Linear Multiuser Detectors for Synchronous Code-Division Multiple-Access Channels

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Preface

- Multiple access interference (MAI) is a factor which limits the capacity and performance of DS-CDMA system. MAI refers to the interference between direct-sequence users. This interference is the result of the random time offsets between signals, which make it impossible to design the code waveforms to be completely orthogonal. While the MAI cause by any one user is generally small, as the number of interferers or their power increases, MAI become substantial. The conventional detector does not take into account the existence of MAI. It follows a single-user detection strategy in which each user is detected separately without regard for other users.
- Because of the interference among users, a better detection strategy is one of multi-user detection (also referred to as joint detection of interference cancellation). Here, information about multiple users is used jointly to better detect each individual user. The utilization of multi-user detection algorithm has the potential to provide significant additional benefits for DS-CDMA systems.

The background of this paper

- Optimum multi-user detection consists of matched filter followed by Viterbi algorithm. The formulation of optimum multi-user detection for AWGN channel, maximum likelihood sequence estimator (MLSE) was published in 1986 by Verdu.
- The decorrelating detector was initially proposed in 1979 by K. S. Schneider and 1983 by R. Kohno. It is extensively analyzed by Lupas and Verdu. This paper has a counterpart which dealing with multi-user detectors in asynchronous channel by Lupas and Verdu.
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Introduction

- The purpose of this paper is to investigate new low-complexity multi-user detection strategies that approach the performance of the optimum detector and to gain further insight into the performance of the optimum multi-user detector. Our attention is focused on symbol-synchronous channels, where the symbol epochs of all users coincide at the receiver.

Asymptotic efficiency

- The main performance measure of interest in digital communications in general, and in multi-user detection in particular, is the bit-error-rate. It is common to average error probability with respect to random quantities such as delays, phases, and received signal-to-noise ratios. When the unknown quantities are slowly time-varying (relative to the data rate), averaging bit-error-rate may be misleading because it may be dominated by particularly unfavorable, but rare, channel conditions.
- In multi-user problems it is often more convenient and intuitively sound to give information concerning the error probability by means of the efficiency, or ratio between the effective signal-to-noise ratio (SNR) and the actual SNR, where the effective SNR is the one required to achieve the same probability of error in the absence of interfering users, and the actual SNR is the received energy of the user divided by the power spectral density level of the background thermal white Gaussian noise (not including interference from other users).

Asymptotic efficiency

- Note that since the single-user error probability is a one-to-one function of the SNR, the efficiency gives the same information as the error probability. Its limit as the background Gaussian noise level goes to zero, the **asymptotic efficiency**, characterizes the underlying performance loss when the dominant impairment is the existence of other users rather than the additive channel noise. Verdu defined the asymptotic efficiency for the kth user as following:

$$\eta_k = \sup\left\{0 \le r \le 1; \lim_{\sigma \to 0} P_k(\sigma)/Q(\sqrt{r w_k}) < +\infty\right\}$$

  where the 2 denotes the power spectral density level of the background white noise, P denotes the error probability of the kth user and denote the energy of the kth user.
- Multi-user asymptotic efficiency depends on the signature waveforms, received signal-to-noise ratio, and the detector employed. In this paper we compare the performance of the various multi-user detectors by means of the asymptotic efficiency.

Near-far resistance

- We show that the optimum multi-user detector and other multi-user detectors with much lower computational complexity are **near-far resistant** under mild conditions on the signal constellation.
- By near-far resistance we mean the asymptotic efficiency minimized over the energies of all the interfering users.

$$\bar{\eta}_k = \inf_{\substack{A_j > 0 \\ j \neq k}} \eta_k$$

- If this minimum is nonzero, and, as a consequence, the performance level is guaranteed no matter how powerful the multi-user interference, then we say that the detector is near-far resistant.

Received signal model

- Suppose that the kth user is assigned a finite energy signature waveform, $S_k(t),\ t \in [0, T]$ and that it transmits a string of bits by modulating that waveform antipodally (BPSK) over a white Gaussian multiple-access channel. The receiver observes:

$$r(t) = \sum^{K} b_k(i) S_k(t - iT) + \sigma n(t) \qquad (1)$$

- where n(t) is a realization of a unit spectral density white Gaussian process and $\{b_k(i)\} = \{-1, 1\}$ is the kth user information sequence.

The conventional single user detector

- The conventional detector for the received signal described in Eq.(1) is a bank of K correlators. Here, each code waveform is regenerated and correlated with the received signal in a separate detector branch.
- The likelihood function depends on the observations only through the outputs of a bank of matched filters:

$$y_k = \int_0^T r(t) S_k(t)\,dt \qquad k = 1, \ldots, K$$

and therefore $y = (y_1, \ldots, y_K)$ are sufficient statistics for demodulating $b = (b_1, \ldots, b_K)$.

and $$y = Hb + n$$

where **H** is the nonnegative definite matrix of crosscorrelations between the assigned waveforms

The conventional single user detector

- Conventional single-user detection is the simplest way to make decisions based on ;demodulation is decoupled and the multi-user interference is ignored, yielding the following decisions for the kth user:

$$\hat{\ }^{c}$$

the superscript c denotes "conventional".

- The <u>asymptotic efficiency of the conventional detector</u> is equal to

$$\eta_k^c = \max\left\{0, 1 - \sum_{i \neq k} |R_{ik}| \frac{\sqrt{w_i}}{\sqrt{\,}}\right\}$$

where **R** is the matrix of normalized (unit-energy) cross correlations, i.e.,

$$1/2 \qquad 1/2$$

where .

The conventional single user detector

- It follows from (2) that the conventional kth user detector is near-far resistant (i.e., its asymptotic efficiency is bounded away from zero as a function of the interfering users' energies) only if $R_{ik} = 0$ for all $i \neq k$ i.e., only if the kth user' s signal is orthogonal to the subspace spanned by the other signals. Otherwise,

$$\bar{\eta}_k^c = \inf_{\substack{w_i \geq 0 \\ i \neq k}} \eta_k^c = 0$$

The optimum multi-user detector

- The optimum multi-user detector selects the most likely hypothesis $\hat{\ }^*$ $\hat{\ }^*$ $\hat{\ }^*$ given the observations, which corresponds to selecting the noise realization with minimum energy, i.e.,

$$\hat{b}^* = \arg \max_{K} 2 v^T b - b^T Hb$$

- The kth user error probability of the optimum multi-user receiver is asymptotically (as   ) equivalent to that of a binary test between the two closest hypotheses that differ in the kth bit. The square of the Euclidean distance between the signals corresponding to these two hypotheses is equal to

$$\min_{b \in \{-1,1\}^K} \min_{d \in \{-1,1\}^K} \left| \sum_{i=1}^{K} b_i S_i(t) - \sum_{i=1}^{K} d_i S_i(t) \right|^2 = 2 \min_{\varepsilon \in \{-1,0,1\}^K} \varepsilon^T H \varepsilon$$

The optimum multi-user detector

- Hence the asymptotic efficiency of the optimum multi-user detector is equal to

$$\eta_k = \frac{1}{w_k} \min_{\substack{\varepsilon \in \{-1,0,1\}^K \\ \varepsilon_k = 1}} \varepsilon^T H \varepsilon$$

- Nevertheless, it is possible to obtain a closed-form expression for the near-far resistance of the optimum multi-user detector. Solution is given by the following result.
- **Proposition 1**: Denote by the Moore-Penrose generalized inverse* of the normalized crosscorrelation matrix . If the signal of the kth user is linearly independent, i.e., it does not belong to the subspace spanned by the other signal, then

$$\bar{\eta}_k = \inf_{\substack{w_i \geq 0 \\ i \neq k}} \eta_k = \frac{1}{R^+_{kk}}$$

Otherwise, $\bar{\eta}_k = 0$

The decorrelating detector

- An important group of multi-user detectors are linear multi-user detectors. These detectors apply a linear mapping T to the soft output of the conventional detector to reduce the MAI seen by each user.
- In general, we consider the set of generalized inverses of the crosscorrelation matrix and analyze the properties of the detector:

which we refer to as a **decorrelating detector**.

The decorrelating detector

- It is interesting to point out that this detector does not require knowledge of the energies of any of the active users. To see this, let $\tilde{y}_k = y_k / \sqrt{w_k}$, i.e., $\tilde{y}_k$ is the result of correlating the received process with the normalized (unit-energy) signal of the kth user. Then

$$\begin{aligned} \operatorname{sgn} H^{-1} y &= \operatorname{sgn} W^{-1/2} R^{-1} W^{-1/2} y \\ &= \operatorname{sgn} W^{-1/2} R^{-1} \tilde{y} \\ &= \operatorname{sgn} R^{-1} \tilde{y} \end{aligned}$$

and therefore, the same decisions are obtained by multiplying the vector of normalized matched filter outputs by the inverse of the normalized crosscorrelation matrix.

The decorrelating detector

- Thus the kth user asymptotic efficiency of a decorrelating detector with matrix is given by

$$\eta_k(H^I) = \max\left\{0, \frac{1}{\sqrt{w_k}} \frac{(H^I H)_{kk} - \sum_{j \neq k} \left|(H^I H)_{kj}\right|}{\sqrt{H^I H H^{I T}}_{kk}}\right\}$$

- **Proposition 2**: If user k is linearly independent every H^I $I(H)$ satisfies

$$\eta_k(H^I) = \frac{1}{R^{+}}$$

Thus for independent users the asymptotic efficiency of the decorrelating detector is independent of the energy of other users and of the specific generalized inverse selected.

The decorrelating detector

- **Proposition 3**: The near-far resistance of the decorrelating detector equals that of the optimum multi-user detector, i.e., for all $H^I \in I(H)$

$$\inf_{\substack{w_j \geq 0 \\ j \neq k}} \eta_k(H^I) = \inf_{\substack{w_j \geq 0 \\ j \neq k}} \eta_k \equiv \overline{\eta}_k$$

- The result of Proposition 3 is of special importance in a near-far environment, where the received signals have different energies and where the energy ratios may vary continuously over a broad scale if the positions of the users evolve dynamically. In this environment any decorrelating detector, with its linear time-complexity per bit, offers the same near-far resistance as the optimum multi-user detector, whose time-complexity per bit is exponential.

The decorrelating detector

Decorrelating detector is shown to have many attractive properties. Foremost among these properties are: [1]

◆Provides substantial performance/capacity gains over the conventional detector under most condition.

◆Does not need to estimate the received amplitudes.

◆Has computational complexity significantly lower than that of the optimum multi-user detector.

◆Corresponds to the optimum multi-user detector when the energies of all users are not known at the receiver.

◆Has a probability of error independent of the signal energies.

◆Yields the optimal value of the near-far resistance performance metric.

◆Can decorrelate one bit at a time.

The decorrelating detector

- A disadvantage of this detector is that it causes noise enhancement. The power associated with the noise term $H^{I}n$ at the output of the decorrelating detector is always great than or equal to the power associated with the noise term at the output of the conventional detector for each bit.
- A more significant disadvantage of the decorrelating detector is that the computations needed to invert the matrix H are difficult to perform in real time, especially in asynchronous channel.

Future Prospect

- During the last ten years multi-user detection (MUD) jointly detecting the signals from different users has been under intense research as a potential method to improve the system performance of CDMA systems.
- Although MUD has many advantages, such as significant improvement in capacity, more efficient uplink spectrum utilization, reduced precision requirement for power control and more efficient power utilization, it still has some disadvantages, i.e., computational complexity and difficulty in implementing MUD on downlink. To commercialize MUD are expected in the future but it depends, as all communication systems, on the performance/cost tradeoff.