Linear Algebra

Spring Semester 2014
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http://shannon.cm.nctu.edu.tw/la.htm

Homework 2 of March 27, 2014

Deadline: April 01, 2014

Problem 1 (20%) ***Column Space v.s. Nullspace***

a) Complete these sentences appropriately for a 3×3 matrix A.

- (2 pts.) If the column space is a (2-dimensional) plane, the nullspace is _____.
- (2 pts.) If the column space is a (1-dimensional) line, the nullspace is _____.
- (2 pts.) If the column space is $\mathbb{R}^3$, the nullspace is _____.
- (2 pts.) If the column space is the zero vector, the nullspace is _____.

b) (12 pts.) Find a 7×7 matrix A whose column space equals its nullspace, or argue briefly such matrix does not exist.

Problem 2 (30%) ***Applications of Echelon Matrix***

Either construct a matrix A or argue that it is impossible, where the nullspace of A is exactly the multiples of $(2, 3, 4, 1)$ and

a) (10 pts.) A is 2 by 4.

b) (10 pts.) A is 3 by 4.

c) (10 pts.) A is 4 by 4.

Problem 3 (20%) ***Special Solutions***

For matrix A below, find the special solutions by setting one of the free variables to 1 and all the other free variables to 0.

$$A = \begin{bmatrix} 1 & 1 & -1 & -3 \\ 2 & 0 & -2 & -2 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Problem 4 (30%) ***Complete Solutions of*** $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$

Find the basis of $\boldsymbol{C}(\mathsf{A})$ and $\boldsymbol{N}(\mathsf{A})$, and the complete solutions to $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\mathsf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -3 \\ 4 & -1 & 1 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}.$$

Please submit your homework either to ED 823 or to the TAs during the Thursday class time