Self-Similar Process in Communications Networks

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Boris Tsybakov and Nicolas D. Georganas

Kai-Lung Hua•u8913508•
Communication Engineering
National Chao-Tung University
Email: *u8913508@cc.nctu.edu.tw*

Outline

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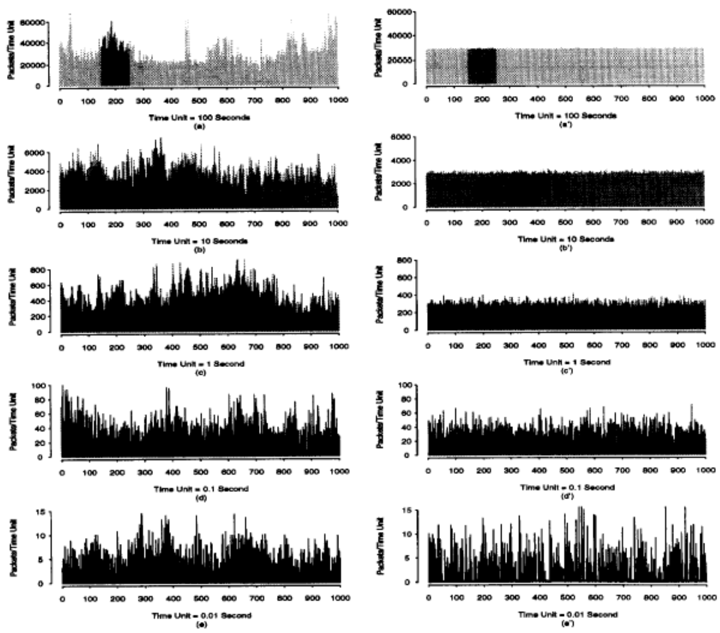

Motivation

- *Recently observed traffic data is long range dependence.*
- *There are no books which consider in detail the second-order self-similar processes and explain their properties.*
- *The main attention of the existing literature oriented to nontelecommunications applications is focused on the self-similar processes which are continuous both in values and in time, whereas cell and packet communications networks demand traffic models which are discrete both in values and in time.*

Key point

- *Compare of the known definitions of self-similar process.*
- *Present a model for ATM and find the conditions of model self similarity.*

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Compare Different Definitions

Definition 1：

A process X is called exactly second-order self similar（es-s） with parameter H=1-（β/2）, 0<β<1,if its correlation coefficient is

$$r(k)=\frac{1}{2}\left\{|k+1|^{2-\beta}-2|k|^{2-\beta}+|k-1|^{2-\beta}\right\}\hat{=}g(k),$$

$$k\in I_1 \hat{=} \{1,2,\cdots\}$$

$$g(k)\sim 1/2(2-\beta)(1-\beta)(k^{-\beta})=H(2H-1)k^{-\beta},k\to\infty$$

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Compare Different Definitions

- *Correlation coefficient* $r(k) \hat{=} \frac{E(X_{t+k}-\mu)(X_t-\mu)}{\sigma^2}$
- *Autocovariance* $b(k) \hat{=} \sigma^2 r(k)$
- *Averaged Process*

$$X^{(m)} = (X_1^{(m)}, X_2^{(m)}, \cdots)$$

$$X_t^{(m)} = \frac{1}{m}(X_{tm-m+1} + \cdots + X_{tm}) \circ m, t \in I_1, ex: X_1^{(2)} = \frac{1}{2}(X_1 + X_2)$$

- *Its correlation coefficient* $r_m(k)$, *autocovariance* $b_m(k) \hat{=} \operatorname{cov}\{X_t^{(m)}, X_{t+k}^{(m)}\}$

Compare Different Definitions

Definition 2：

A process X is called exactly second-order self-similar（es-s）with parameter H=1-(β/2), 0<β<1, if

$$b_m(k) = b(k) m^{-\beta}, k \in I_0, m \in I_2 \hat{=} \{2,3,...\}.$$

$$\Updownarrow$$

$$r_m(k) = r(k)$$

Traffic Model And Conditions of Its Self-Similarity

- *A. Cell Traffic Model*
- *B. Conditions for Self-Similarity of Traffic Y*

Cell Traffic Model

- *Traffic Y is assumed to be a stream of cells.*
- *The sources are enumerated by s.*
- *A source s starts to generate cells at time denoted by ω_s. The moment ω_s is called the arrival epoch of source s.*
- *The source s generates $?_s(i) \in I_0$ cells at time $\omega_s + i-1$ in time interval $\omega_s, \ldots, \omega_s + \tau_s - 1, i \in \{1, \ldots, \tau_s\}$.*

- *Let ξ_t be the number of sources with arrival epochs being equal to t. ξ_t are independent and they are identically Poisson distributed $Pr[\xi_t = k] = e^{-\lambda}(\lambda^k / k!)$ where $0 < \lambda < \infty$ is the parameter of the Poisson distribution.*
- *The considered traffic $Y=(\dots, Y_{-1}, Y_0, Y_1, \dots)$ is an aggregation of cells generated by different sources. $Y_t = \Sigma\ ?_s(t - \omega_s + 1)$ $t \in I_{-\infty}$*

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- *Split Y into an infinite number of independent processes $Y(l), l \in I_1$.*
- *Given active-period length $\tau_s = l$ for all its sources and with the number of sources arrived at t equal to $\xi_{t,l}$, the Poisson random variable with parameter $\lambda_l = E\xi_{t,l} = \lambda\, Pr[\tau = l]$ .This means*

$$Y = \sum_{1=1}^{\infty} Y(l) \quad \xi_t = \sum_{l=1}^{\infty} \xi_{t,l} \quad \sum_{1=1}^{\infty} \lambda_1 = \lambda$$

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Conditions for Self-Similarity of Traffic Y

Theorem：
The mean and autocovariance function of process Y are

$$EY_t = \lambda \sum_{l=1}^{\infty} l \Pr\{\tau = l\} \mu^{(l)}$$

$$w(k) \triangleq \text{cov}\{Y_t, Y_{t+k}\} = \lambda \sum_{l=k+1}^{\infty} (l-k) \Pr\{\tau = l\} B^{(l)}(k),$$

Theorem：
The process Y is as-s with parameter H=1-（β/2）, 0<β<1, if , are such that

$$\Pr\{\tau = l\}, \mu^{(l)}, \text{and } B^{(l)}(k)$$

$$\Pr\{\tau = l\}\, B^{(l)}(l) \propto L(l) l^{-(\beta+2)}, l \to \infty$$

$$\sum_{l=1}^{\infty} \Pr\{\tau = l\}\, B^{(l)}(0) < \infty, \sum_{l=1}^{\infty} l \Pr\{\tau = l\}\, \mu^{(l)} < \infty$$

where L(x) is a slowly varying function

Example

Let Pr[τ=l] be negative exponential Distribution, Pr[τ=l] ~ $C_0 e^{-\varphi l}, l \to \infty$

Where φ is a constant, then Y is as-s with H=1-(β/2),0< β <1,if

$$\mu^{(l)} = const \text{ and}$$

$$B^{(l)}(l) = B^{(l)} \propto L(l) l^{-(\beta+2)} e^{\varphi l}, l \to \infty$$

Achievement

- *Prove that the different known definitions of second-order self-similar processes are the same.*
- *Give a model for ATM cell traffic, the conditions for its self-similarity.*

Future Work

Nowadays, the research direction of self-similar is stressed on the modeling of the self-similar. However, in some other papers, like reference [10], Dr. Peha also did some simulations which shows that some protocols （e.g. re-trasmission） also produce self-similar. Therefore, the research of self-similar should be in two directions. One is on the modeling, and the other is on improving protocols to reduce self-similar phenomenon.

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