



LINEAR ALGEBRA

Spring Semester 2014
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<http://shannon.cm.nctu.edu.tw/la.htm>

Homework 2 of April 17, 2014

Deadline: April 24, 2014

Problem 1 (30%)

Rank of Projection

P is an n by n projection matrix. Compute the ranks of matrices A , B , and C shown below:

- a) (10 pts.) $A = (I - P)P$.
- b) (10 pts.) $B = (I - P) - P$.
- c) (10 pts.) $C = (I - P)^{1024} + P^{512}$.

Problem 2 (30%)

Projection Matrix

- a) (10 pts.) What matrix P projects every vectors in \mathbb{R}^3 onto the line passes through origin and the coordinate point $\mathbf{a} = (3, 4, 5)$?
- b) (10 pts.) What is the nullspace of matrix P ?
- c) (10 pts.) What is the row space of matrix P^2 ?

Problem 3 (20%)

The Least Square Solutions

- a) (10 pts.) Suppose $\hat{\mathbf{x}}$ is a least square approximation to $A\mathbf{x} = \mathbf{b}$ and $\hat{\mathbf{y}}$ is a least square approximation to $A\mathbf{y} = \mathbf{c}$. Give a least square approximation $\hat{\mathbf{z}}$ to $A\mathbf{z} = \mathbf{b} + \mathbf{c}$?
- b) (5 pts.) If Q is an m by n matrix with orthonormal columns, find a least square approximation $\hat{\mathbf{x}}$ to $Q\mathbf{x} = \mathbf{d}$.
- c) (5 pts.) If $B = QR$, where R is invertible and Q is the same as in (b), find a least square approximation $\hat{\mathbf{x}}$ to $B\mathbf{x} = \mathbf{e}$.

Problem 4 (20%)

QR Decomposition

Write A as QR in terms of QR decomposition, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{bmatrix}$.