On Deterministic Traffic Regulation and Service Guarantees :A Systematic Approach by Filtering

IEEE Transactions on Information Theory
Vol. 44, pp. 1097-1110, May 1998

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Abstract

- 1.A filtering theory for deterministic traffic regulation and service guarantees under the(min,+)-algebra
- 2.Traffic regulators that generate f -upper constrained outputs can be implemented optimally by a linear time invariant filter with the impulse response f* under the (min,+)-algebra,where f * is the subadditive closure
- Analogous to the classical filtering theory,there is an associate calculus,including feedback,concatenation, "filter bank summation" and performance bounds

Min-plus algebra

In such an algebra,one replaces the usual addition operator by the min operator and the usual multiplication operator by the addition operator .Denote by ⊕ the min operator and ⊗ the + operator.

ie. $A \oplus B = \min\{A,B\}$ $A \otimes B = A+B$

Definition

- (Subadditive closure) For any $f \in F$,define f^* via the following recursive equation:

  $f^*(0)=0$, $f^*(t)=\min_{0<s<t}[f(t),\min[f^*(s)+f^*(t-s)]],t>0$

  we call f^* the subadditive closure of f.
- For the equation $B=A \oplus (B*f)$, $B=A*f^*$ is the maximum solution

Definition

- For an increasing sequence $A\alpha\{A(t), t=0,1,\ldots\}$ $A(0)=0$, we called A is f -upper constrained for some function f if $A(t2)-A(t1) \le f(t2-t1)$
- A sequence $f \equiv \{f(t), t=0,1,2,\ldots.\} \in F$(resp.F) satisfies $f(0) \ge 0$(resp. $f(0)=0$)and $f(s) \le f(t)$ for all t.

(I)(min) the pointwise minimum of two sequence:
$(f \oplus g)(t)=\min[f(t),g(t)]$

(II)(convolution) the convolution of two sequences under the (min,+)-algebra:

$(f*g)(t)=\min\,[f(s)+g(t-s)], 0 \le s \le t$

Suppose that A, $f \in F_0$.Construct a sequence $B=A * f^*$,

i.e. ,$B(t)=\min_{0 \le s \le t}\,[A(s)+f^*(t-s)]$

- (Traffic regulation) B is f^*-upper constrained and thus f-constrained
- (Optimality) For any f-upper constrained sequence $\tilde{B}$ that satisfies $\tilde{B} \le A$,one has $\tilde{B} \le B$
- (Conformity) A is f-upper constrained if and only if

  B=A.

(Maximal f-regulator)For $f \in F_0$,the construction in

,$B(t)=\min_{0 \le s \le t} [A(s)+f^*(t-s)]$ is called the maximal f regulator (for the input A).

(Concatenation)A concatenation of the maximal f_1-regulator

And the maximal f-regulator,independent of the order,is the maximal f-regulator,where $f= f_1 * f_2$ is the convolution of f_1 and f_2 under the (min,+)-algebra,i.e.,

$f(t)=\min_{0 \le s \le t} [f(s)+f(t-s)]$

(Filter bank summation)A “filter bank summation”of the

maximal the maximal f_1-regulator and the maximal f_2 regulator is the maximal f-regulator with $f= f_1 \oplus f_2$,if

$f_1^* \oplus f_2^*$ is subadditive.

We show that leaky buckets are indeed maximal regulator and they can be realized by a systematic approach under the (min,+)-algebra.

Traffic regulation for periodic constraint functions can be efficiently implemented by the FIR-IIR realization.

One possible application for such a regulator is traffic policing for video sequence,e.g. MPEG,where a certain periodic structure exists.

(f-server) A server guarantees service curve $f \in F$ for a Input sequence $A \in F$ if its output sequence satisfies $B \geq A * f$,i.e.,$B(t) \geq= \min_{0 \leq s \leq t} [A(s)+f(t-s)]$, for all t,Such a server is called an f-server for A.

(Window flow control)Let A be the input to a network and B be the output.Suppose that the network is a universal f-Server and that the network also enforces a window flow Control for the input A with the window size $w > 0$.

Thus ,the effective input,denoted by A1 ,satisfies

$A1(t)=\min[A(t),B(t)+w]$

Conclusions and future research

- In the following,we discuss possible extensions of the filtering theory.

(i)Matrix operations:one may consider square n×n matrices with entries in F. Such an extension could be used for modelling filtering with multiple inputs and outputs.

(ii)Transfer functions under the(min,+)-algebra:one may define the so called Z-transform under the (min,+)-algebra by $F(z)=\min_t[f(t)+zt]$

(iii)Time varying filtering

(iv)Constrained filtering