Linear Algebra

Spring Semester 2014
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http://shannon.cm.nctu.edu.tw/la.htm

Homework 8 of May 22, 2014

Deadline: May 29, 2014

Problem 1 (30%) ***Square Root of a Matrix***

Find the matrix $\sqrt{\mathsf{A}} = \mathsf{A}^{1/2}$, where

$$\mathsf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

(Hint: $\cos\theta + \imath \sin\theta = e^{\imath\theta}$, $\cos\theta - \imath \sin\theta = e^{-\imath\theta}$, and $\imath^2 = -1$. Also, $\mathsf{A} = \mathsf{S}\Lambda\mathsf{S}^{-1}$.)

Problem 2 (25%) ***The Exponential of a Matrix***

Put $\mathsf{A} = \begin{bmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & -2 \end{bmatrix}$ into the infinite series to find $e^{\mathsf{A}t}$.

Problem 3 (25%) ***Diagonalizing a Matrix***

Diagonalize matrix $\mathsf{A} = \begin{bmatrix} 7 & 12 \\ -4 & -7 \end{bmatrix}$, and find A^{99}.

Problem 4 (30%) ***Equivalent Conditions***

Let A be an $n \times n$ square matrix. Please show the following two statements are equivalent:

a) The column vectors of A are orthonormal to each other.

b) The row vectors of A are orthonormal to each other.

Please submit your homework either to ED 823 or to the TAs during the Thursday class time