



LINEAR ALGEBRA

Spring Semester 2014
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Homework 2 of March 06, 2014

Deadline: March 13, 2014

Problem 1 (30%)

Properties of Inverses

Suppose that A is an invertible matrix and has one of the following properties:

- a) A is a triangular matrix.
- b) A is a symmetric matrix.
- c) All the entries of A are integers (i.e. $a_{i,j} \in \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$).

Which of the above would also be true of the inverse of A ? Prove your answer if it is true, or find a counterexample if it is false.

Problem 2 (20%)

Inverse of Matrix

Find A^{-1} and B^{-1} by elimination on $[A \ I]$ and $[B \ I]$:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & -1 \\ 2 & 2 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

Problem 3 (25%)

LDU Factorization

Find $A = LDU$ factorization for

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 4 \\ 2 & 4 & 6 \end{bmatrix}.$$

(Hint: If $A^T = A$, then $A = LDL^T$.)

Problem 4 (25%)

Using LU to Solve $Ax = b$

Suppose $A = LU$ and $Ax = b$, where L , U , and b are respectively given as follows. Please solve x without knowing/computing A .

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$