Previous work :

Kennedy:

Capacity of an infinite bandwidth Rayleigh fading channel is the same as that of an infinite bandwidth AWGN channel with the same average received power.

Gallager & Me'dard:

If the channel is such that the fading process at different frequency are independent, then the mutual information achievable over this channel approaches zero with increasing bandwidth if spread-spectrum input signals are used.

Capacity and Mutual Information of Wideband Multipath Fading Channels

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Capacity of a multipath fading channel

Channel modeling:

$$y(t) = \sum_{l=1}^{L} a_l(t) x(t - d_l(t)) + z(t)$$

$x(t)$: input waveform

$y(t)$: channel output

L : number of path

$a_l(t)$: gain of path l at time t

$d_l(t)$: delay of the path l at time t

$z(t)$:white Gaussian noise with power spectral density $N_0 / 2$

Assumption:

- $\{a_l(t)\}$, $\{d_l(t)\}$ are fixed on time intervals $[nT_c,(n+1)T_c)$, $n \in Z$
- $\{a_l(nT_c)\}$ and $\{d_l(nT_c)\}$ are stationary and ergodic discrete-time stochastic process, and independent of each other.
- Delay spread is much less than the coherence time of the channel.
- Average received power is constrained to P
- Bandwidth of the input signal is constrained to be W
- T_d is the delay spread
- T_c is the coherent time

Capacity via Frequency-Shift Keying

- *Theorem 1:* The capacity of the multipath fading channel without bandwidth constraint is at least

$$\left(1-2\frac{T_d}{T_c}\right)\frac{P}{N_0}$$

if $T_d << T_c$, the capacity of the is close to P/N_0

Mutual information for white-Like signals

- Discrete time channel model:

Shift to baseband and sample at a rate of $1/W$

$$Y_i = \sqrt{\frac{\varepsilon}{K_c}} \sum_{l=1}^{\tilde{L}} G_l X_{(i-D_l)} + Z_i, \qquad i = 0, \cdots, K_c - 1$$

where $K_c = \lfloor WT_c \rfloor$ $\varepsilon = PT_c / N_0$

$(n) \equiv n \bmod K_c$

Upper bound on mutual information

$I(X;Y \mid D)$

$X := (X_0, \cdots, X_{K_c - 1})$, $Y := (Y_0, \cdots, Y_{K_c - 1})$ and

$D = (D_1, \cdots, D_{\tilde{L}})$

- *Theorem 2:* Assume that the input process $\{X_i\}$ satisfies either:
  - $\{X_i\}$ is an i.i.d. complex-valued process, such that $E(X_i) = E(X_i^2) = 0$ and $E(|X_i|^4) < \infty$

or

– There exist a universal constant d , not dependent on the bandwidth, such that for any realization of the input process $\{X_i\}$

$$\left|C(m,n)-\delta(n-m)\right| \le \frac{d}{K_c}$$

for all *m,n.*

where

$$C(m,n) := \frac{1}{K_c}\sum_{i=0}^{K_c-1} X_{(i-m)}X_{(i-n)}$$

is *empirical autocorrelation function*

Then as the bandwidth $W \to \infty$, the following asymptotic upper bound holds:

$$I(X;Y \mid D) \le \sum_{l=1}^{\tilde{L}} \log E\exp\left(\varepsilon^2 |H_l|^2 |G_l|^2\right) + O\left(\frac{1}{\sqrt{W}}\right)$$

where H_l has the same distribution as and is independent of G_l

if $E\left(|G_l|^2\right) = 1/\tilde{L}$ for all l, the upper bound on the mutual information per unit time is

$$\frac{P^2 T_c}{N_0^2 \tilde{L}}$$

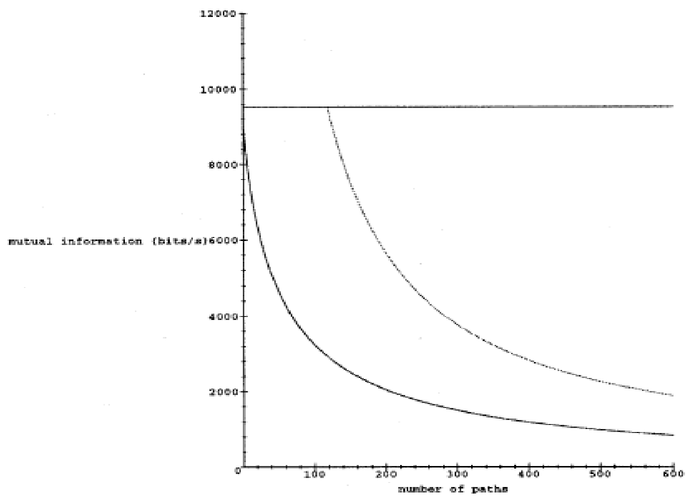
Lower bound on mutual information

- *Theorem 3:* If the input $\{X_i\}$ is i.i.d. complex circular symmetric Gaussian, then

$$I(X;Y \mid D) \geq \varepsilon - \widetilde{L}\log\left(1+\frac{\varepsilon}{\widetilde{L}}\right) + O\left(\frac{1}{W}\right)$$

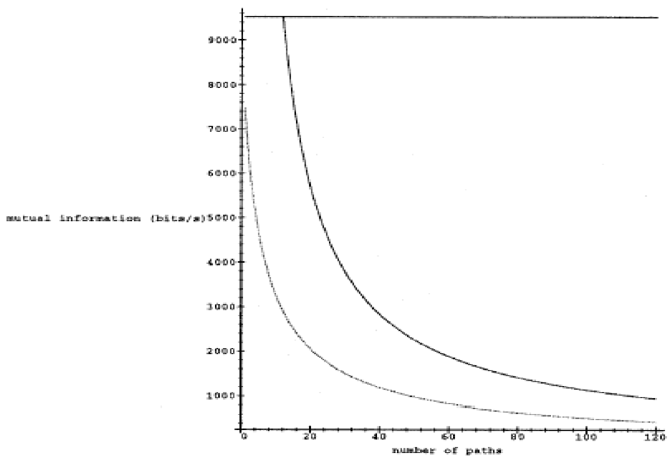
As $\widetilde{L} \rightarrow \infty$, the asymptotic upper bound:

$$\frac{P^2 T_c}{2N_0^2 \widetilde{L}}$$



Carrier frequency 1 GHz, vehicle speed of 60 mi/h

A voice user with a nominal AWGN capacity of 9.6 kbits/s



Carrier frequency 1 GHz,

Timing Uncertainty

- If delay of each path is not a *priori* assumed the mutual information goes to zero with increasing bandwidth even when there is only one path.

- *Theorem 4:* If $\{X_i\}$ is an i.i.d. complex-valued process such that $E(X_i) = E(X_i^2) = 0$, then as the bandwidth $W \to \infty$, the following asymptotic upper bound holds:

$$I(X;Y) \leq \frac{1}{WT_d}\left[\exp\left(\frac{2PT_c'}{N_0}\right) - 1\right] + O\left(\frac{1}{W^2}\right)$$

- As the bandwidth W becomes large, the upper bound decays to zeros like $1/W$, the decay is due to the necessity to track the path timing accurately.

Conclusion

- As the bandwidth gets large, one can achieve communication rates over a multipath fading channel equal to the capacity of an infinite bandwidth AWGN channel of the same SNR without fading. This can be achieve by FSK and noncoherent detection.

- In contrast, if one uses “ spread-spectrum” white-like signals, then the mutual information is inversely proportional to the number of resolvable paths. This result holds even when the receiver can track perfectly the timing of each path.
- Without side information about the timing of the paths, if one uses spread-spectrum signals, the mutual information approaches zeros with increasing bandwidth even when there is a only a single fixed gain path with random time varying delay.