Linear Algebra

Homework 5 of April 03, 2014

Spring Semester 2014
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Deadline: April 10, 2014

Problem 1 (20%) ***Bases and Rank***

a) (10%) U comes from A by subtracting row 1 from row 3:

$$\mathsf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } \mathsf{U} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find bases for the two column spaces, $\mathbf{C}(\mathsf{A})$ and $\mathbf{C}(\mathsf{U})$. Find bases for the two row spaces, $\mathbf{R}(\mathsf{A})$ and $\mathbf{R}(\mathsf{U})$. Find bases for the two nullspaces, $\mathbf{N}(\mathsf{A})$ and $\mathbf{N}(\mathsf{U})$.

b) (10%) For which numbers c and d do the below two matrices have exactly rank 2?

$$\mathsf{B} = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix} \quad \text{and} \quad \mathsf{C} = \begin{bmatrix} c & d \\ d & c \end{bmatrix}.$$

Problem 2 (20%) ***Finding a Basis***

Find a basis for each of the following subspaces of $\mathbb{R}^4$:

a) (5%) All vectors whose components are equal.

b) (5%) All vectors whose components add to zero (also explain why it is a vector space).

c) (5%) All vectors that are perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ (also explain why this forms a vector space).

d) (5%) The column space and the nullspace of $\mathsf{I}_{4\times 4}$.

Problem 3 (20%) ***Basis of a Vector Space of Matrices***

Find a basis and the dimension for each of the spaces formed by the following matrices. Also explain why they are vector spaces.

a) (5%) All diagonal 3×3 matrices.

b) (5%) All symmetric 3×3 matrices ($\mathsf{A}^\intercal = \mathsf{A}$).

c) (5%) All 3×3 skew-symmetric matrices ($\mathsf{A}^\intercal = -\mathsf{A}$).

d) (5%) All 2×3 matrices whose columns add to zero.

Problem 4 (20%) ***Extension to Matrix Space***

Determine all the matrices whose nullspace is spanned by $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

Hint: Read Slides 3-57 and 3-58.

Problem 5 (20%) ***Determination of the Left Nullspace***

a) (10%) Check whether all solutions to matrix equation $\mathsf{A}\mathbf{x} = \mathbf{0}$ are perpendicular to the rows of $\mathsf{R} = \text{rref}(\mathsf{A})$, where

$$\mathsf{A} = \tilde{\mathsf{E}}\mathsf{R} = \begin{bmatrix} 4 & 0 & 0 \\ 8 & 1 & 0 \\ 12 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{4} \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

b) (10%) Find a basis of the left nullspace of A. Based on Slides 3-60 and 3-61, explain why one choice of bases of the left nullspace of A can be the last row of $\mathsf{E} \triangleq \tilde{\mathsf{E}}^{-1}$.