A Distance Spectrum Interpretation of Turbo Codes

Lance C. Perez, Member, IEEE

Jan Seghers, Fellow, IEEE

Daniel J. Costello, Jr., Fellow, IEEE

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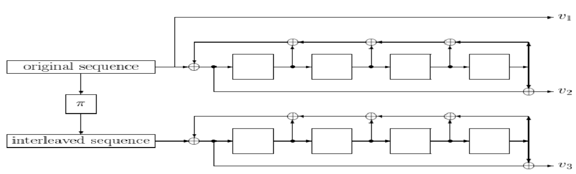
Introduction (1/2)

- The birth of turbo coding
  Year: 1993
  Authors: Berrou, Glavieux and Thitimajshima
  Paper Title: Near Shannon Limit Error-Correcting Coding and Decoding:Turbo Codes

Turbo encoders



Introduction (2/2)

- Issue 1:
  What causes the "error floor" for high SNR's?

- Issue 2:
  What is it that allows Turbo codes to achieve a BER of 10^{-5} at a SNR of 0.7dB, which is only 0.7dB from the Shannon limit?

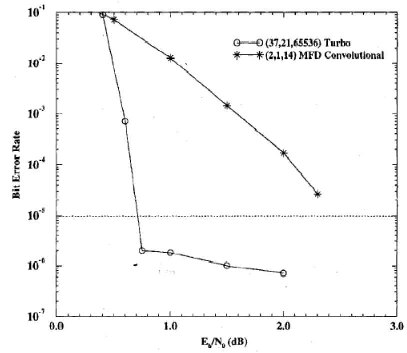


Fig. 1. Simulation results for a(37, 21, 65536) Turbo code and a (2, 1, 14) MFD convolutional code.

Issue 1 (1/3)

- The BER performance of a convolutional code with maximum-likelihood(ML) decoding on an AWGN channel can be upper-bounded using a union bound technique by

$$P_b \le \sum_{i=1}^{2^N} \frac{w_i}{N} Q\left(\sqrt{d_i \frac{2RE_b}{N_0}}\right)$$

$$\Rightarrow P_b \le \sum_{d=d_{free}}^{2(v+N)} \frac{N_d \widetilde{w}_d}{N} Q\left(\sqrt{d \frac{2RE_b}{N_0}}\right)$$

Issue 1 (2/3)

- For Turbo codes the asymptotic performance approaches

$$P_b \approx \frac{N_{free}\tilde{w}_{free}}{N} Q\left(\sqrt{d_{free}\frac{2RE_b}{N_0}}\right)$$

$$P_{t-free} \approx \frac{3\times 2}{65536} Q\left(\sqrt{6\frac{E_b}{N_0}}\right)$$

$$P_{c-free} \approx 137 Q\left(\sqrt{18\frac{E_b}{N_0}}\right)$$

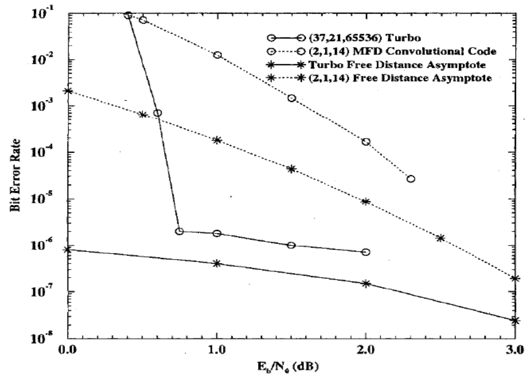


Fig. 4. Simulation results for a (37, 21, 65536) Turbo code and a (2, 1, 14) MFD convolutional code along wiht their free-distance asymptotes.

Issue 1 (3/3)

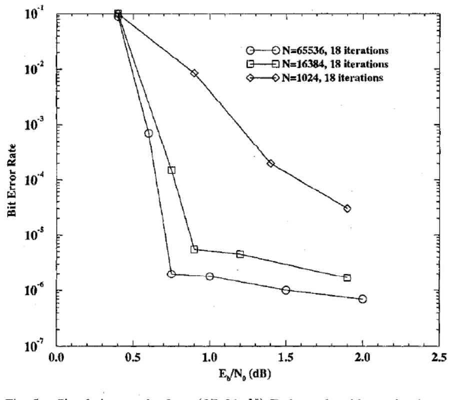


Fig. 5. Simulation results for a (37, 21, N) Turbo code with varying interleaver size N.

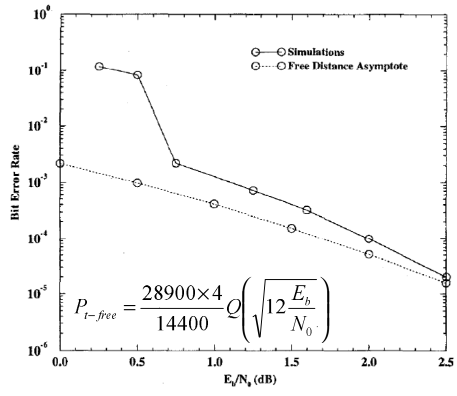


Fig. 6. Simulation results and the free-distance asymptote for the (37, 21, 14400) Turbo code with a 120 × 120 rectangular interleaver.

Issue 2 (1/4)

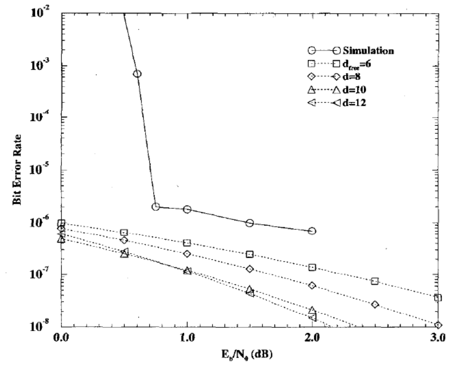


Fig. 8. Performance of a (37, 21, 65536) Turbo code decomposed by spectral line.

d	N_d	W_d
6	4.5	9
8	11	22
10	20.5	41
12	75	150

Issue 2 (2/4)

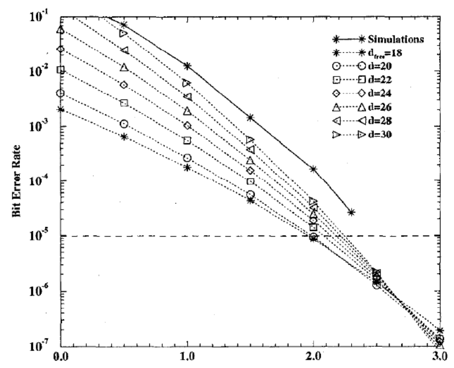


Fig. 9. Performance of the (2, 1, 14) MFD convolutional code decomposed by spectral line.

d	N_d^0	W_d^0
18	33	187
20	136	1034
22	835	7857
24	4787	53994
26	27941	361762
28	162513	2374453
30	945570	15452996
32	5523544	99659236

Issue 2 (3/4)

- This process of **spectral thinning** is represented graphically in Fig. 10, which depicts the thinning of low-weight codewords as the size of the interleaver increases for hypothetical distance spectra. It is this thinning of the distance spectrum that enables the free-distance asymptote of a Turbo code to dominate the performance for low SNR and thus to achieve near-capacity performance.

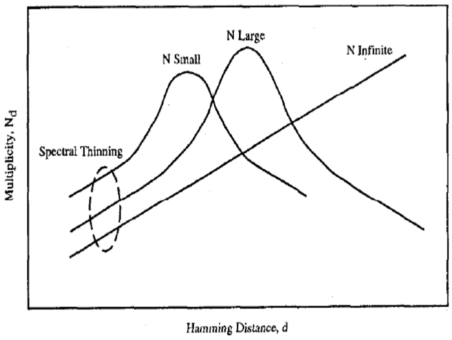


Fig. 10. Graphical representation of spectral thinning for increasing interleaver size.

Issue 2 (4/4)

- we compare a (37,21,400) Turbo code which has K = 5 and free distance dfree = 6 to a (23,35,400) Turbo code. Both codes are punctured. The feedback polynomial ho = 23 in the second Turbo code is a primitive polynomial of degree v = 4 and thus l/ho has a period of K = 2" - 1 = 15. The free distance of this Turbo code was found to be dfree = 10

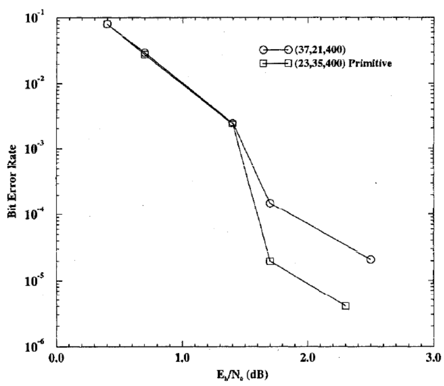


Fig. 11. Simulation results for a (37, 21, 400) Turbo code and a (23, 35, 400) primitive Turbo code.

Conclusion (1/2)

- The “error floor” observed in simulations of Turbo codes is a manifestation of the free-distance asymptote. Since Turbo codes have relatively low free distances, the free-distance asymptote has a shallow slope, and thus the performance curves flatten out at moderate to high SNR’ s.
- The exceptional performance of Turbo codes at low SNR’ s is due to the sparse distance spectrum and the resultant ability of the code to follow the free-distance asymptote at moderate to low SNR’ s. Thus spectral thinning results in few low-weight codewords and a large number of codewords of “average” weight.

Conclusion (2/2)

- In a more philosophical light, Turbo codes remind us that information-theoretical arguments imply that long block lengths, but not necessarily large free distances, are required to achieve capacity at moderate BER’ s. Thus like convolutional codes, Turbo codes are a class of codes that achieve long block lengths, but without the corresponding increased density of the distance spectrum common to convolutional codes, and for which a practical, albeit nontrivial, decoding algorithm exists.