A Bound on the Financial Value of Information

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Abstract and Introduction

- It will be shown that each bit of information at most doubles the resulting wealth in the general stock market setup.
- This information bound on the growth of wealth is actually attained for certain probability distributions on the market investigated by Kelly.
- The bound will be shown to be a special case of the result that the increase in exponential growth of wealth achieved with true knowledge of the stock market distribution F over that achieved with incorrect knowledge G is bounded above by D(F||G).

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Abstract and Introduction

· Let $\mathbf{X} \geq 0$, $\mathbf{X} \in R^m$ denote a random stock market vector, with the interpretation that X_i is the ratio of the price of the ith stock at the end of and investment period to the price at the beginning.

· Let $B = \{b \in R^m : b_i \geq 0, \Sigma_{i=1}^m b_i = 1\}$, be the set of all portfolios b, where b_i is the proportion of wealth invested in the ith stock.

· The wealth is

$$S = \sum_{i=1}^{m} b_i X_i = \mathbf{b^t X}.$$

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Doubling Rate

· For $F(X)$, the probability distribution function of the stock vector $\mathbf{X}$, define the doubling rate $W(\mathbf{X})$ by

$$W(\mathbf{X}) = \max_{b \in B} \int \log b^t x dF(x)$$

· (Kuhn-Tucker conditions) Necessary and sufficient conditions for $\mathbf{b}$ to maximize $E(\log \mathbf{b^t X})$ are

$$E\left(\frac{X_i}{\mathbf{b^t X}}\right) = 1, \qquad for \;\; b_i > 0$$

$$E\left(\frac{X_i}{\mathbf{b^t X}}\right) < 1, \qquad for \;\; b_i = 0$$

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Doubling Rate

· If current wealth is reallocated according to $\mathbf{b}^*$ in repeated independent investments against i.i.d. stock vectors $\mathbf{X_1}, \mathbf{X_2}, \ldots$ according to $F(x)$, then the wealth S_T^* at time T is

$$S_T^* = \prod_{t=1}^{T} \mathbf{b}^{*\mathbf{t}} \mathbf{X_t}.$$

· According to strong law of large numbers,

$$(S_T^*)^{\frac{1}{T}} = 2^{(1/T)\Sigma_{t=1}^{T} \log \mathbf{b}^{*\mathbf{t}} \mathbf{X_t}} \rightarrow 2^W \quad a.s$$

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Side Information

· Suppose side information $\mathbf{Y}$ is available. The doubling rate for side inforamtion is

$$W(\mathbf{X}|\mathbf{Y}) = \max_{\mathbf{b}(y)} \int\int \log \mathbf{b}^{\mathbf{t}}(y) x dF(x, y)$$

· In repeated investments against $\mathbf{X_1}, \mathbf{X_2}, \ldots, \mathbf{X_T}$ where $(\mathbf{X_t}, \mathbf{Y_t})$ are i.i.d. from $F(x, y)$, and $\mathbf{b}^*(\mathbf{Y_t})$ is the portfolio used at invetment time t given side information $\mathbf{Y_t}$, then the wealth is

$$S_T^{**} = \prod_{t=1}^{T} b^{*t}(\mathbf{Y_t})\mathbf{X_t}.$$

Moreover,

$$(S^{**})^{\frac{1}{T}} \rightarrow 2^{W(\mathbf{X}|\mathbf{Y})}$$

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Side Information

$$(\frac{S_T^{**}}{S_T^*})^{\frac{1}{T}} \rightarrow 2^{W(\mathbf{X}|\mathbf{Y})-W(\mathbf{X})}$$

$$\Delta = W(\mathbf{X}|\mathbf{Y}) - W(\mathbf{X})$$

Δ is the increment in doubling rate due to the side information $\mathbf{Y}$.

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The Information Bound for Side Information

$$0 \leq \Delta \leq I(\mathbf{X} : \mathbf{Y})$$

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Related and Further Problems and Conclusion

- Universal portfolios with side information [T.M. Cover & E. Ordentlich, 1996]
- The study on effeciency of investment information [E. Erkip & T.M. Cove, 1998]
- Consider stochastic calculus of information to deal with the financial problem, e.g., option pricing.
- Information theory provides a different viewpoint of the interpretation of the problems and situations in financail engineering.

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