1}^{2} t_{mj} r_{nj} \sum_{i=1}^{N_1} P_{s_j}(k \mid i) \int_{A_{s_m i}} p(x) dx}$$

Definition

- For noisy observation at the transmitter
  - x replace by $\tilde{x}(y)$
  - D replace by $D' - E\left[(X - \tilde{X}(Y))^2\right]$
- i.e. Lagrangian becomes

$$L = \sum_{j=1}^{2} \lambda_j \sum_{m=1}^{2} \sum_{n=1}^{2} t_{mj} r_{nj} \sum_{i=1}^{N_1} \sum_{k=1}^{N_2} p_{s_j}(k \mid i) \int_{A_{mi}} p(y)(\tilde{x}(y) - g_n(k))^2 dx$$
$$- \lambda\left(D' - E\left[(X - \tilde{X}(Y))^2\right]\right)$$

Conclusion

- Minimize the MSE in the normal mode of operation subject to a constraint on the MSE when the channel is in the other state.
- Noisy observation can be separated into two stage:

  stage 1: estimating the parameter of interest in the mean-squared sense

  stage 2: using the quantizer designed for no observation noise

Problem

- Complexity
- Parameters decide
- Null set
- Initial encoder structure

Problem

TABLE I
AN EXAMPLE OF A 16-LEVEL QUANTIZER DESIGN

index	A	g	index	A	g
0000	$(-1.7893, -1.0618)$	-1.3551	1000	$(-0.5468, -0.3900)$	-0.4730
0001	$(-\infty, -1.7893)$	-2.0965	1001	—	-0.1877
0010	—	-0.5531	1010	$(-1.0618, -0.5468)$	-0.7642
0011	—	0.0789	1011	$(-0.3900, 0.0550)$	-0.1705
0100	—	0.0994	1100	$(0.7068, 1.0447)$	0.8311
0101	—	0.4523	1101	$(0.3450, 0.7068)$	0.5174
0110	$(1.7417, \infty)$	2.0679	1110	—	0.1929
0111	$(1.0447, 1.7417)$	1.3118	1111	$(0.0550, 0.3450)$	0.2181